GPS 자세각 추정을 위한 쿼터니언 기반 최소자승기법의 성능평가

원종훈*, 김형철*, 고선준*, 이자성**
*아주대학교 대학원 전자공학과,** 아주대학교 전자공학부

Performance Analysis of Quaternion-based Least-squares Methods for GPS Attitude Estimation

Jong-Hoon Won*, Hyung-Cheol Kim*, Sun-Jun Ko*, Ja-Sung Lee**
*Dept. of Electronic Eng., Ajou Univ., **School of Electronics Eng., Ajou Univ.

Abstract - In this paper, the performance of a new alternative form of three-axis attitude estimation algorithm for a rigid body is evaluated via simulation for the situation where the observed vectors are the estimated baselines of a GPS antenna array. This method is derived based on a simple iterative nonlinear least-squares with four elements of quaternion parameter. The representation of quaternion parameters for three-axis attitude of a rigid body is free from singularity problem. The performance of the proposed algorithm is compared with other eight existing methods, such as, Transformation Method (TM), Vector Observation Method (VOM), TRIAD algorithm, two versions of QUaternion ESTimator (QUEST), Singular Value Decomposition (SVD) method, Fast Optimal Attitude Matrix (FOAM), Slower Optimal Matrix Algorithm (SOMA).

1. Introduction

The Global Positioning System (GPS) provides us with very precise range information such as carrier phase measurements. By measuring differenced carrier phase between the main antenna and the sub-antennas, a receiver can determine the three-axis attitude of a rigid body from the relative position between a pair of antennas [1-3].

The problem of attitude determination is to find the rotation matrix such as Direction-Cosine-Vector (DCM) or a set of orientation parameters which rotates the baseline vectors in the reference frame into the corresponding vectors in the body frame from a set of measurements [4]. In 1965 Wahba showed that the problem of attitude determination is equivalent to finding a proper orthogonal transformation attitude matrix A that minimizes the scalar weighted norm-squared residuals given by [5]

$$J(A) \equiv \frac{1}{2}\sum_{i=1}^{n}\varpi_i \left|\underline{r}_i - A\cdot\underline{l}_i\right|^2 \tag{1}$$

where ϖ_i represents a scalar weighting factor for the i-th baseline, $\underline{l}_i$ denotes the vector representation in the local-level frame of the direction to some observed objects and $\underline{r}_i$ is the previously defined vector representation of the corresponding observations in the vehicle body frame. This Wahba's problem is shown to be a special case of nonlinear least-squares when the vector measurement has a scalar weighting factor [6].

This paper compares the performance of the quaternion-based nonlinear least-squares method with that of other eight existing attitude determination algorithms.

2. Quaternion-based Least-squares Method [6]

The problem of three-axis attitude determination can be formulated as a linear least-squares problem with norm constraint on the solution [7]. This attitude determination problem can be solved by using simple quaternion parameterization of the rotation matrix.

From a pair of reference-observation vectors, a nonlinear vector measurement equation for any baseline i can be given by

$$\underline{l}_i = A^T\cdot\underline{r}_i + \underline{w}_i = h_i(X) + \underline{w}_i, \quad \text{for } i=1,2,\dots,n \tag{2}$$

where $\underline{w}_i$ represents the vector of observation noise, h_i denotes a nonlinear vector equation for baseline i and X denotes the vector of unknowns $(q_0, q_1, q_2, q_3)^T$. The transformation matrix A^T derived from these quaternion parameters is free from the singularity problem.

After Linearizing Eq. (2) about nominal values X^* and $\underline{l}_i^*$ and expanding the result to m noncolinear baselines case, we can obtain the optimal solution by applying a simple iterative least-squares with the following linear error equation:

$$\delta\hat{X} = (H^T Q^{-1} H)^{-1} H^T Q^{-1}\cdot\delta L \tag{3}$$

where

$$\delta L = \left[\delta\underline{l}_1^T \quad \delta\underline{l}_2^T \quad \cdots \quad \delta\underline{l}_m^T\right]^T,$$
$$H = \left[H_1^T \quad H_2^T \quad \cdots \quad H_m^T\right]^T,$$

and δX is the vector of corrections to quaternion parameters, $\delta\underline{l}_i$ represents the residual baseline vector, Q denotes the measurement noise covariance matrix and H_i is the matrix of partial derivatives which has a general form as:

$$H_i = \frac{\partial h_i(X)}{\partial X}\bigg|_{X=X^*} \tag{4}$$

The attitude matrix obtained from the unconstrained least-squares solution in Eq. (3) is not proper orthogonal in general because of the noisy measurements. This problem can be overcome by using either a constrained least-squares solution method with orthogonal constraint of quaternion parameters or an ad hoc approach for dealing with normalization constraint equation [6-8].

