A New Negative Impedance Stabilizing Control Technique for Switching Power Supplies with Constant Power Loads

A. Emadi

Power Electronics and Motor Drives Laboratory
Electrical and Computer Engineering Department
Illinois Institute of Technology
Chicago, IL 60616-3793, USA
Phone: +1/(312) 567-8940; Fax: +1/(312) 567-8976
EML: emadi@iit.edu
URL: http://www.ece.iit.edu/~emadi/

***Abstract*—In this paper, an approach to the design of negative impedance stabilizing controllers for PWM DC/DC converters that are used in DC switching power supplies with constant power loads is presented. The control approach is based on the feedback linearization technique. Because of the negative impedance destabilizing characteristics of constant power loads, classical linear control methods have stability limitations around the operating points. However, the proposed stabilizing technique improves large-signal stability and dynamic responses. The proposed controllers are simulated and their responses under different operations are studied. Stability of the control technique is also verified using the second theorem of Lyapunov.**

***Keywords*—DC-DC power conversion, feedback linearization, Lyapunov methods, negative impedance instability, state space averaging.**

I. INTRODUCTION

The current through a constant power load decreases/ increases when the voltage across it increases/decreases. As a result, constant power loads have negative impedance characteristics. This is a destabilizing effect known as negative impedance instability. The most popular examples of constant power loads are tightly regulated power electronic converters and motor drives. In fact, power electronic loads when tightly regulated tend to behave as constant power loads [1], [2].

Basic PWM DC/DC converters, such as Buck, Boost, Buck-Boost, and Cuk converters, with constant power loads are unstable. This is because the poles of the small-signal transfer functions have positive real parts [2]. The transfer functions are obtained from the linearized state space averaged models of the converters [3], [4].

Because of the non-linearity and time-dependency of the power electronic converters and because of the negative impedance destabilizing characteristics of constant power loads, classical linear control methods, which are often used to design controllers for DC/DC converters, have stability limitations around the operating points. To overcome the instability problem, an approach to the design of stabilizing controllers for PWM DC/DC converters with constant power loads using sliding-mode control is presented in [5]. However, the main disadvantage is that the output voltage is not fixed.

Small-signal properties of the Buck converter with a constant power load are studied in [6]. The line-to-output and control-to-output transfer functions are derived, for voltage mode control and current mode control techniques. The results show that the open loop Buck converter with constant power load is unstable. However, no stabilizing controller is proposed in [6]. The effects of such loads in an induction motor based electric propulsion system as well as a small distribution system consisting of a generation system, a transmission line, a DC/DC converter load, and a motor drive load are studied in [7] and [8]. Both systems are unstable due to the negative impedance characteristics. [7] and [8] also propose a nonlinear stabilizing control to manipulate the input impedance of the converter/motor drive.

In [9] and [10], for driving constant power telecom loads, which are in parallel with batteries, it has been shown that the use of rectifiers with constant power output characteristics results in power supply systems which are better than those with the constant current output characteristics. A PID controller is designed and simulated to stabilize a telecom power supply with constant power loads and backup batteries in [11].

In this paper, a new negative impedance stabilizing control technique for switching power supplies with constant power loads is presented. In Section II, constant power loads and their characteristics are explained. Negative impedance destabilizing effects on PWM DC/DC Flyback, Forward, Double-ended, and Push-pull converters are introduced in Section III. In addition, these converters are modeled by the state space averaging method. In Section IV, negative impedance stabilizing techniques are discussed and a new approach based on the feedback linearization theory [12], [13] is presented. The stability of the proposed controller is verified using the second theorem of Lyapunov in Section V. Section V also presents the simulation results. Finally, conclusion remarks are given in Section VI.

II. CONSTANT POWER LOADS AND THEIR CHARACTERISTICS

An example of constant power loads is a DC/AC inverter which drives an electric motor and tightly regulates the speed when the rotating load has one-to-one torque-speed characteristics. The simplest form of one-to-one torque-speed characteristics is a linear relation between torque and speed. In this system, the controller tightly regulates the speed; therefore, the speed (ω) is almost constant. Since the rotating load has one-to-one torque-speed characteristics, for every speed there is one and only one torque. As a result, for a constant speed (ω), torque (T) is constant and power, which is the multiplication of speed and torque, is constant. If we assume a constant efficiency for the drive system, considering the constant power of the rotating load, the input power of the DC/AC inverter will be constant. Therefore, the DC/AC inverter presents constant power load characteristics to its power supply.

