2 자유도 헬리콥터 시스템의 제어를 위한 퍼지 모델 기반 제어기

장욱*, 이회재*, 주영훈**, 박진배*
* 연세대학교 전기및전자 공학과 ** 군산대학교 전자정보공학부

A fuzzy-model-based controller for a helicopter system with 2 degree-of-freedom in motion

Wook Chang*, Ho Jae Lee*, Young Hoon Joo**, Jin Bae Park*
* Dept. of Electrical and Electronic Eng., Yonsei Univ., Seodaemoonku, Seoul 120-749, Korea Tel: +82-2-2123-2773; Fax: +82-2-362-4539
School of Electronics & Information Eng., Kunsan Univ., Kunsan, Chonbuk, 573-701, Korea Tel: +82-63-469-4706; Fax: +82-63-469-4706

Abstract

This paper deals with the control of a nonlinear experimental helicopter system by using the fuzzy-model-based control approach. The fuzzy model of the experimental helicopter system is constructed from the original nonlinear dynamic equations in the form of an affine Takagi-Sugeno (TS) fuzzy system. In order to design a feasible switching-type fuzzy-model-based controller, the TS fuzzy system is converted to a set of uncertain linear systems, which is used as a basic framework to synthesize the fuzzy-model-based controller.

1 Introduction

This paper deals with the fuzzy control of an experimental helicopter system in our control engineering laboratory. The research and experimental works are based on the experimental helicopter system with 2 degree-of-freedom (DOF) in motion, which is manufactured by Quanser Inc. [1]. Although this experimental helicopter system is not a real system but a simplified one, it contains most main characteristics of the real helicopter system, i.e., intrinsic instabilities, highly nonlinearities, and the cross-coupled dynamics. Due to these difficulties, for a control engineer, the control of the experimental helicopter system is difficult yet challenging at the technical standpoint and mentally taxing.

2 Fuzzy Modeling of the Experimental Helicopter System

The experimental helicopter system consists of a main body of the helicopter which is equipped with two DC motors and a base which supports the main body of the helicopter. The helicopter system has two propellers driven by DC motors. The main propeller and the tail propeller are used to control the pitch angle and the yaw angle of the main body of the experimental helicopter system, respectively. Spatial motions with 2 DOF are measured as relative degrees from the initial values using two encoders which are attached on the main body and the base, respectively.

$$\begin{cases} J_p\ddot{p}(t) + B_p\dot{p}(t) = R_pF_p(V_p(t)) - M_eg(h\sin(p(t)) \\ \qquad\qquad +R_a\cos(p(t))) + G_p(\tau_y(V_y(t)), p(t)) \\ J_y\ddot{y}(t) + B_y\dot{y}(t) = R_yF_y(V_y(t)) + G_y(\tau_p(V_p(t))) \end{cases} \tag{1}$$

where $p(t)$ is the pitch angle relative to tho horizontal axis and $y(t)$ is the yaw angle, in which reference is not relevant and the positive direction is definod by the counter clockwise (CCW) looking from the above view. Let the state vector and input vector be $x(t) = \begin{bmatrix} p(t) & y(t) & \dot{p}(t) & \dot{y}(t) \end{bmatrix}^t$ and $u(t) = \begin{bmatrix} V_p(t) & V_y(t) \end{bmatrix}^t$, then the TS fuzzy system can be constructed as follows:

Plant Rules:

$$R^1 : \text{ IF } x_1(t) \text{ is about } -M, \text{ THEN } \dot{x}(t) = A_1x(t) + B_1u(t) + d_1$$
$$R^2 : \text{ IF } x_1(t) \text{ is about } M, \text{ THEN } \dot{x}(t) = A_2x(t) + B_2u(t) + d_2 \tag{2}$$