3. Simulation Results

The performance of the new quaternion-based least-squares attitude estimation algorithms were evaluated by the numerical simulation. Six test cases were simulated as shown in Table 1. Eight existing attitude determination algorithms, TM, VOM, TRIAD, two versions of QUEST, and three variations of QUEST were coded to compare the performance of least-squares methods. These five QUEST-based algorithms are applicable to an arbitrary number of reference-observation vector pairs and provide us with the optimal solution and a simple analytical expression of the covariance matrix [9-13]. The characteristics of the attitude determination algorithms are summarized in Table 1. The computational steps of implemented algorithms and the more complete survey of other attitude determination algorithms are summarized in References 14 and 15, respectively.

Four versions of quaternion-based least-squares algorithms were included in simulation. They are Euler angle-based least-squares (ELS), quaternion-based unconstrained least-squares (QULS), quaternion-based constrained least-squares (QCLS) and quaternion-based constrained least-squares with ad hoc constraint approach (QACLS).

Figure 1 represents the time history of true Euler angles used in the simulation. Euler angle errors of eight existing attitude determination algorithms in the test case 1 are shown in Figures 2 through 5. Note that the error from TRIAD is relatively large because its accuracy depends on the choice of the first observation-reference vector pair; i. e., the more accurate solution is obtained when a set of the observation-reference vector pair of greater accuracy is chosen as the first vector pair. The output of ELS in cases 5 and 6 is subject to large errors when the yaw reaches 180 degrees because of its singularity problem near 0 or 180 degrees as shown in Figure 6. Figures 7 through 9 show the results of quaternion-based least-squares algorithms. They provide accurate solution comparable to the results of the QUEST-based algorithms in Figure 5. Relative characteristics of quaternion-based least-squares algorithms in terms of accuracy and computational load are summarized in Table 3.