Another example of constant power loads is a DC/DC converter which feeds an electric load and tightly regulates the voltage when the electric load has one-to-one voltage-current characteristics. The simplest form of these loads is a resistor, which has a linear relation between voltage and current.

There is, in fact, a tendency in tightly regulated power electronic loads to be constant power. In addition, some of the loads of DC switching power supplies, such as electric motors, actuators, and power electronic converters, have to be controlled such that constant output power is maintained for them. As a result, these loads also present constant power characteristics to their power supply.

In constant power loads, although the instantaneous value of impedance is positive; but, the incremental impedance is always negative. In fact constant power loads have negative impedance characteristics, which might impact the power quality and system stability. Fig. 1 depicts the negative impedance behavior of constant power loads.

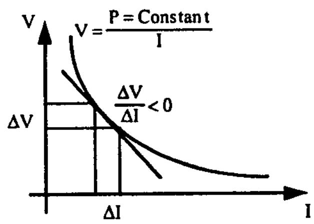

Fig. 1. The negative impedance behavior of constant power loads.

III. SWITCHING POWER SUPPLIES WITH CONSTANT POWER LOADS

In DC switching power supplies, in order to provide electrical isolation between the input and output, a transformer is used in the switching scheme of the DC/DC converter. Main advantages are ability of achieving very high switching frequencies, which, in turn, reduces the weight and volume of the transformer, and added design flexibility due to the turns ratio of the transformer. Usually, Flyback, Forward, Double-ended forward, and Push-pull converters are used in DC switching power supplies. In this Section, stability of these converters with constant power loads is studied.

A. Flyback Converter

A PWM DC/DC Flyback converter is shown in Fig. 1. We assume that the converter is operating with the switching period T and duty cycle d. In Fig. 2, L_m is the magnetizing inductance of the transformer.

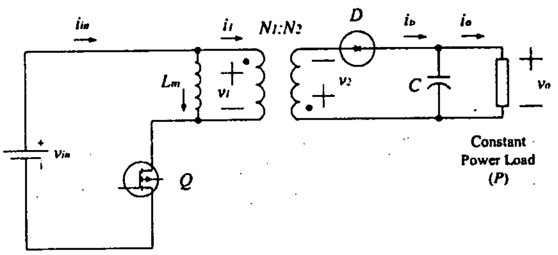

Fig. 2. A PWM DC/DC Flyback converter with constant power load.

During continuous conduction mode of operation, the space equations when switch Q is ON and diode D is OFF are given by,

$$\begin{cases} \dfrac{di_{L_m}}{dt} = \dfrac{1}{L_m}\left[v_{in}\right] \\ \dfrac{dv_o}{dt} = \dfrac{1}{C}\left[-\dfrac{P}{v_o}\right] \end{cases} \tag{1}$$

and when switch Q is OFF and diode D is ON are represented by,

$$\begin{cases} \dfrac{di_{L_m}}{dt} = \dfrac{1}{L_m}\left[\dfrac{N_1}{N_2}(-v_o)\right] \\ \dfrac{dv_o}{dt} = \dfrac{1}{C}\left[\dfrac{N_1}{N_2}i_{L_m} - \dfrac{P}{v_o}\right] \end{cases} \tag{2}$$

Using the state space averaging method [3], [4], these sets of differential equations can be shown by,

$$\begin{cases} \dfrac{dx_1}{dt} = \dfrac{1}{L_m}\left[dv_{in} - \dfrac{N_1}{N_2}(1-d)x_2\right] \\ \dfrac{dx_2}{dt} = \dfrac{1}{C}\left[\dfrac{N_1}{N_2}(1-d)x_1 - \dfrac{P}{x_2}\right] \end{cases} \tag{3}$$

where x_1 and x_2 are the moving averages of i_{Lm} and v_o, respectively.