where,

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{M_ega}{J_p} & 0 & -\frac{B_p}{J_p} & 0 \\ 0 & 0 & 0 & -\frac{B_y}{J_y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -18.8361 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{M_egb}{J_p} & 0 & -\frac{B_p}{J_p} & 0 \\ 0 & 0 & 0 & -\frac{B_y}{J_y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -12.5030 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{R_pK_{pg}}{J_p} & -\frac{K_{py}}{J_p} \\ -\frac{K_{yp}}{J_y} & \frac{R_yK_{yg}}{J_y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 4.9708 & -0.3257 \\ -0.8486 & 2.4703 \end{bmatrix},$$

$$d_1 = \begin{bmatrix} 0 \\ 0 \\ -\frac{M_ega\beta}{J_p} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -13.5828 \\ 0 \end{bmatrix}, \quad d_2 = \begin{bmatrix} 0 \\ 0 \\ -\frac{M_egb\beta}{J_p} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -9.003 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix};$$

and the membership functions are

$$\Gamma_1(x_1(t)) = \frac{a\sin(x_1(t)+\beta) - b(x_1(t)+\beta)}{(a-b)(x_1(t)+\beta)},$$
$$\Gamma_2(x_1(t)) = \frac{a(x_1(t)+\beta) - a\sin(x_1(t)+\beta)}{(a-b)(x_1(t)+\beta)}.$$

The state-space equation of the TS fuzzy system (2) of the experimental helicopter system is denoted by

$$\dot{x}(t) = \sum_{i=1}^{2} \eta_i(z(t))((A_i + D_i F_i(t) E_{i1})x(t) + B_i u(t)) + d(\mu(z(t))), \tag{3}$$

where $D_1 = D_2 = \begin{bmatrix} 0 & 0 & 0.6291 & 0 \end{bmatrix}^t$, $E_{11} = E_{21} = \begin{bmatrix} 0.0663 & 0 & 0 & 0 \end{bmatrix}$, and $F_i(t)^t F_i(t) \le I$, $i = 1, 2$.

3 Fuzzy Controller Design

Throughout this paper, the reference signal to be tracked is the output $r(t)$ generated by the exogenous system given below:

$$\dot{w}(t) = Fw(t) \tag{4}$$

$$r(t) = Gw(t) \tag{5}$$

where $w(t) \in \Re^k$ is the state of the exogenous system and $r(t) \in \Re^p$ is the output to be tracked by the output of the TS fuzzy system (2). It is assumed that $w(t)$ is uniformly bounded for practical reasons.

Problem 1 *Determine a TS fuzzy-model-based control input $u(t)$ which stabilizes the plant described by the affine TS fuzzy system (2) and tracks the reference signal vector $r(t)$ such that the tracking error*

$$\begin{aligned} e(t) &:= y(t) - r(t) \\ &= Cx(t) - Cw(t) \end{aligned} \tag{6}$$

goes to zero as $t \to \infty$.

First, consider the following non-affine TS fuzzy system, in which the offset terms in consequent parts of the fuzzy rules are ignored:

Rule i

IF $z_1(t)$ is F_1^i and $\cdots$ and $z_n(t)$ is F_n^i

$$\text{THEN} \quad \begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) \\ y(t) &= Cx(t) \end{aligned}, \quad i = 1, 2, \cdots q, \tag{7}$$

The case of the affine TS fuzzy system will be discussed later in this section.

In order to achieve the control objective, here, we establish an error dynamic system. A new state vector is defined as,

$$\zeta(t) := x(t) - T_i w(t), \quad i = 1, 2, \ldots, q \tag{8}$$

where T_i is a solution to the following matrix equations:

$$\begin{bmatrix} A_i & B_i \\ C & 0 \end{bmatrix} \begin{bmatrix} T_i \\ L_i \end{bmatrix} = \begin{bmatrix} T_i & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} F \\ G \end{bmatrix}, \quad i = 1, 2, \ldots, q \tag{9}$$

The matrix equations (9) are assumed to be solvable. To solve (9), following assumption must be satisfied [2].

