2001년도 대한전기학회 하계학술대회 논문집 2001.7.18-20

연속형 상태 방정식에 대한 최소최대 필터

권욱현, 한수희
서울대학교 전기·컴퓨터 공학부

Minimax Filter for Continuous-Time State Space Models

Wook Hyun Kwon, Soo Hee Han
School of Electrical Engineering & Computer Science, Seoul National University

Abstract - In this paper, a new robust deadbeat minimax FIR filter (DMFF) is proposed for continuous-time state space signal models. Linearity, deadbeat property, FIR structure, and independence of the initial state information will be required in advance, in addition to a performance index of the worst case gain between the disturbance and the current estimation error. The proposed DMFF is obtained by directly minimizing a performance index with the deadbeat constraint. The proposed DMFF is represented first in a standard FIR form and then in an iterative form. The DMFF will be shown to be used also for the IIR structure. It is shown that the DMFF is similar in form to the existing receding horizon unbiased FIR filter (RHUFF) with some noise covariances. The former is a deterministic filter, while the latter is a stochastic filter.

1. Introduction

Filters are designed for the identification of unknown, information-bearing parameters or variables in a physical or mathematical model from quantities that can be measured. For the frequency domain based filter design, linear phase filters are often preferred because the original signal can be tracked without distortion. In the time domain based filter design, FIR filter can provide the exact estimation when there are no noises. These filters can be called deadbeat FIR filters which are often sensitive to uncertainties. Among filters, some filters can be designed considering a worst case. The majority of filters considering a worst case are classified as minimax filters for stochastic systems and H_∞ filters for deterministic systems. In this paper, a new deadbeat FIR filter design for deterministic continuous-time systems is suggested considering the worst case. It will turn out that these filters are insensitive and robust to disturbances. FIR filters often depend on information at the initial time of the horizon. The suggested filter will be independent of the initial state information.

2. Deadbeat Minimax FIR Filters

Consider a linear continuous-time state space model with control input: (5)

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t), \quad (5)$$
$$y(t) = Cx(t) + Dw(t) \quad (6)$$

where $w(t)$ is the disturbance. $GD^T = O$ and $DD^T = I$ are satisfied to decouple the system disturbance and the measurement disturbance.

The systems (5) and (6) will be represented on the most recent time interval $[t-T, t]$, called the horizon. The current state $x(t)$ is given by a solution of (5) as follows:

$$x(t) = e^{A(t-\sigma)}x(\sigma) + \int_\sigma^t e^{A(t-\tau)}Bu(\tau)d\tau + \int_\sigma^t e^{A(t-\tau)}Gw(\tau)d\tau, \quad t-T \le \sigma \le t. \quad (7)$$

Therefore, on the horizon $[t-T, t]$, the finite measurements and inputs can be expressed in terms of the state $x(t)$ at the current time t as follows:

$$\begin{aligned} y(\sigma) &= Cx(\sigma) + Dw(\sigma) \\ &= C[e^{A(\sigma-t)}x(t) - \int_\sigma^t e^{A(\sigma-\tau)}Bu(\tau)d\tau \\ &\quad - \int_\sigma^t e^{A(\sigma-\tau)}Gw(\tau)d\tau] + Dw(\sigma). \end{aligned} \quad (8)$$

The output $y(\sigma)$ and the integral term including input $u(\tau)$ are assumed to be known. Hence, known and unknown parts can be separated as

$$y(\sigma) + C\int_\sigma^t e^{A(\sigma-\tau)}Bu(\tau)d\tau = C[e^{A(\sigma-t)}x(t) - \int_\sigma^t e^{A(\sigma-\tau)}Gw(\tau)d\tau] + Dw(\sigma).$$

Using known variables, the DMFF for the current state $x(t)$ can be expressed as a linear functional of the finite measurements and inputs on the horizon $[t-T, t]$ as follows:

$$\hat{x}(t|t) = \int_{t-T}^{t} H(t-\sigma)[y(\sigma) + C\int_{\sigma}^{t} e^{A(\sigma-\tau)}Bu(\tau)d\tau]d\sigma$$
$$= \int_{t-T}^{t} H(t-\sigma)y(\sigma)d\sigma + \int_{t-T}^{t} L(t-\sigma)u(\sigma)d\sigma \quad (9)$$

where

$$L(t-\sigma) = \int_{t-T}^{\sigma} H(t-\tau)Ce^{A(\tau-\sigma)}Bd\tau. \quad (10)$$