Differential equations (3) are nonlinear. For studying the small-signal stability of the Flyback converter of Fig. 1, small perturbations in the state variables due to small disturbances in the input voltage and duty cycle are considered. The stability of this small-signal model can be determined by obtaining the transfer functions and their pole locations [12]-[14]. The small-signal transfer functions of the system of differential equations (3) are as follows.

$$H_1(s)=\frac{\tilde{x}_2(s)}{\tilde{d}(s)}=\frac{-\frac{N_1}{N_2}X_1 s+\frac{N_1}{N_2}\frac{(1-D)}{CL_m}\left(V_{in}+\frac{N_1}{N_2}X_2\right)}{s^2-\frac{P}{CX_2^2}s+\left(\frac{N_1}{N_2}\right)^2\frac{(1-D)^2}{CL_m}} \tag{4}$$

$$H_2(s)=\frac{\tilde{x}_2(s)}{\tilde{v}_{in}(s)}=\frac{\frac{N_1}{N_2}\frac{D(1-D)}{CL_m}}{s^2-\frac{P}{CX_2^2}s+\left(\frac{N_1}{N_2}\right)^2\frac{(1-D)^2}{CL_m}} \tag{5}$$

The poles of the transfer functions $H_1(s)$ and $H_2(s)$ have positive real parts. Therefore, the Flyback converter is unstable as the effect of the constant power load.

B. Forward Converter

A PWM DC/DC Forward converter is shown in Fig. 2. We assume that the converter is operating with the switching period T and duty cycle d. L_m is the magnetizing inductance of the three-winding transformer.

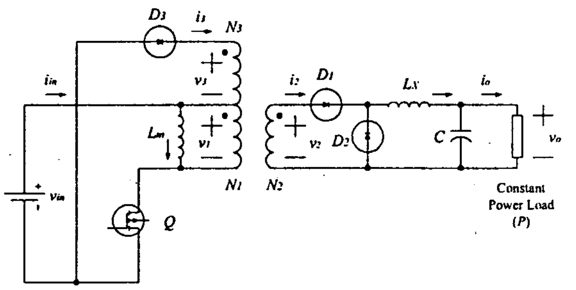

Fig. 3. A PWM DC/DC Forward converter with constant power load.

During continuous conduction mode of operation, the space equations when switch Q and diode D_1 are ON and diodes D_2 and D_3 are OFF are given by,

$$\begin{cases}\frac{di_{L_X}}{dt}=\frac{1}{L_X}\left[\frac{N_2}{N_1}v_{in}-v_o\right]\\ \frac{dv_o}{dt}=\frac{1}{C}\left[i_{L_X}-\frac{P}{v_o}\right]\end{cases} \tag{6}$$

and when switch Q and diode D_1 are OFF and diode D_2 is ON are represented by,

$$\begin{cases}\frac{di_{L_X}}{dt}=\frac{1}{L_X}\left[-v_o\right]\\ \frac{dv_o}{dt}=\frac{1}{C}\left[i_{L_X}-\frac{P}{v_o}\right]\end{cases} \tag{7}$$

Using the state space averaging method [3], [4], these sets of differential equations can be shown by,

$$\begin{cases}\frac{dx_1}{dt}=\frac{1}{L_X}\left[\frac{N_2}{N_1}dv_{in}-x_2\right]\\ \frac{dx_2}{dt}=\frac{1}{C}\left[x_1-\frac{P}{x_2}\right]\end{cases} \tag{8}$$

where x_1 and x_2 are the moving averages of i_{LX} and v_o, respectively.

The small-signal transfer functions of the system of differential equations (8) assuming small perturbations in the state variables due to small disturbances in the input voltage and duty cycle are as follows.

$$H_1(s)=\frac{\tilde{x}_2(s)}{\tilde{d}(s)}=\frac{\frac{N_2}{N_1}\frac{1}{CL_X}V_{in}}{s^2-\frac{P}{CX_2^2}s+\frac{1}{CL_X}} \tag{9}$$

$$H_2(s)=\frac{\tilde{x}_2(s)}{\tilde{v}_{in}(s)}=\frac{\frac{N_2}{N_1}\frac{D}{CL_X}}{s^2-\frac{P}{CX_2^2}s+\frac{1}{CL_X}} \tag{10}$$

The poles of the transfer functions $H_1(s)$ and $H_2(s)$ have positive real parts. Therefore, the Forward converter is unstable as the effect of the constant power load.

