# The Application of Fuzzy Reaching Law Control in

# **AC Position Servo System**

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Abstract: In this paper, a novel method of reaching law variable structure control based on fuzzy rules is present, which is that the reaching law parameters is on-line adjusted by fuzzy rules. This method is used in a digital ac position servo system, the experiment results show that the system designed by this method has both satisfactory quality and very smaller chattering.

**Key words:** variable structure control, reaching law, fuzzy rules, quality control, chattering

### 1 Introduction

Variable Structure Controllers with Sliding Mode Control (SMC) was first proposed in early 1950's. SMC has become more popular and it enjoys a wide variety of application areas because of its attractive superior properties such as good control performance even in the case of nonlinear systems, applicability to MIMO systems, etc. The best property of SMC is its robustness—the sliding mode is insensitive to parameter changes or external disturbances, see (Hung 1993).

Many optimal performance controllers for AC servo systems are designed by SMC. But the main disadvantage of SMC is the high-frequency oscillations of controller output, termed "chattering", which limit the applications of SMC. In AC servo control systems based on SMC, the stable quality is very poor because of the chattering of controller output, the stable position error exists here. Because of this, some researchers have proposed some methods for eliminating the high-frequency chattering. Palm (1992) and Glower et al. (1997) proposed a

design method for fuzzy controllers called Fuzzy Sliding Mode Control (FSMC), which eliminates the high-frequency chattering and meanwhile, destroys the sliding mode. Hwang (1992) proposed a controller with FSMC that can guarantees system stability, but cannot ensure the dynamic quality. Gao and Cheng (1989) presented a new method of the reaching law, which can not only guarantee the quality of the system before reaching the switching surface but also reduce the chattering to great extent. But it is very difficult to obtain the reaching law parameters, which can ensure the high reaching speed and reduce the high-frequency chattering. In the AC servo control system, it is more difficult to select the parameters because the parameters (for example, resistance inductance) of motor can be changed along with the varying temperature.

Fuzzy system and control based on fuzzy logic have received many applications recently (Tong, R.M, 1997). The fuzzy logic has been proved to be a powerful tool applied to ill-defined and complex system. The main feature of fuzzy systems is the fuzzy sets instead of numbers. The fuzzy controller is based on the expert experience not the plant model. The fuzzy control is more suitable to control the systems with qualitative, uncertain and inaccurate information than general control method.

In this paper, the physical concept and qualitative of the reaching law parameters are first analyzed. Based on the analysis results, a novel method called fuzzy reaching law is applied to the AC Position Servo System, in

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which the reaching law parameters are selected by the fuzzy rules to eliminate the high-frequency chattering and keep the optimal quality of the system.

This paper is organized as follows. In section 2, we analyze of the conflicting relations of the reaching law parameters in the discrete control system. In section 3, we present the new idea of fuzzy reaching law and the detailed designing procedure of the fuzzy controllers. In section 4, the hardware of the AC position servo system based on the TMS320F240 DSP and the detail procedure for designing the fuzzy reaching law position controller are presented, and the experimental results have been analyzed. Conclusions are drawn in section 5.

# 2 Reaching law controller for the discrete control system

Consider the linear nth-order discrete control system

$$X(k + 1) = AX(k) + Bu(k)$$
  
y(k) = g(x(k)) (1)

where  $X \in R^n$ ,  $u \in R^m$ , (A, B) satisfies the controllability. The vector

$$X = [x_1, x_2, K, x_n]^T$$

is the system state variable. y is the output and u is the input.

Define the linear function of the sliding mode as  $S(k) = C^T X(k)$  (2)

where  $C = [c_1, c_2]$  is the constant matrix.

It is well known that the design of VSC can satisfy a reaching condition, which can be denoted by inequalities. Obviously, it is very difficult to solve control laws under such conditions. So Gao and Cheng (1989) proposed a method that denotes the reaching condition by equalities. This approach established the reaching mode by using the reaching law, called reaching law method. The main feature of this method is that it is very easy to obtain the

control law of the VSC, not only establishes the reaching mode but also specifies the dynamic characteristic of the system during the reaching mode, at the same time the chattering is limited.

The reaching law equation is:

$$S(k + 1) - S(k) = -\varepsilon sgnS(k) - qTS(k)$$
 (3)  
where  $\varepsilon > 0$  is the reaching rate, q>0 is the reaching rate exponent, T is the sample interval.

