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입력 지연 TS 퍼지 시스템의 지능형 디지털 재설계

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Intelligent Digital Redesign for Continuous-Time TS Fuzzy Systems with Input Delay

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Abstract– This paper proposes a novel intelligent digital redesign technique for a class of nonlinear systems represented by input-delayed Takagi-Sugeno (TS) fuzzy systems. The digitally redesigned controller can show good performance provided that the analog controller is well-designed. The developed digital redesign technique is based on the 'state-matching', so the control performance is guaranteed as well as the stability of the system. An simulation example is included to ensure the effectiveness of the proposed method.

1. Introduction

Since the most practical systems and industrial control processes are formulated by a continuos-time framework, it is natural to design a controller in the continuous-time domain. At the same time, as the computing technology has been rapidly grown, some attempts that control the continuous-time system via digital computer. This means that the hybrid control technique should be developed. There are a few techniques to design a digital controller for an analog system. One of them is named as "digital redesign," which is to find a digital control gain matrix such that the controlled state with the existing analog controller is close to that with the digital controller [1,2].

However, in physical systems, there exist time delay phenomena inevitably occur. It is well known that the existence of time delay makes the closed-loop stabilization more difficult. Moreover most systems have nonlinearities. These factors makes the synthesis of the controller difficult and the problem of digital redesign for the input-delayed Takagi-Sugeno (TS) fuzzy system is fully open.

This paper explores the intelligent digital redesign for input-delayed TS fuzzy systems. It should be noted that the digital redesign for nonlinear system is extremely complicated, thus, we only consider the local dynamics of the TS fuzzy system. However, it provides somewhat positive solution to the considered nonlinear hybrid control problem. First we briefly review the input-delayed TS fuzzy system in Section 2. The discretization of the closed-loop input-delayed TS fuzzy system is discussed in 3. The intelligent digital redesign of the input-delayed TS fuzzy system is developed in Section 4. An example is demonstrated to ensure the effectiveness of the proposed method in 5. Finally some remarks are drawn in Section 6.

2. Input-Delayed TS fuzzy systems

Consider the following continuous-time local dynamics:

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(x(t))(A_i x(t) + B_i u(t-\tau)), \tag{1}$$

$$x(t) = \phi(t), \tag{2}$$

where $\phi(t)$ is defined in Banach space $\mathbb{C}\left[-\tau,\ 0\right]$. τ is the input delay. Throughout this paper, we assume the continuous-time TS fuzzy-model-based control law of the form:

$$u(t) = \sum_{i=1}^{q} \mu_i(x(t)) K_i x(t). \tag{3}$$

3. Discretization of the Closed-Loop of the Input-Delayed TS Fuzzy System

Let us assume that we find the control gain matrix K_i for the delayed control input $u(t) = K_i x(t)$ such that the following closed-loop system is asymptotically stable

$$\dot{x}(t) = A_i x(t) + B_i K_i x(t-\tau). \tag{4}$$

The detailed discussion on this problem can be found in [3]. The goal of this section is to find the approximated solution evaluated at $t = kT + \alpha T$ with the initial state $x(kT)$, where T is the sampling time and $0 \leq \alpha < 1$. The general solution is represented as

$$\begin{aligned} &x(kT+\alpha T) \\ &= G_i(\alpha)x(kT) + \int_{kT}^{kT+\alpha T} e^{A_i(kT+\alpha T-\lambda)} B_i K_i x(\lambda - \tau) d\lambda, \end{aligned} \tag{5}$$

where $G_i(\alpha) = e^{\alpha T}$. Because the convolution integral in (5) is difficult to be evaluated exactly, the principle of equivalent areas is used to obtain an approximate one.

$$\begin{aligned} &\int_{kT}^{kT+\alpha T} e^{A_i(kT+\alpha T-\lambda)} B_i K_i x(\lambda-\tau) d\lambda \\ &\approx \int_{kT}^{kT+\alpha T} e^{A_i(kT+\alpha T-\lambda)} d\lambda \frac{B_i K_i}{\alpha T} \int_{kT}^{kT+\alpha T} x(\lambda-\tau) d\lambda. \end{aligned} \tag{6}$$

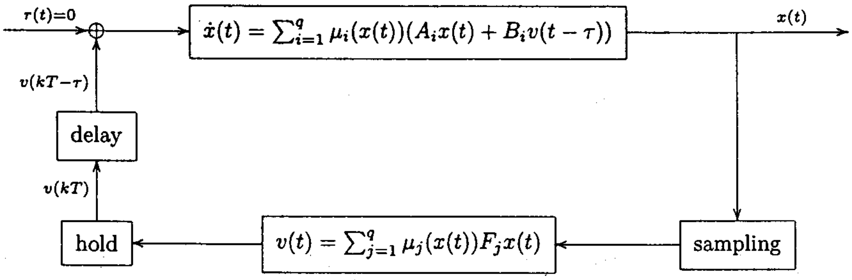

Figure 1: Block diagram of the intelligent digitally redesigned control system.