Other PWM DC/DC converters that are used in switching power supplies, such as Double-ended forward and Push-pull converters, are also unstable with constant power loads. Figs. 3 and 4 depict Double-ended forward and Push-pull converters, respectively. Flyback converter is usually used up to *150W*. On the other hand, Forward converter can be used up to *300W*. This is mainly because the magnetizing current resets to zero in each period in a Forward converter which, in turn, allows a smaller core for the transformer. Therefore, it can be used at higher power ratings than Flyback converter. Double-ended forward and Push-pull converters can be used up to *500W* and *1kW*, respectively. For power ratings higher than *1kW*, full-bridge DC/DC converters are usually used. However, this converter is also unstable with constant power loads.

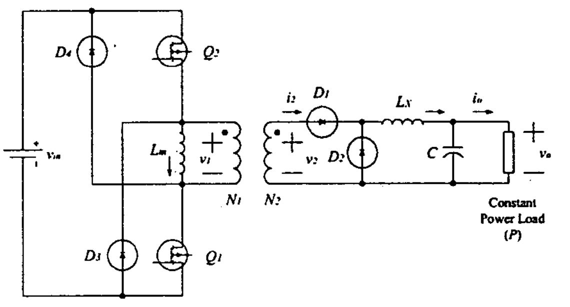

Fig. 4. A PWM DC/DC Double-ended forward converter with constant power load.

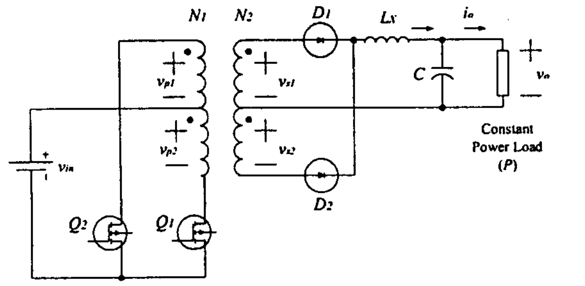

Fig. 5. A PWM DC/DC Push-pull converter with constant power load.

IV. NEGATIVE IMPEDANCE STABILIZING CONTROLLER

In this Section, we present a negative impedance stabilizing controller which regulates the output voltage of the DC switching power supply even with the presence of a constant power load. However, if the output voltage regulation is not necessary for the constant power load, a conventional control technique such as *PI* or state variable feedback can be used. Yet, the controller must regulate the power instead of the voltage. In this case, sliding mode control theory may be used to achieve better dynamic performances [15], [16].

A. Feedback Linearization Technique

In this section, we use feedback linearization technique to stabilize and regulate the output voltage of the DC/DC converters [12]. As the example, we present this method for the PWM DC/DC Forward converter of Fig. 3. For other converters used in switching power supplies, a similar approach should be followed.

We look for a nonlinear feedback to cancel out the nonlinearity in the set of differential equations (8). In (8), there is no direct relation between the control input and the nonlinearity; therefore, the following change of variables is considered.

$$\begin{aligned} z_1 &= x_1 - \frac{P}{x_2} \\ z_2 &= x_2 - V_{o,ref.} \end{aligned} \tag{11}$$

Where $V_{o,ref.}$ is the output reference voltage for the Forward converter. Using change of variables (11), state equations (8) can be written as follows.

$$\begin{cases} \dfrac{dz_1}{dt} = \dfrac{\frac{N_2}{N_1} dv_{in}}{L_X} - \dfrac{z_2 + V_{o,ref.}}{L_X} + \dfrac{P z_1}{C\left(z_2 + V_{o,ref.}\right)^2} \\ \dfrac{dz_2}{dt} = \dfrac{1}{C} z_1 \end{cases} \tag{12}$$

In order to cancel out the nonlinearity in (12), the following nonlinear feedback is proposed.