Assumption 1 *Assume that*

$$\text{rank} \begin{bmatrix} A_i & B_i \\ C & 0 \end{bmatrix} = n + p \tag{10}$$

so that the matrix equations (9) is solvable. This assumption is satisfied if each nominal subsystem in the set of uncertain linear systems (3) is controllable and the number of outputs is less than or equal to the number of the inputs, i.e., $p \le m$.

Assuming that T_i and L_i have been found to satisfy (9), consider the switching-type fuzzy-model-based tracking control law of the following form:

$$u(t) = \sum_{i=1}^{q} \eta_i(z(t))(L_i w(t) + K_i \zeta(t)) \tag{11}$$

where K_i remains still to be found.

Using the newly defined state $\zeta(t)$, the global dynamics of the closed-loop system of (7) with the control law (11) is easily obtained and given by

$$\begin{aligned} \dot{\zeta}(t) &= \sum_{i=1}^{q} \eta_i(z(t)) (A_i + B_i K_i + D_i F_i(t) (E_{i1} + E_{i2} K_i)) \zeta(t), \\ e(t) &= C\zeta(t). \end{aligned} \tag{12}$$

Associated with (12), define the cost function as follows:

$$J = \int_0^\infty \zeta^t(t) Q \zeta(t) dt. \tag{13}$$

The main result on the tracking of the non-affine TS fuzzy system (7) with the fuzzy-model-based controller (11) is summarized in the following theorem.

Theorem 2 *If there exist symmetric positive definite matrices, W_i, a symmetric positive definite matrix, Q, and matrices, M_i such that the following LMIs are satisfied, then the TS fuzzy system (7) is asymptotically stable, and the output of TS fuzzy system (7) can track the reference signals $r(t)$ via the switching-type fuzzy-model-based controller (11) with the guaranteed-cost (13).*

$$\begin{bmatrix} \Psi_i & * & * & * \\ W_i & -Q^{-1} & * & * \\ E_{i1}W_i + E_{i2}M_i & 0 & -\epsilon_i I & * \\ D_i^t & 0 & 0 & -\epsilon_i^{-1} I \end{bmatrix} < 0, \quad i = 1, 2, \ldots, q \tag{14}$$

where

$$\Psi_i = W_i A_i^t + A_i W_i + M_i^t B_i^t + B_i M_i,$$

and $W_i = P_i^{-1}$, $M_i = K_i P_i^{-1}$, and $*$ denotes the transposed elements in the symmetric positions.

Proof: It follows directly from Theorem 1 in [3] with setting $\Delta \mathbf{A}_i = 0$, and $\Delta \mathbf{B}_i = 0$.

Next, let us define the integral of the state which is generated by the following differential equation:

$$\dot{x}_I(t) = y(t) = Cx(t) \tag{15}$$

The augmented state-space representation is constructed by appending the integral of the state $x_I(t)$ to the state $x(t)$. The augmented state equation of of the affine TS fuzzy system (2) is then

$$\dot{x}_a(t) = \sum_{i=1}^{q} \mu_i(z(t))(A_{ai}x_a(t) + B_{ai}u_a(t) + d_{ai}), \tag{16}$$

$$y_a(t) = C_a x_a. \tag{17}$$

where,

$$x_a(t) = \begin{bmatrix} x(t) \\ x_I(t) \end{bmatrix}, \quad A_{ai} = \begin{bmatrix} A_i & 0_{n\times p} \\ C & 0_{p\times p} \end{bmatrix},$$

$$B_{ai} = \begin{bmatrix} B_i \\ 0_{p\times m} \end{bmatrix}, \quad d_{ai} = \begin{bmatrix} d_i \\ 0_{p\times 1} \end{bmatrix}, \quad C_a = \begin{bmatrix} C & 0_{p\times p} \end{bmatrix}.$$

$$\dot{x}_a(t) = \sum_{i=1}^{q} \eta_i(z(t))((A_{ai} + D_{ai}F_{ai}(t)E_{ai1})x_a(t) + (B_{ai} + D_{ai}F_{ai}(t)E_{ai2})u_a(t) + d_a(\mu(z(t)))), \tag{18}$$