Note that $H(t-\sigma)$ and $L(t-\sigma)$ are gain matrices of a linear filter. It is noted that the filter defined in (9) is an FIR structure without any a priori statistical information on the horizon initial state $x(t-T)$. The gain matrix $H(t-\sigma)$ will be designed such that $\hat{x}(t|t)$ is an deadbeat estimation filter of the current state $x(t)$ as

$$\hat{x}(t|t) = \int_{t-T}^{t} H(t-\sigma)[y(\sigma) + C\int_{\sigma}^{t} e^{A(\sigma-\tau)}Bu(\tau)d\tau]d\sigma$$
$$= \int_{t-T}^{t} H(t-\sigma)[Ce^{A(\sigma-t)}x(t) - C\int_{\sigma}^{t} e^{A(\sigma-\tau)}Gw(\tau)d\tau + w(\sigma)]d\sigma$$

If there are no disturbances,

$$\hat{x}(t|t) = \int_{t-T}^{t} H(t-\sigma)Ce^{A(\sigma-t)}x(t)d\sigma.$$

In order for $\hat{x}(t|t) = x(t)$, the following constraint on $H(t-\sigma)$ is required:

$$\int_{t-T}^{t} H(t-\sigma)Ce^{A(\sigma-t)}d\sigma = I \quad (11)$$

which will be called the deadbeat constraint. It is noted that constraint (11) must hold regardless of the information on the horizon initial state $x(t-T)$ on the horizon $[t-T, t]$. This constraint may be too strict, but surprisingly, we were able to obtain the solution.

The objective now is to obtain the best gain matrix $H_B(t-\sigma)$, subject to the deadbeat constraint (11), based on the following criterion:

$$H_B(t-\sigma) = \arg\min_{H(t-\sigma)} \max_{w(\cdot)\neq 0} \left\{ \frac{[x(t)-\hat{x}(t|t)]^T[x(t)-\hat{x}(t|t)]}{\int_{t-T}^{t} w^T(\tau)Fw(\tau)d\tau} \right\}. \quad (12)$$

To solve the above state estimation problem with deadbeat constraint, an optimization problem with constraints will be introduced. It will be shown that the constraints consist of an algebraic equation and a differential equation.

Replacing $y(\sigma)$ with the right side of (8), we have the estimate as follows:

$$\hat{x}(t|t) = \int_{t-T}^{t} H(t-\sigma)[C\{e^{A(\sigma-t)}x(t) - \int_{\sigma}^{t} e^{A(\sigma-\tau)}Bu(\tau)d\tau$$
$$- \int_{\sigma}^{t} e^{A(\sigma-\tau)}Gw(\tau)d\tau\} + Dw(\sigma)$$
$$+ C\int_{\sigma}^{t} e^{A(\sigma-\tau)}Bu(\tau)d\tau]d\sigma.$$

Using the deadbeat constraint (11) and rearranging the terms, the error between the real current state and the estimate can be expressed as

$$x(t) - \hat{x}(t|t) = \int_{t-T}^{t} H(t-\sigma)\{C\int_{\sigma}^{t} e^{A(\sigma-\tau)}Gw(\tau)d\tau - Dw(\sigma)\}d\sigma \quad (14)$$

In solving for $H(t-\sigma)$, it will be convenient to define $H(t-\sigma)$ consisting of the row vector $h_i^T(t-\sigma)$ for $1 \leq i \leq n$ as

$$H(t-\sigma) = \begin{bmatrix} h_1^T(t-\sigma) \\ h_2^T(t-\sigma) \\ h_3^T(t-\sigma) \\ \vdots \\ h_n^T(t-\sigma) \end{bmatrix} \quad (15)$$