Integrating (4) from time $t = kT$ to $t = kT + \alpha T$ and using the well-known trapezoidal rule we get

$$
\begin{aligned}
& B_i K_i \int_{kT}^{kT+\alpha T} x(\lambda - \tau) d\lambda \\
&= x(kT + \alpha T) - x(kT) - \frac{A_i \alpha T}{2}(x(kT + \alpha T) + x(kT)) \\
&= \left(I - \frac{A_i \alpha T}{2}\right) x(kT + \alpha T) - \left(I + \frac{A_i \alpha T}{2}\right) x(kT) . \quad (7)
\end{aligned}
$$

Inserting (7) into (6), and its results into (5) again, yields

$$
\begin{aligned}
x(kT + \alpha T) &= G_i(\alpha) x(kT) + \int_{kT}^{kT+\alpha T} e^{A_i(kT+\alpha T-\lambda)} d\lambda \\
&\times \frac{1}{\alpha T}\left(\left(I - \frac{A_i T}{2}\right) x(kT + \alpha T) - \left(I + \frac{A_i T}{2}\right) x(kT)\right) .
\end{aligned} \quad (8)
$$

Rearranging (8) results in

$$
x(kT + \alpha T) = \Gamma_i(\alpha) x(kT) , \quad (9)
$$

where

$$
\begin{aligned}
\Gamma_i(\alpha) &= \left(I - (G_i(\alpha) - I) A^{-1} \left(\frac{1}{\alpha T} - \frac{A_i}{2}\right)\right)^{-1} \\
&\times \left(G_i(\alpha) - (G_i(\alpha) - I) A_i^{-1} \left(\frac{1}{\alpha T} + \frac{A_i}{2}\right)\right) .
\end{aligned}
$$

4. Digital Redesign of the Input-Delayed Systems

Consider the following local dynamics of (2)

$$
\dot{x}(t) = A_i x(t) + B_i u(t - \tau) , \quad (10)
$$

then its solution evaluated at $t = kT + T + \tau$ with an initial condition $x(kT + \tau)$ is given by

$$
\begin{aligned}
& x(kT + T + \tau) \\
&= G_i(1) x(kT + \tau) + \int_{kT+\tau}^{kT+T+\tau} e^{A_i(kT+T+\tau-\lambda)} B_i u(\lambda - \tau) d\lambda \\
&\approx G_i(1) x(kT + \tau) + H_i(1) \frac{1}{T} \int_{kT+\tau}^{kT+T+\tau} u(\lambda - \tau) d\lambda . \quad (11)
\end{aligned}
$$

where $H_i(1) = (G_i(1) - I) A_i^{-1} B_i$.

Assumption 1 *Throughout this paper, the input delay τ is assumed to be not larger than the sampling time T for simplicity of analysis. This assumption is not restrictive and we can always straightforwardly extend to the case $\tau > T$.*

Consider the following system with delayed digital control law

$$
\dot{\zeta}(t) = A_i \zeta(t) + B_i v(t - \tau) , \quad (12)
$$

where $v(t-\tau)$ is a piecewise constant input for $kT+\tau \le t < kT + T + \tau$. The block diagram of the digitally redesigned control system for TS fuzzy system is shown in Fig. 1. Its corresponding discretized system with the sampling period T is given by

$$
\begin{aligned}
& \zeta(kT + T + \tau) \\
&= G_i(1) \zeta(kT + \tau) + \int_{kT+\tau}^{kT+T+\tau} e^{A_i(kT+T+\tau-\lambda)} B_i v(\lambda - \tau) d\lambda \\
&= G_i(1) \zeta(kT + \tau) + H_i(1) v(kT) \\
&= G_i(1) \zeta(kT + \tau) + H_i(1) v(kT - T + \tau) , \quad (13)
\end{aligned}
$$

The last equality of the above equation is natural since $v(kT)$ is a piecewise constant input for the time interval $[kT - T + \tau, \ \ kT + \tau)$.

Comparing the two above equations, if $v(kT - T + \tau) \approx \frac{1}{T} \int_{kT+\tau}^{kT+T+\tau} u(\lambda - \tau) d\lambda$ then, $x(kT + \tau) = \zeta(kT + \tau)$ obviously holds. Using the trapezoidal approximation rule, the above equation can be approximately evaluated as

$$
\begin{aligned}
v(kT - T + \tau) &= \frac{1}{T} \int_{kT+\tau}^{kT+T+\tau} u(\lambda - \tau) d\lambda \\
&\approx \frac{1}{2}(u(kT + T) + u(kT)) \\
&= \frac{K_i}{2}(x(kT + T) + x(kT)) \\
&= \frac{1}{2} K_i (I + \Gamma_i(1)) x(kT) \\
&= \frac{1}{2} K_i (I + \Gamma_i(1)) \Gamma_i(1 - \tau) x(kT - T + \tau) \\
&\approx \frac{1}{2} K_i (I + \Gamma_i(1)) \Gamma_i(1 - \tau) \zeta(kT - T + \tau) .
\end{aligned} \quad (14)
$$