Consider the following state vector

$$\zeta_a(t) = x_a(t) - T_{ai}w(t), \quad i = 1, 2, \cdots, q \tag{19}$$

where T_{ai} is a solution to the following matrix equations

$$\begin{bmatrix} A_{ai} & B_{ai} \\ C_a & 0 \end{bmatrix} \begin{bmatrix} T_{ai} \\ L_i \end{bmatrix} = \begin{bmatrix} T_{ai} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} F \\ G \end{bmatrix}, \quad i = 1, 2, \cdots, q$$

and

$$T_{ai} = \begin{bmatrix} T_i \\ T_{Ii} \end{bmatrix}$$

Here, T_i is a solution of the equations in (9) and T_{Ii} is obtained by solving

$$CT_i = T_{Ii}F, \quad i = 1, 2, \cdots, q$$

Now, consider the switching-type integral fuzzy-model-based control law of the form

$$u_a(t) = \sum_{i=1}^{q} \eta_i(z(t))(L_i w(t) + K_{ai}\zeta_a(t)) \tag{20}$$

Using (20), the controlled system is given by

$$\dot{\zeta}_a(t) = \sum_{i=1}^{q} \eta_i(z(t))(A_{ai} + B_{ai}K_{ai} + D_{ai}F_{ai}(t)(E_{ai1} + E_{ai2}K_{ai}))\zeta_a(t) + d_a(\mu(z(t))) \tag{21}$$

$$e = C_a\zeta_a(t) = C\zeta(t)$$

where $\dot{\zeta}_a(t) = \dot{x}_a(t) - T_{ai}\dot{w}(t) = \begin{bmatrix} \dot{x} \\ \dot{x}_I \end{bmatrix} - \begin{bmatrix} T_i \\ T_{Ii} \end{bmatrix} Fw(t) - \begin{bmatrix} \dot{\zeta}(t) \\ Cx(t) - r(t) \end{bmatrix} \triangleq \begin{bmatrix} \dot{\zeta}(t) \\ \dot{e}_I(t) \end{bmatrix}, \quad z \in S_i$.

Theorem 3 *If there exist symmetric positive definite matrices, W_{ai}, a symmetric positive definite matrix, Q_a, and matrices, M_i such that the following LMIs are satisfied, then the affine TS fuzzy system (2) is uniformly stable, and the output of TS fuzzy system (2) can track the reference signals $r(t)$ via the switching-type fuzzy-model-based controller (20) with the guaranteed-cost $J_a = \int_0^\infty \zeta_a(t)^t Q_a \zeta_a(t) dt$ and the small error variations.*

$$\begin{bmatrix} \Psi_i & * & * & * \\ W_{ai} & -Q_a^{-1} & * & * \\ E_{ai1}W_{ai} + E_{ai2}M_{ai} & 0 & -\epsilon_i I & * \\ D_{ai}^t & 0 & 0 & -\epsilon_i^{-1} I \end{bmatrix} < 0, \tag{22}$$

$$i = 1, 2, \ldots, q$$

where

$$\Psi_i = W_{ai}A_{ai}^t + A_{ai}W_{ai} + M_{ai}^t B_{ai}^t + B_{ai}M_{ai},$$

*and $W_{ai} = P_{ai}^{-1}$, $M_i = K_{ai}P_{ai}^{-1}$, and * denotes the transposed elements in the symmetric positions.*

4 Conclusions

In this paper, we have discussed the modeling and control of the experimental helicopter system with 2 DOF in motion by using the fuzzy-model-based control approach. The fuzzy model of the experimental helicopter system is derived from the original nonlinear dynamic equations, which is represented as the affine TS fuzzy system. The obtained TS fuzzy systems is further rearranged to give a set of uncertain linear systems. Based on the developed alternative representation technique, the output tracking problem as well as the stabilization problem are formulated in the LMI format and solved by a convex optimization technique. During the design procedure, a control strategy for the affine TS fuzzy system was also rigorously studied and proved that admissible control performance is guaranteed.

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