Then, the error of the i-th state $x_i(t)$ can now be expressed in terms of the vector components of $H(t-\sigma)$ as follows:

$$\{x_i(t) - \hat{x}_i(t|t)\}^2$$
$$= [\int_{t-T}^{t}\{\int_{t-T}^{\sigma} h_i(t-\tau)Ce^{A(\tau-\sigma)}Gd\tau - D\}w(\sigma)d\sigma]^2.$$

By the Causchy-Schwartz inequality, the following relation is obtained:

$$\frac{\{x_i(t) - \hat{x}_i(t|t)\}^2}{\int^{t_{t-T}} w^T(\tau)w(\tau)d\tau} \leq$$
$$[\int_{t-T}^{t}\{\int_{t-T}^{\sigma} h_i(t-\tau)Ce^{A(\tau-\sigma)}Gd\tau - D\}^2 d\sigma].$$

Thus, note that an equality is satisfied for some $w(\tau)$ which is linearly dependent on an error. So,

$$\max_{w(\cdot)\neq 0} \frac{\{x_i(t) - \hat{x}_i(t|t)\}^2}{\int^{t_{t-T}} w^T(\tau)w(\tau)d\tau}$$
$$= [\int_{t-T}^{t}\{\int_{t-T}^{\sigma} h_i(t-\tau)Ce^{A(\tau-\sigma)}Gd\tau - D\}^2 d\sigma].$$

Rearranging the terms, we obtain the somewhat simplified form requiring only minimization as follows:

$$\min_{h_i(t-\sigma), f(\sigma)} \int_{t-T}^{t} f_i^T(\sigma)GG^T f_i(\sigma)d\sigma + \int_{t-T}^{t} h^{T_i}(t-\sigma)h_i(t-\sigma)d\sigma$$

subjec to

$$\dot{f}_i^T(\sigma) = h^{T_i}(t-\sigma)C - f^{T_i}(\sigma)A, f_i^T(t) = e^{T_i}$$

where

$$f_i(\sigma) = \int_{t-T}^{\sigma} e^{A^T(\tau-\sigma)}C^T h_i(t-\tau)d\tau, \ f_i(t-T) = 0$$

and e_i is the i th unit vector such that $e_i=[0,\cdots,0,1,0,\cdots,0]^T$ with the nonzero element in the i th position. To solve the above optimization problem, the key theorem is introduced.

theorem 1

To extremize the integral

$$I=\int_{t_1}^{t_2} f(x_1,x_2,\cdots,\dot{x}_1,\dot{x}_2,\cdots,t)dt \quad (16)$$

with respect to the continuously differentiable functions $x_1,x_2,\cdots$ which achieve the prescribed values $t=t_1$ and $t=t_2$, and satisfy the given equation $G(x_1,x_2,\cdots,\dot{x}_1,\dot{x}_2,\cdots,t)=0$, the following differential equations must be satisfied

$$\frac{\partial F}{\partial x_i}-\frac{d}{dt}\left(\frac{\partial F}{\partial \dot{x}_i}\right)=0 \quad (17)$$

where

$$F(x_1,\cdots,\dot{x}_1,,\cdots,t)=f(x_1,\cdots,\dot{x}_1,\cdots,t) +\lambda(t)G(x_1,\cdots,\dot{x}_1,\cdots,t)$$

for function $\lambda(t)$ which is determined to extremize I.

To use the result of Theorem (1), define the following notation:

$$F(\sigma)=f_i^T(\sigma)GG^Tf_i(\sigma)+h^T(t-\sigma)h_i(t-\sigma) +\lambda_i^T(\dot{f}_i(\sigma)-C^Th_i(t-\sigma)+A^Tf_i(\sigma)).$$

Applying the theorem (1), we need to calculate the following value:

$$\frac{F}{\partial f_i} = 2GG^Tf_i(\sigma)+A\lambda_i(\sigma) \quad (18)$$

$$\frac{F}{\partial \dot{f}_i} = \lambda_i(\sigma) \quad (19)$$

$$\frac{F}{\partial h_i} = 2h_i(t-\sigma)-C\lambda_i(\sigma). \quad (20)$$

From equation (17), we can obtain $f_i(\sigma)$ in a Hamiltonian matrix form

$$\frac{d}{d\sigma}\begin{bmatrix} f_i(\sigma) \\ \lambda_i(\sigma)\end{bmatrix}=\begin{bmatrix} -A^T & \frac{1}{2}C^TC \\ 2GG^T & A\end{bmatrix}\begin{bmatrix} f_i(\sigma) \\ \lambda_i(\sigma)\end{bmatrix}=H\begin{bmatrix} f_i(\sigma) \\ \lambda_i(\sigma)\end{bmatrix}$$

