Lyapunov-Based Fuzzy Control Scheme for Switched Reluctance Motor Drives

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***Abstract*—In this paper, the classical Lyapunov synthesis method for designing controllers is extended to fuzzy logic. This control technique is then applied to the design of a novel tracking controller for reluctance motor drives. The main features of the method are small rule base, simplicity of construction, and low cost. The proposed controller has been simulated for a model case. In addition, its dynamic performances have been shown to be satisfactory. Capabilities of the proposed technique in controlling the highly nonlinear systems of reluctance motors with much simplicity are also verified.**

***Keywords*—Fuzzy logic control, Lyapunov methods, switched reluctance motor.**

I. INTRODUCTION

Due to the simple and rugged construction, hazard free operation, and high speed/high torque capabilities, Switched Reluctance Motor (SRM) drives have been considered among the best candidates for many adjustable speed drive systems in industrial and commercial applications [1]-[4].

On the other hand, fuzzy logic based systems have attracted much attention during the past few decades. They have, in fact, found various fields of applications ranging from signal and image processing to control and power systems. Reluctance motor drives are among the most widely used test beds both for classical and non-classical control methodologies such as fuzzy logic control.

In recent years, a large number of papers have been published on the fuzzy control of reluctance motor drives [1]-[8]. Different issues, such as torque ripple minimization, current profiling, modeling, estimation, and prediction have also been investigated extensively [9]-[12].

The most challenging task in designing fuzzy controllers is the construction of the rule base [13], [14]. The methods commonly used to derive these rules are mainly based on a good understanding of the plant and the experience of the human operators; however, there are a few methods for constructing the rule base systematically. Lyapunov-based fuzzy approach is a relatively simple method to systematically derive the rules, while ensuring system stability [14].

In this paper, a novel Lyapunov-based fuzzy control scheme for reluctance motor drives is introduced. The method is based on a Lyapunov function candidate. The control approach and the requirements to indeed make the candidate controller a Lyapunov function are presented in Section II. In Section III, the nonlinear model of reluctance motor is described. Furthermore, the controller design approach is outlined and the method adopted to derive the rules is demonstrated in Section III. It is also shown that because partial knowledge of the plant is assumed to be at hand, the requirements are stated as IF-THEN rules and, hence, a fuzzy system. In Section IV, the proposed method is applied to a model case and a Lyapunov-based fuzzy tracking controller is designed. Simulation results are also presented and analyzed. Finally, we give conclusion remarks in Section V.

II. LYAPUNOV-BASED FUZZY CONTROLLER SYNTHESIS

As was explained, main idea behind the Lyapunov-based fuzzy control approach is to extend the classical method to the domain of partial and fuzzy knowledge [14]. This section briefly outlines the proposed control method.

The system is assumed to be described by a set of state equations as follows.

$$\dot{x} = F(x,u) \qquad (1)$$
$$y = h(x)$$

where x, y, and u are the state vector, the output, and the input of the system, respectively. In order to design the stabilizing controller, a Lyapunov function candidate $V(x)$, is considered. The function $V(x)$ is indeed a Lyapunov function if the following requirements are satisfied:

$$V(0) = 0$$
$$V(x) > 0; x \neq 0 \qquad (2)$$
$$\dot{V} = \frac{\partial V}{\partial x_i} \cdot \dot{x}_i < 0; x \neq 0$$

It is worth noting that $x = 0$ has been assumed to be the equilibrium point, without loss of generality.

Since V is a function of the state variables of the system and, hence, of the input, the input can be designed such that V satisfies the requirements of (2).

It will be explained in the following sections that partial (fuzzy) knowledge of the exact model (1), will result in a number of IF-THEN rules of the form of fuzzy rule bases. In this paper, Mamdani-type fuzzy inference system with rules of the following form, is adopted [13].

IF x_1 is < Ling. Val.> and/or x_2 is <Ling. Val.>... and/or x_n is <Ling. Val.> THEN u is <Ling. Val.>.

where Ling. Val. is a linguistic value.

The nonlinear model of the reluctance motor is presented in the next section and the method outlined above will be applied to it in order to design a tracking controller for the rotor position.