$$\frac{\frac{N_2}{N_1} dv_{in}}{L_X} = k_1 z_1 + k_2 z_2 - \frac{\hat{P} z_1}{C\left(z_2 + V_{o,ref.}\right)^2} + \omega \tag{13}$$

$$\omega = \frac{V_{o,ref.}}{L_X}$$

Where k_1, k_2, and $\hat{P}$ are the parameters of the controller to be designed. With the nonlinear feedback, the state equations of (12) can be shown by,

$$\begin{cases} \dfrac{dz_1}{dt} = k_1 z_1 + \left(k_2 - \dfrac{1}{L_X}\right) z_2 + \dfrac{z_1}{C\left(z_2 + V_{o,ref.}\right)^2}\left(P - \hat{P}\right) \\ \dfrac{dz_2}{dt} = \dfrac{1}{C} z_1 \end{cases} \tag{14}$$

If $\hat{P} = P$, the system is linear. Other parameters of the controller, i.e., k_1 and k_2, can be designed such that the resulted system has poles at appropriate places. However, we do not have any control over the loads, which may, in fact, change. In deed, *P* varies according to the load changes. Therefore, in the next Section, we design the controller to guarantee the stability of the system in the presence of load changes.

V. STABILITY ANALYSIS

In order to evaluate the stability of the converter, a continuously differentiable positive definite function *V(z)* needs to be determined. We define *V(z)* as follows.

$$V(z_1, z_2) = \frac{1}{2} K z_1^2 + \frac{1}{2} K C\left(\frac{1}{L_X} - k_2\right) z_2^2 \tag{15}$$

where

$$K > 0 \quad \& \quad k_2 < \frac{1}{L_X} \tag{16}$$

Therefore, *V(z)* is a positive definite function. The derivative of *V(z)* is given by,

$$\dot{V}(z_1, z_2) = K k_1 z_1^2 + K \frac{z_1^2}{C\left(z_2 + V_{o,ref.}\right)^2}\left(P - \hat{P}\right) \tag{17}$$

In order to guarantee the stability of the converter, the derivative of *V(z)* needs to be negative definite. Therefore, the parameters of the controller are chosen as follows.

$$k_1 < 0, \quad P - \hat{P} < 0, \quad \& \quad k_2 < \frac{1}{L_X} \tag{18}$$

Considering upper and lower limits of *P* as well as variations in the values of the inductor and capacitor of the converter, control parameters for a robust design are given by,

$$k_1 < 0, \quad \hat{P} > P_{\max.}, \quad \& \quad k_2 < \frac{1}{L_{X,\max.}} \tag{19}$$

$L_{X,max.}$ is the maximum value that L_X can hold. Therefore, $\dot{V}(z)$ is a negative definite function and, consequently, $V(z)$ is a Lyapunov function. As a result, the closed loop system is asymptotically stable and the operating point is a stable equilibrium point.

A negative impedance stabilizing controller based on the proposed feedback linearization technique and *PI* controller has been designed and simulated for a Forward converter. The parameters of the converter and designed robust controller are given in Table 1.

Table 1. Parameters of the Forward converter and stabilizing controller.

Parameter	Value
V_{in}	*20v*
$V_{o,ref.}$	*10v*
f	*10kHz*
C	*10mF*
L_X	*1mH*
L_m	*40uH*
$P_{max.}$	*20W*
N_1/N_2	*1.0*
N_1/N_3	*1.5*
PI controller	$K_p=2,\ K_I=180$
Feedback Linearization	$k_1=-200,\ k_2=950,\ \hat{P}=25$

In order to study the converter dynamic performances under load variations, step changes in constant power load have been investigated. Fig. 6 depicts one of the simulation results. As is shown in Fig. 6, the system is stable and the controller performs satisfactory. It must be mentioned that most of the practical constant power loads at the starting phase of their operations have positive incremental impedance characteristics. Therefore, Their power is increasing until they reach the nominal power. After they hit the nominal power, the input power will be constant and they behave as constant power sinks, which have negative incremental impedance characteristics. In our simulations, we have considered linear *v-i* characteristics for the starting period of the constant power loads.

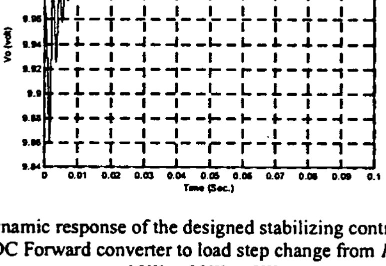

Fig. 6. Dynamic response of the designed stabilizing controller for the PWM DC/DC Forward converter to load step change from *10W* to *12W* to *15W* to *20W* to *5W*.