From (20), $h_i(t-\sigma)$ is of the form

$$\begin{aligned} h_i(t-\sigma) &= \frac{1}{2}C\lambda_i(\sigma) \\ &= \frac{1}{2}C[\,0 \;\; I\,]e^{H(\sigma-t+T)}\begin{bmatrix} f_i(t-T) \\ \lambda_i(t-T)\end{bmatrix}\end{aligned}$$

Using $f_i(t-T)=0$ and $e^{H(\sigma-t+T)}$ defined by

$$e^{H(\sigma-t+T)}=\begin{bmatrix} X(\sigma-t+T) & Y(\sigma-t+T) \\ Z(\sigma-t+T) & W(\sigma-t+T)\end{bmatrix} \quad (21)$$

$h_i(t-\sigma)$ can be expressed as

$$h_i(t-\sigma)=\frac{1}{2}CW(\sigma-t+T)\lambda_i(t-T).$$

Using the deadbeat condition

$$\int_{t-T}^{t}\frac{1}{2}\lambda_i^T(t-T)W^T(\sigma-t+T)C^TCe^{A(\sigma-t)}d\sigma=e_i^T,$$

we have

$$\lambda_i^T(t-T)=e_i^T\left[\int_{t-T}^{t}\frac{1}{2}W^T(\sigma-t+T)C^TCe^{A(\sigma-t)}d\sigma\right]^{-1}$$

where the inverse exists as follows. By tedious calculation using (21),

$$\int_{t-T}^{t}\frac{1}{2}W^T(\sigma-t+T)C^TCe^{A(\sigma-t)}d\sigma$$

can be replaced by $Y^T(T)$. If (A,C) is observable, $Y(T)$ is guaranteed to be nonsingular.

theorem 2

The DMFF for the observable system (5) and (6) can be expressed as

$$\hat{x}(t|t)=\int_{t-T}^{t}H(t-\sigma)y(\sigma)d\sigma+\int_{t-T}^{t}L(t-\sigma)u(\sigma)d\sigma$$

where $H(t-\sigma)$ and $L(t-\sigma)$ are as follows:

$$H(t-\sigma)=\left[\int_{t-T}^{t}W^T(\tau-t+T)C^TCe^{A(\tau-t)}d\tau\right]^{-1} W^T(\sigma-t+T)C^T$$

and

$$L(t-\sigma)=\int_{t-T}^{\sigma}H(t-\tau)Ce^{A(\tau-\sigma)}Bd\tau$$

where $W(\tau-t+T)$ is given by (21).

It is surprising that there exists a closed form solution even under the strong condition (11) and that the gain $H(\cdot)$ is independent of the initial state information.

[참 고 문 헌]

[1] Sergio Verdo and H. Vincent Poor "Minimax L Observers and Regulators for Stochastic Systems Uncertain Second-Order Statistics", IEEE Trans. Contr, AC-29 no. 6, pp.499-510, 1984.

[2] J. C. Darragh and D. P. Looze, "Noncausal M Linear State Estimation for Systems wiht Unc Second-order Statistics", IEEE Trans. Auto. Contr, no. 6, pp.555-557, 1984.

[3] Arthur J. Krener, "kalman-Bucy and Mi Filtersing", IEEE Trans. Auto. Contr, AC-25 n pp.291-292, 1980.

[4] M. Fu, C. E. de Souza,, "H_∞ estimation for un systems ", Int. J. Robust and Nonliner Contr., vol 87-105, 1992

[5] S. H. Han, W. H. Kwon, and P. S. Kim "New de Minimax Filters for Deterministic State Space M without the Requirement of A Priori Initial Information", Submitted to IEEE Trans. Automatic. 2001