III. RELUCTANCE MOTOR MODEL AND LYAPUNOV-BASED FUZZY CONTROLLER

A variable reluctance motor is described by the following state equations [13]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \sigma_l(y)\theta_l + \left(\sum_{i=1}^{L}\sigma_i(y)\mathbf{B}^T(u_i)\right)\theta_u \\ y &= x_1 \end{aligned} \tag{3}$$

where x_1 and x_2 are the rotor position and speed, respectively. u_i $(i = 1, 2, ..., L)$ are the phase currents that are considered as the inputs. L is the number of phases. The term $\sigma_l(y)\theta_l$ represents the load acceleration and the term $\sigma_i(y)B^T(u_i))\theta_u$ represents the acceleration due to the i^{th} phase current. The functions used are defined as the following.

$$\begin{aligned} \sigma_i(y) &= \sin\left(Py - \frac{2\pi(i-1)}{L}\right) \\ B(u_i) &= [b_1(u_i) \quad b_2(u_i) \quad \dots \quad b_M(u_i)]^T \\ b_j(u_i) &= n\left(\frac{u_i-(j-1)\delta}{3\delta}\right) \\ n(s) &= \begin{cases} 0, s<0, s>1 \\ 9s^2/2, s\in[0,1/3] \\ (-18s^2+18s-3)/s, s\in(1/3,2/3] \\ (3-3s)^2/2, s\in(2/3,1] \end{cases} \end{aligned} \tag{4}$$

where P is the number of poles, M is the number of spline intervals and δ is the interval size.

The model is obviously highly nonlinear and, therefore, the commonly used control strategies are applied with difficulty. In the following, the problem of designing a tracking controller, using the aforementioned Lyapunov-based fuzzy approach is considered. The problem is to find the phase currents such that the rotor position x_1 follows a reference trajectory y_c. In order to design the controller, we define the tracking error e and the candidate Lyapunov function V.

$$e = x_1 - y_c \tag{5}$$

$$V = \frac{1}{2}(e^2 + \dot{e}^2)^2 \tag{6}$$

The first two requirements of (2) apparently hold. Therefore, the third one has to be considered only.

$$\dot{V} = e\dot{e} + \dot{e}\ddot{e} = e\dot{e} + \dot{e}(\dot{x}_2 - \ddot{y}_c) \tag{7}$$

We define

$$w = \dot{x}_2 - \ddot{y}_c \tag{8}$$

As a result,

$$\dot{V} = e\dot{e} + \dot{e}w \tag{9}$$

In order for V to be a true Lyapunov function, we need

$$\dot{V} = e\dot{e} + \dot{e}w < 0 \tag{10}$$

It is clear that if e and $\dot{e}$ are of opposite signs, setting w = 0, causes (2) to hold; if e and $\dot{e}$ are both positive, we should have $w < -e$, and if e and $\dot{e}$ are both negative, we should have $w > -e$.

These deductions could be formulated in the following rule base.

IF e is *positive* and $\dot{e}$ is *positive* THEN w is *negative big.*

IF e is *negative* and $\dot{e}$ is *negative* THEN w is *positive big.*

IF e is *positive* and $\dot{e}$ is *negative* THEN w is *zero.*

IF e is *negative* and $\dot{e}$ is *positive* THEN w is *zero.*

The next step is to derive the phase currents from w. We can observe that

$$\dot{x}_2 = w + \ddot{y}_c = \sigma_l(y)\theta_l + \left(\sum_{i=1}^{L}\sigma_i(y)\mathbf{B}^T(u_i)\right)\theta_u \tag{11}$$

Therefore, the problem of finding the phase currents reduces to solving the nonlinear equation of (11). The method proposed in [15], [16] is adopted to tackle the problem. The method involves the steps of determining the phase(s) to be excited and finding the phase current(s) needed. The details of the method are beyond the scope of this paper and are not presented here.

IV. SIMULATION RESULTS

In this section, simulation results for a model case are presented. The characteristics of the motor are listed in Table 1. The reference trajectory is considered to be $y_c = \pi - \pi \, Cos(\pi t)$ which rotates the rotor smoothly between 0 and 2π *rad.*

Table 1. Reluctance motor characteristics.

Parameter	Value
Number of spline functions	M = 5
Interval size	δ = 0.25
Load coefficient	$\theta_l = 67$
Winding coefficients	$\theta_i = 50\, i$
Number of poles	P = 8
Number of phases	L = 4

The membership functions used for the variables are as the following.

$$\begin{aligned}
\mu_{positive}(x) &= \frac{1}{1+\exp(-30x)} \\
\mu_{negative}(x) &= \frac{1}{1+\exp(30x)} \\
\mu_{negative-big}(u) &= \exp(-(u+5)^2) \\
\mu_{zero}(u) &= \exp(-u^2) \\
\mu_{positive-big}(u) &= \exp(-(u-5)^2)
\end{aligned} \qquad (12)$$

The fuzzy inference system uses product inference engine and center of area defuzzifier [13].

The reference and actual rotor positions for an initial conditions as $x_1(0) = 30$ *deg*, $x_2(0) = 0.3$ *rad/ sec* are shown in Fig. 1 and the tracking error is shown Fig. 2.