VI. CONCLUSIONS

In this paper, the concept of negative impedance characteristics of constant power loads for DC/DC converters used in DC switching power supplies was described. It was shown that constant power loads affect the stability and dynamics of Flyback and Forward converters. They even destabilize the supplying systems. By applying the feedback linearization technique to the state space averaged models of the converters, negative impedance stabilizing controllers have been designed. The responses of the proposed controllers under different operations and in the presence of significant variations in load were studied. Large-signal control, simplicity of the construction, and high reliability were described as the main advantages.

REFERENCES

[1] A. Emadi and M. Ehsani, "Dynamics and control of multi-converter DC power electronic systems," in *Proc. IEEE 2001 Power Electronics Specialist Conference*, Vancouver, Canada, June 2001.

[2] A. Emadi and M. Ehsani, "Negative impedance stabilizing controls for PWM DC/DC converters using feedback linearization techniques," in *Proc. 35th Intersociety Energy Conversion Engineering Conference*, Las Vegas, Nevada, July 2000.

[3] R. D. Middlebrook and S. Cuk, "A general unified approach to modeling switching converter power stages," in *Proc. IEEE Power Electronics Specialist Conf.*, June 1976, pp. 18-34.

[4] J. Mahdavi, A. Emadi, M. D. Bellar, and M. Ehsani, "Analysis of power electronic converters using the generalized state space averaging approach," *IEEE Trans. on Circuits and Systems I: Fundamental Theory and Applications*, vol. 44, no. 8, Aug. 1997.

[5] A. Emadi, B. Fahimi, and M. Ehsani, "On the concept of negative impedance instability in advanced aircraft power systems with constant power loads," *Society of Automotive Engineers (SAE) Journal*, Paper No. 1999-01-2545, 1999.

[6] V. Grigore, J. Hatonen, J. Kyyra, and T. Suntio, "Dynamics of a Buck converter with constant power load," in *Proc. IEEE 29th Power Electronics Specialist Conf.*, Fukuoka, Japan, May 1998, pp. 72-78.

[7] S. F. Glover and S. D. Sudhoff, "An experimentally validated nonlinear stabilizing control for power electronics based power systems," *Society of Automotive Engineers (SAE) Journal*, Paper No. 981255, 1998.

[8] S. D. Sudhoff, K. A. Corzine, S. F. Glover, H. J. Hegner, and H. N. Robey, "DC link stabilized field oriented control of electric propulsion systems," *IEEE Trans. on Energy Conversion*, vol. 13, no. 1, pp. 27-33, March 1998.

[9] A. S. Kislovski, "Optimizing the reliability of DC power plants with backup batteries and constant power loads," in *Proc. IEEE Applied Power Electronics Conf.*, Dallas, TX, March 1995, pp. 957-964.

[10] A. S. Kislovski and E. Olsson, "Constant-power rectifiers for constant-power telecom loads," in *Proc. 16th International Telecommunications Energy Conf.*, Nov. 1994, pp. 630-634.

[11] I. Gadoura, V. Grigore, J. Hatonen, and J. Kyyra, "Stabilizing a telecom power supply feeding a constant power load," in *Proc. 10th International Telecommunications Energy Conf.*, Nov. 1998, pp. 243-248.

[12] H. A. Khalil, *Nonlinear Systems*, Prentice-Hall, Upper Saddle River, NJ, 1996.

[13] J. E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice-Hall, Upper Saddle River, NJ, 1991.

[14] P. A. Ioannou and J. Sun, *Robust Adaptive Control*, PTR Prentice-Hall, Upper Saddle River, NJ, 1996.

[15] V. I. Utkin, *Sliding Modes and Their Application in Variable Structure Systems*, MIR, Moscow, Russia, 1974.

[16] J. Mahdavi, A. Emadi, and H. A. Toliyat, "Application of state space averaging method to sliding mode control of PWM DC/DC converters," in *Proc. IEEE 1997 Industry Application Society Annual Meeting*, New Orleans, LA, Oct. 1997, pp. 820-827.