From (1) and (3), we can obtain that:

$$S(k+1)-S(k) = C^{T}X(k+1)-C^{T}X(k)$$

$$= C^{T}AX(k)+C^{T}Bu(k)-C^{T}X(k)$$

$$= -\varepsilon T \operatorname{sgn}S(k)-q T S(k)$$
(4)

Assume  $C^TB \neq 0$ , from (4) we can obtain the control input u as:

$$u(k) = -(C^{T}B)^{-1}[C^{T}(A-I+qTI)X(k) + \varepsilon TsgnS(k)]$$
(5)

In order to obtain the parameter C, (1) can be transformed to a reduced one as:

$$\begin{cases} x_1(k+1) = A_{11}x_1(k) + A_{12}x_2(k) \\ x_2(k+1) = A_{21}x_1(k) + A_{22}x_2(k) + b_2u(k) \end{cases}$$
 (6)

where 
$$A = \begin{bmatrix} A_{11} \\ A_{22} \end{bmatrix}$$
,  $B = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}$ , thus, on the

switching surface, the order of the VSC system is reduced from n to n-1. Where  $c_2=1$ . If the system type is a controllability form, it is very easy to select the parameter  $c_1$  to make the system

$$x_1(k+1) = (A_{11} - A_{12}c_1^T)x_1(k)$$

stable.

The sliding mode can be determined as follows:

$$S(k) = C^{T}X(k) = [c_{1}^{T}, 1]X(k)^{[8]}$$
 (7)

It is obvious that there are some problems for the VSC using the reaching law described in (3) and (4):

1) From (3) or (4), we can see that increasing the reaching rate  $\varepsilon$  may result in the increase of reaching speed to the sliding mode, but this will

cause severe chattering; on the other hand, decreasing of  $\varepsilon$  will lead to the decrease of the reaching speed. The response of the system will become bad.

2) From (3) or (4), we can also see that the increasing of q may shorten the reaching time to the sliding mode and keep the optimal quality of the system, but from (5), we can see that this will lead to very high gain control input when

|S(k)| is large, on the contrary, decreasing q can

reduce the strength of the control input, but will lengthen the reaching time to the sliding mode.

The unavoidable contradiction and a trade-off between the dynamic quality and the high frequency chattering obviously exist in the procedure of the reaching law VSC design. To solve these problems, we apply the fuzzy logic to the reaching law, referred to as fuzzy reaching law.

## 3 Fuzzy reaching law controller

From (3), we obtain:

$$S(k + 1) = (1 - qT)S(k) - \varepsilon T sgnS(k)$$
 (8)  
where the reaching rate is:

$$\varepsilon = \frac{\left|S(k+1) - S(k)\right|}{T} \tag{9}$$

Analyze (8) and (9), we can see that:

- 1) When S(k) < 0, it is obvious that the chattering is reduced by decreasing  $\epsilon$  and the high reaching speed is kept by increasing q. this is the same to the condition when S(k) > 0.
- 2) When |S(k)| is large, increasing q may increase the reaching speed to the sliding mode, but the control input will have too high gain to realize. It is contrary to  $\varepsilon$ .
- 3) when  $\left|S(k)\right|$  is very small and q is constant,  $\left|S(k+1)-S(k)\right|$  and reaching rate are also small, so it is necessary to increase q to ensure to reach the sliding mode. To  $\epsilon$ , it is contrary.

From the discussions above, we can draw the conclusions as follows: when |S(k)| is large, a large value  $\epsilon$  and a small value q can be selected to keep the control input small and the reaching speed very fast. When |S(k)| is small, a small value  $\epsilon$  and a large value q can be selected to maintain the reaching rate fast. We can induce two types of fuzzy rules as follows:

1) if 
$$|S(k)|$$
 is  $\widetilde{A}$  , then  $\epsilon$  is  $\widetilde{B}$ 

2) if 
$$|S(k)|$$
 is  $\widetilde{A}$  , then q is  $\widetilde{C}$ 

Define the adjusting coefficients of  $\epsilon$  and q are  $k_\epsilon$  and  $k_q$ . The linguistic value of  $|S(k)|,\ k_\epsilon$  and  $k_q$  are VB (very big),B(big),M medium  $\lambda$  PS( small  $\lambda$  VS( very small ).The fuzzy rules are given in Table 1.