It is clear that the fuzzy controller has successfully caused the rotor position to rapidly follow the reference trajectory, despite the considerable initial deviation.

It is common practice not to measure x_2 and to estimate it through approximate differentiation as

$$\hat{x}_2 = \frac{w_0^2 s}{s^2 + 1.4 w_0 s + w_0^2} x_1 \qquad (13)$$

where $\hat{x}_2$ denotes the estimate of x_2, and the cutoff frequency w_0 is large relative to the system dynamics.

Figs. 3 and 4 show the reference and actual rotor positions and the tracking error, respectively, when x_2 is estimated using the filter of (13) with a cutoff frequency of 150 *rad/sec*. The initial conditions have been assumed unchanged.

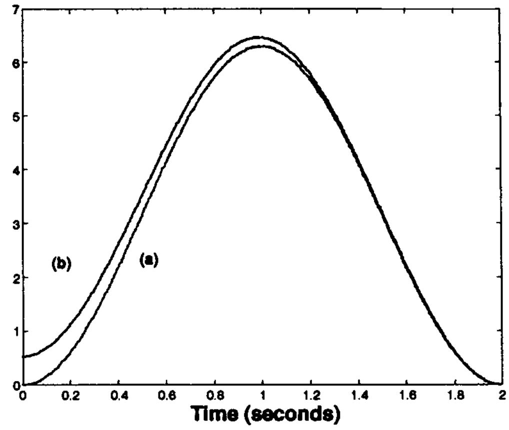

Fig. 1. Rotor position (rad.) (a) reference trajectory (b) actual rotor position.

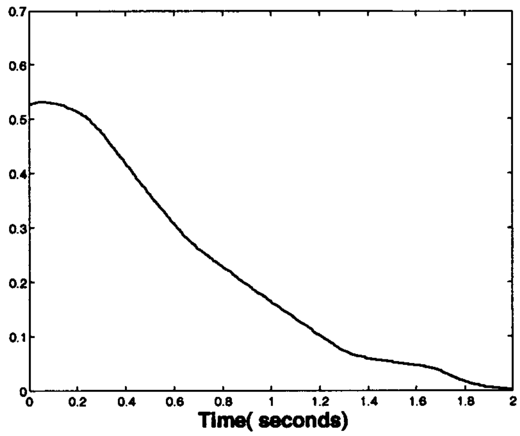

Fig. 2. Tracking error (rad.).

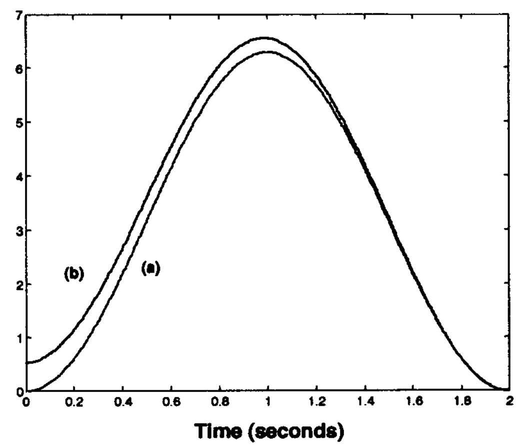

Fig. 3. Rotor position (rad.) (a) reference trajectory and (b) actual rotor position.

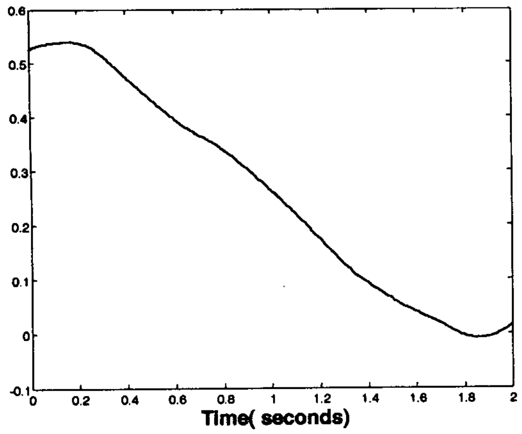

Fig. 4. Tracking error (rad.).

It is observed that the fuzzy controller still performs satisfactorily, which is a promising sign of its robustness to system uncertainties. In addition, speed estimation (13) can be implemented to achieve a sensorless operation for the reluctance motor drive. Therefore, there is no need for the speed sensor. Position sensor can also be eliminated by adding a position estimator to the proposed Lyapunov-based fuzzy controller. Hardware simplicity, low cost, and high reliability are the main advantages of the sensorless drives [17].