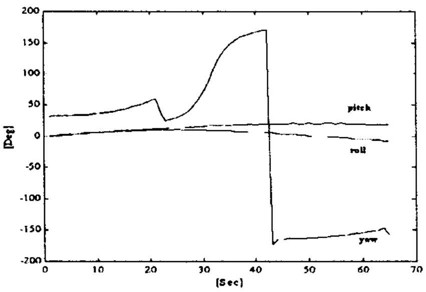

Figure 1. True Euler angles

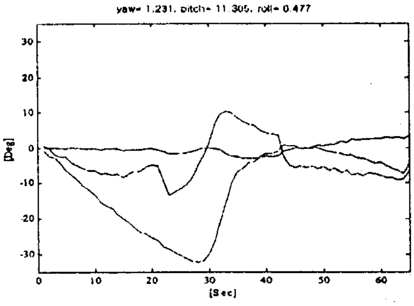

Figure 2. Euler angle errors of TM

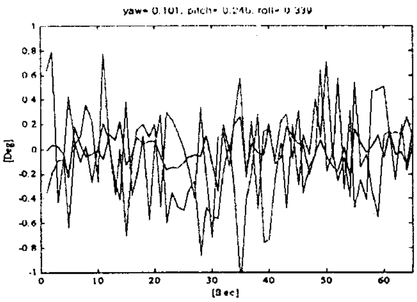

Figure 3. Euler angle errors of VOM

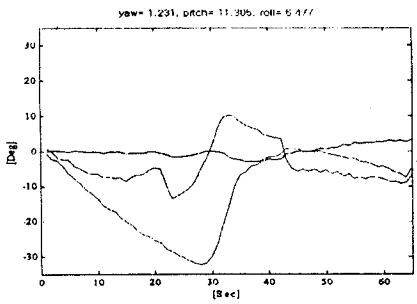

Figure 4. Euler angle errors of TRIAD

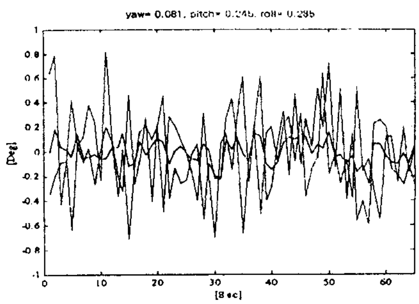

Figure 5. Euler angle errors of QUEST-based algorithms

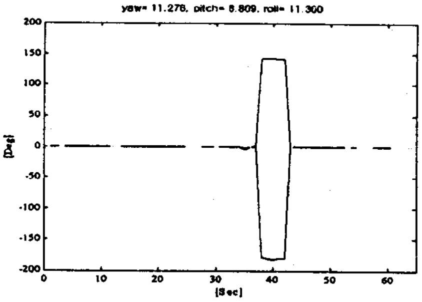

Figure 6. Euler angle errors of ELS

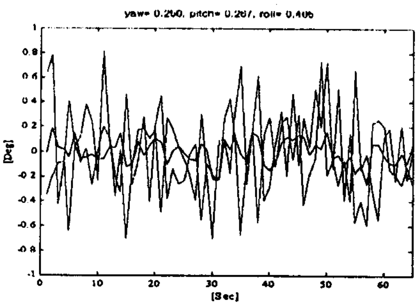

Figure 7. Euler angle errors of QULS

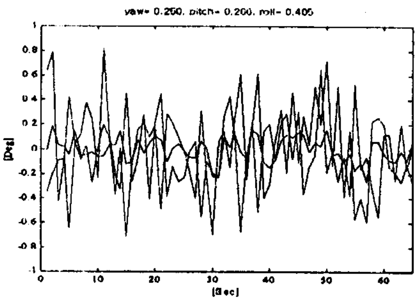

Figure 8. Euler angle errors of QCLS

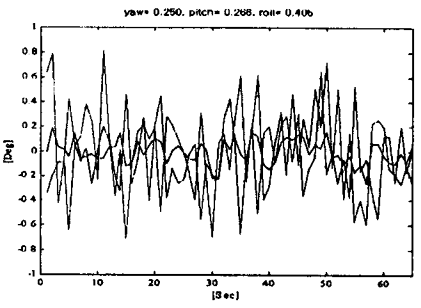

Figure 9. Euler angle errors of QACLS

Table 1. Simulation test cases

Case	Body Frame Vector (m) $\underline{r}_1^T$	$\underline{r}_2^T$	σ_Φ (mm)
1	[1,0,0]	[0,1,0]	2
2	[1,0,0]	[0,1,0]	20
3	[1,0,0]	$[1/\sqrt{2}, 1/\sqrt{2}, 0]$	2
4	[1,0,0]	$[1/\sqrt{2}, 1/\sqrt{2}, 0]$	20
5	$[\sqrt{3}/\sqrt{5}, 1/\sqrt{5}, 1/\sqrt{5}]$	$[1/\sqrt{5}, \sqrt{3}/\sqrt{5}, 1/\sqrt{5}]$	2
6	$[\sqrt{3}/\sqrt{5}, 1/\sqrt{5}, 1/\sqrt{5}]$	$[1/\sqrt{5}, \sqrt{3}/\sqrt{5}, 1/\sqrt{5}]$	20

Table 2. Comparison of eight existing algorithms

	Advantages	Disadvantages
VOM	· simple deterministic solution · fast computational speed	· non-optimal solution · only applicable to two baselines (three GPS antennas) · trigonometric functions used · antenna configuration restricted
TM	· simple deterministic solution · fast computational speed	· non-optimal solution · at least three baselines required
TRIAD	· simple deterministic solution · fast computational speed	· non-optimal solution · only applicable to two baselines (three GPS antennas)
QUESTq	· quaternion-based QUEST (free from singularity problem)	· high computational burden
QUESTg	· fast computational speed	· Gibbs vector-based QUEST (singular near 180 degrees)
SVD	· easy tool for theoretical analysis · provides attitude uncertainty (maximum eigen value and eigen vector)	· high computational burden · analysis purpose
FOAM	· robust computing without SVD process · fast computational speed	· iterative method
SOMA	· robust computing without SVD process	· low computational speed

Table 3. Characteristics of quaternion-based least-squares methods

	Accuracy	Computation burden
QULS	high	less
QCLS	highest	more
QACLS	highest	moderate

4. Conclusions

This paper presented the performance evaluation of a new quaternion-based least-squares method for three-axis attitude estimation of a rigid body with vector measurements from GPS antenna array. Simulation results showed that the new algorithm provides the accuracy comparable to that of QUEST-based methods in various test cases.

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