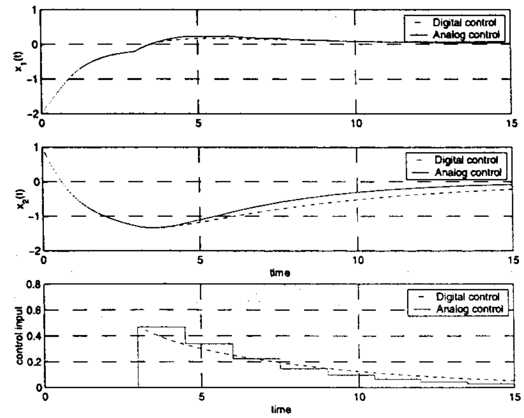

Figure 2: The controlled results.

Thus, the digitally redesigned control gain matrix is given by

$$F_i = \frac{1}{2}K_i(I + \Gamma_i(1))\Gamma_i(1-\tau)\,. \tag{15}$$

5. An Example

In this section, we show the effectiveness of the proposed digital control technique for the input-delayed TS fuzzy system. Consider a TS fuzzy system constructed with following system matrices.

$$A_1 = \begin{bmatrix} -0.5 & 0.2 \\ 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -0.1 & 0.1 \\ 1 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} -0.5 & 0.15 \\ 1 & 0 \end{bmatrix},$$

$$B_1 = B_2 = B_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

and the associated membership functions are

$$\Gamma_1(x(t)) = \begin{cases} 1 & x_1(t) \le -3 \\ -\frac{1}{2}x_1(t) - \frac{1}{2} & -3 < x_1(t) < -1 \\ 0 & -1 \le x_1(t) \end{cases}$$

$$\Gamma_2(x(t)) = \begin{cases} 0 & x_1(t) \le -2 \\ 1 - \frac{1}{2}|x_1(t)| & -2 < x_1(t) < -2 \\ 0 & -2 \le x_1(t) \end{cases}$$

$$\Gamma_3(x(t)) = \begin{cases} 0 & x_1(t) \le 1 \\ \frac{1}{2}x_1(t) + \frac{1}{2} & 1 < x_1(t) < 3 \\ 1 & 1 \le x_1(t) \end{cases}$$

From [3], the continuous-time control gain matrices for the analog fuzzy-model-based controller are given by

$$K_1 = \begin{bmatrix} -0.7554 & -0.5499 \end{bmatrix}, K_2 = \begin{bmatrix} -0.3322 & -0.2904 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -1.0655 & -0.8691 \end{bmatrix}.$$

With the above control gain matrices, the given TS fuzzy system can be stabilized with input delay τ not large than 0.2055 sec.. Based on the intelligent digital redesign technique, the digital control gain matrices are simply calculated as

$$F_1 = \begin{bmatrix} -0.9295 & -0.6935 \end{bmatrix}, F_2 = \begin{bmatrix} -0.3164 & -0.3157 \end{bmatrix},$$
$$F_3 = \begin{bmatrix} -1.3409 & -1.0208 \end{bmatrix},$$

where the sampling time is $T = 1.5$ sec., which is sufficiently large sampling time to implement the physical system. The simulation results are shown in Fig. 2. The control input is activated at $t = 3$ sec.. Before the control is activated, the system trajectories tend to diverse to infinity. After the control input is activated, the trajectories are guided to the origin. One can see that digitally controlled trajectories are similar to the analogously controlled ones. It is reasonable since the developed digital redesign method is based on the so-called 'state-matching' technique. Indeed, from the simulation results, one can see that the intelligent digital redesign method have a good performance.

6. Conclusion

In this paper, an effective digital controller design technique has been proposed for the input-delayed TS fuzzy systems. The digital control gain matrices for the fuzzy-model-based controller has been obtained via simple algebraic calculation. The control performance by the redesigned digital controller is guaranteed, because the digital redesign technique is based on the 'state-matching.' For the simplicity of the synthesis of the digital redesign, we only considered the local dynamics of the TS fuzzy system, since the global analysis is extremely complex. This topic is still open and the future research will be devoted to it.

Acknowledgement

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References

[1] S. M. Guo, W. Wang, and L. S. Shieh, "Discretisation of two degree-of-freedom controller and system with state, input and ouput delays, *IEE Proc.-Control Theory Appl.*, Vol. 147, No. 1, pp. 87-96, Jan., 2000.

[2] C. S. Shieh, J. S. H. Tsai, and Y. Y. Sun, "Digital modeling and hybrid control of sampled-data uncertain system with input time delay using the law of mean," *Applied Mathematical Modeling*, Vol. 23, pp. 131-152, 1999.

[3] H. J. Lee, J. B. Park, and Y. H. Joo, "Stabilization of the input-delayed TS fuzzy systems," *Proc. of KFIS*, pp. 140-143, 2001.