V. CONCLUSIONS

Lyapunov-based fuzzy control technique has been adopted to tackle the problem of position control of reluctance motor drives. The approach shows the merits of both classical Lyapunov synthesis and the fuzzy control design. This is the main advantage of the proposed controller. Computer simulations have been carried out to investigate the effectiveness of the controller in tracking position reference trajectories. Simulation results confirm the excellent performance as well as the capability of the method in controlling highly nonlinear plants with much simplicity.

REFERENCES

[1] D. S. Reay, M. Mirkazemi-Moud, T. C. Green, and B. W. Williams, "Switched reluctance motor control via fuzzy adaptive systems," *IEEE Control Systems Magazine*, vol. 15, no. 3, pp. 8-15, June 1995.

[2] M. T. DiRenzo, M. K. Masten, and C. P. Cole, "Switched reluctance motor control techniques," in *Proc. American Control Conf.*, June 1997, pp. 272-277.

[3] T. T. Borges, H. R. de Azevedo, D. A. de Andrade, and A. C. S. Goncalves, "An application of fuzzy logic for reluctance motor drive," in *Proc. IEEE International Electric Machines and Drives Conf.*, 1997, pp. TB1/10.1-10.3.

[4] M. G. Rodrigues, W. I. Suemitsu, P. Branco, J. A. Dento, and L. G. B. Rolim, "Fuzzy logic control of a switched reluctance motor," in *Proc. IEEE International Symposium on Industrial Electronics*, 1997, pp. 527-531.

[5] N. Abut, B. Cakir, N. Inanc, A. B. Yildiz, and M. Z. Bilgin, "Switched reluctance motor drive by using fuzzy controller," in *Proc. IEEE International Conf. on Industrial Electronics*, 1997, pp. 348-353.

[6] N. Abut, B. Cakir, U. Akca, and N. Erturk, "Fuzzy control application of SR motor drive for transportation systems," in *Proc. IEEE Conf. on Intelligent Transportation Systems*, 1997, pp. 135-140.

[7] B. Singh, V. K. Sharma, and S. S. Murthy, "Performance analysis of adaptive fuzzy logic controller for switched reluctance motor drive system," in *Proc. IEEE 33rd Industry Applications Society Annual Meeting*, 1998, pp. 571-579.

[8] C. C. Chan, Y. J. Zhan, and K. T. Chau, "Stability analysis of fuzzy sliding mode controlled switched reluctance motor drives," in *Proc. IEEE Power Electronics Specialist Conf.*, 1998, pp. 1283-1289.

[9] A. D. Cheok and N. Ertugrul, "Use of fuzzy logic for modeling, estimation, and prediction in switched reluctance motor drives," *IEEE Trans. on Industrial Electronics*, vol. 46, no. 6, pp. 1207-1224, Dec. 1999.

[10] S. Mir, M. Elbuluk, and I. Husain, "Torque ripple minimization in switched reluctance motors using adaptive fuzzy logic," *IEEE Trans. on Industry Applications*, vol. 35, no. 2, pp. 461-468, March/April 1999.

[11] A. Derdiyok, N. Inanc, V. Ozbulur, and Y. Ozgolu, "Optimal phase current profiling of SRM by fuzzy logic controller to minimize torque ripple," in *Proc. 12th IEEE International Symposium on Intelligent Control*, July 1997, pp. 77-82.

[12] N. Inanc, A. Derdiyok, and V. Ozbulur, "Fuzzy logic control of a switched reluctance motor including mutual inductances and operating in the linear region," in *Proc. 12th IEEE International Symposium on Intelligent Control*, July 1997, pp. 73-76.

[13] L. X. Wang, *A Course in Fuzzy Systems and Control*, Prentice-Hall, Englewood Cliffs, NJ, 1997.

[14] M. Margaliot and G. Langholz, "Fuzzy Lyapunov-based approach to the design of fuzzy controllers," Fuzzy Sets and Systems 106, 1999.

[15] S. A. Bortoff, R. R. Kohan, and R. Milman, "Adaptive control of variable reluctance motors: a spline function approach," *IEEE Trans. on Industrial Electronics*, vol. 45, no. 3, pp. 433-444, June 1998.

[16] R. Milman and S. A. Bortoff, "Observer-based adaptive control of a variable reluctance motor: experimental results," *IEEE Trans. on Control Systems Technology*, vol. 7, no. 5, pp. 613-621, Sep. 1999.

[17] B. Fahimi, G. Suresh, and M. Ehsani, "Review of sensorless control methods in switched reluctance motor drives," in *Proc. IEEE 35th Industry Applications Society Annual Meeting*, 2000, pp. 1850-1857.