Table 1 the fuzzy rules for  $~k_{\epsilon}$  and  $~k_{\alpha}$ 

S(k)	VB	В	М	S	vs
kε	VB	В	М	S	VS
k <sub>q</sub>	VS	S	М	В	VB

The membership function is used as triangular shaped. The defuzzification algorithm used is the simple center-average method. Define the reaching law function as (11),

$$S(k+1)-S(k)=-\epsilon_f sgn(S(k)) q_f S(k)$$
 (11)  
thus, we can get the control input:

$$u(k) = -(C^{T}B)^{-1}[C^{T}(A - I + q_{f}TI)X(k) + \epsilon_{f}TsgnS(k)]$$

where 
$$\varepsilon_f = k_{ef} \varepsilon \varepsilon$$
,  $q_f = k_{of} q$ .

#### 4 AC position servo system

To test the proposed method, we apply it to a Digital AC Position Servo System based on a Brushless Director Current Motor (BLDCM) controlled by TMS320F240 DSP.

The simple structure of the system is shown in Fig 1.

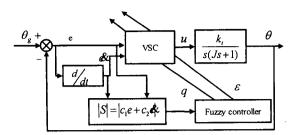


Fig 1 The simple structure of the system

We use the reaching law method and fuzzy reaching law method to design the position loop controller. The experimental results with 8000 pulses input is as follows:

Responses of the reaching law controller

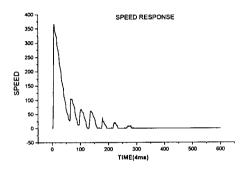


Fig 2 speed response

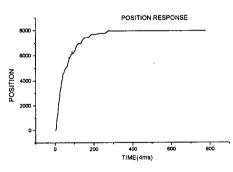


Fig 3 position response

 Responses of the fuzzy reaching law controller

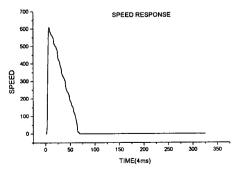


Fig 4 speed response

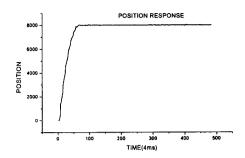


Fig 5 position response

From the Fig 2, we can see that there is high frequency chattering in the speed response of reaching law controller, which has bad influence on the position response, as shown in Fig 4. This is because that in order to maintain the high reaching speed to the sliding mode, the reaching rate  $\varepsilon$  can not be too small, so in the neighbor of the switching surface |S(k)| = 0, the

response has high value chattering, which leads to position error and long regulating time, about 1.0s. But when using the fuzzy reaching law controller, the reaching law parameters can be online adjusted by fuzzy controller, so the chattering is reduced and the optimal quality is maintained. The position error is very small, about  $\pm 2$  pulse, and the regulating time is shortened, about 0.28s, as shown in Fig 4 and Fig 5

### 5 Conclusions

In this paper, a new approach of the fuzzy controller for the AC position servo system has been presented. The core of this approach is the fuzzy reaching law method. It incorporates fuzzy logic into the reaching law VSC. The experimental results show that the AC position servo system designed by this method greatly reduces the chattering and also maintains the optimal dynamic response.

## References

O.Kaynak, and H.Hashimoto, "Variable structure systems theory applied to sub-time optimal position control with an invariant trajectory"

Trans.IEEE Japan, vol.104, no.3/4, Mar./Apr.,

- pp.47-51, 1984。
- Gao, W,B.(1990)." Foundation of Variable Structure Control
  Theory". Chinese Science and Technology Press,
  Beijing.
- Gao, W,B and M.Cheng(1989)."Quality control of variable structure control systems". Cont.Decision, vol 4 no.4,pp.1-6.
- Tong, R.M(1997). "A control engineering review of fuzzy systems". Automation 13,559-568.
- Hung ,J.Y(1993) ."Variable Structure Control: A survey".

  IEEE Trans. On Industrial Electronics,
  Vol.40,no.1,February.
- Hwang, G.C. and S.C.Lin (1992). "A stability approach to fuzzy control design for nonlinear systems". Fuzzy Sets and Systems. Vol.48, pp.279-287.