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Analytics of PIV Measurement and Its Application for Higher Performances

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Abstract Present paper describes the principles of PIV measurement approaching from the analytical view, which enables to explain the general form of principles covering all the PIV measurement, and that gives theoretical basis for its higher measurement performances. The explanation of the measurement principles started from the definition of governing equation in differential form as same as the gradient method, and the integral along the particle path line was executed to show the principle of the correlation method with same basis. The integral processes clearly shows the analytical reason why the correlation peak gives the terminal point of path line, and how the effects of deformation and rotation of fluid appears in the correlation map. These results have no differences from our experiences and understandings of the conventional PIV measurement definition in final form. However, the analytical approach enable to understand those facts a priori, and it makes easy to achieve the innovative higher performances of measurement. Analytical explanation clearly shows the behavior of the residual errors caused by the fluid motion, and it enables to analyze the measurement uncertainty theoretically.

1. Introduction

The correlation method has been a most popular and conventional algorithm of PIV measurement, and it is widely used in commercial PIV systems. The origin of the correlation method is the optical approach using speckle-pattern and interference of laser light that is realized in the digital-image processing by Fourier transformation. The basic idea of usual correlation method is the identification of similar particle pattern in two PIV images, which is expressed by the correlation peak. This process works properly so fur as the so-called frozen condition is satisfied, and it gives the robust feature on the method that contributes to the present popularization of PIV.

Recent demands on the higher performance of PIV measurement require considering the detail of image analysis. The sub-pixel analysis is the essential for the accuracy of velocity vector itself. The conventional Gaussian fitting sub-pixel algorithm has been widely used, but the theoretical background is weak and the uncertainty band cannot satisfy the higher demands for its performance. The higher spatial resolution is another feature of highly demanded item. The conventional FFT approach correlation method has the difficulties to increase the spatial resolution with its limitation in frequency domain and the Euler type approach. Several higher resolution techniques have been proposed and the theoretical supported compress of the low error vector rate is essential to those attempts.

The theoretical backbone of measurement is vital for the higher performance of PIV. The deep understandings of the mechanism enable to produce further capability of measurement and to give proper amendments. In this paper, the general explanation on the principles of PIV measurement was made, that covers most of the existing PIV algorithms. The basis of the mathematical approach is the constraint of image, and it shows the mathematical local image transportation in strict form of differential equation, which is usually used as the governing equation of gradient method. The theory has been extended to cover the principle of correlation method with the same basis, and it gives the integral form of governing equation that enables to re-consider the measurement principle analytically. The analytics of PIV measurement has been shown in this paper that produces the idea of new higher performance PIV algorithms, and that shows the basis for consideration of higher order term effects.

2. Measurement principles of differential approach and correlation method

Differential approach to the principle of PIV measurement

PIV systems estimate the flow velocity by detecting the displacement of particle images. The movement of image can be expressed by a differential equation under the assumption of the small time interval and the smooth change of luminance function. The differential form of governing equation has been used as the measurement governing equation of the gradient method. (Horn et al., 1981, Ando, 1986, Okuno, 1991, 1995) Figure 1 shows the concept of image transfer. When a particle image at $P(x-\Delta x, y-\Delta y)$ moves to $P'(x+\Delta x, y+\Delta y)$ in time interval $2\Delta t$, the luminance change of particle image is expressed as follows.

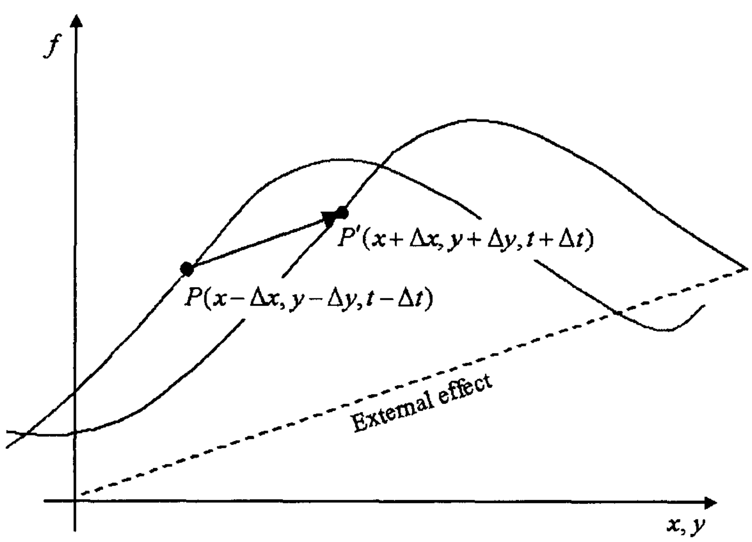


Figure 1 Transfer of particle image luminance.

$$f(x+\Delta x, y+\Delta y) - f(x-\Delta x, y-\Delta y) = \Delta g(x, y, t, \Delta t), \quad (1)$$

where f shows the luminance of image as a function of position (x, y) and time t. g shows the effects of back ground image and variance of illumination distribution, and Δg the difference of two particle image. When the change of luminance is continuous and smooth, the two-luminance values can be expressed by the Taylor's expansion around (x, y, t).

$$f(x+\Delta x, y+\Delta y, t+\Delta t) = f(x,y,t) + \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial t}\Delta t + O(\delta^2). \tag{2}$$

$$f(x-\Delta x, y-\Delta y, t-\Delta t) = f(x,y,t) - \frac{\partial f}{\partial x}\Delta x - \frac{\partial f}{\partial y}\Delta y - \frac{\partial f}{\partial t}\Delta t + O(\delta^2). \tag{3}$$

Substituting Eqs.(2),(3) into (1) and dividing $2\Delta t$, following first order governing equation at finite time interval is obtained.

$$\frac{\partial f}{\partial x}\cdot\frac{\Delta x}{\Delta t} + \frac{\partial f}{\partial y}\cdot\frac{\Delta y}{\Delta t} + \frac{\partial f}{\partial t} = \frac{\Delta g(x,y,t,\Delta t)}{2\Delta t} + O(\delta). \tag{4}$$

The residual terms less than or equal to order δ shows the higher order effects caused by the luminance gradient change in finite time interval and displacement. (Nishio et al, 1992) Figure 2 shows the effects of higher order terms. When the change of luminance function is gentle, the effective area of first order equation becomes large, which shows the dependency of the dynamic range to the spatial frequency of image.

As the definition of $(\Delta x, \Delta y)$, the displacement shows particles image movement droved by the flow speed in the time interval Δt. Then $(\Delta x, \Delta y)$ can also expressed by the Taylor's expansion using the flow velocity (u,v) and its derivatives at $t = t$ as shown in Eqs.(5),(6).

$$\Delta x = u\Delta t + \frac{1}{2}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial u}{\partial t}\right)(\Delta t)^2 + O(\delta^3). \tag{5}$$

$$\Delta y = v\Delta t + \frac{1}{2}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial v}{\partial t}\right)(\Delta t)^2 + O(\delta^3) \tag{6}$$

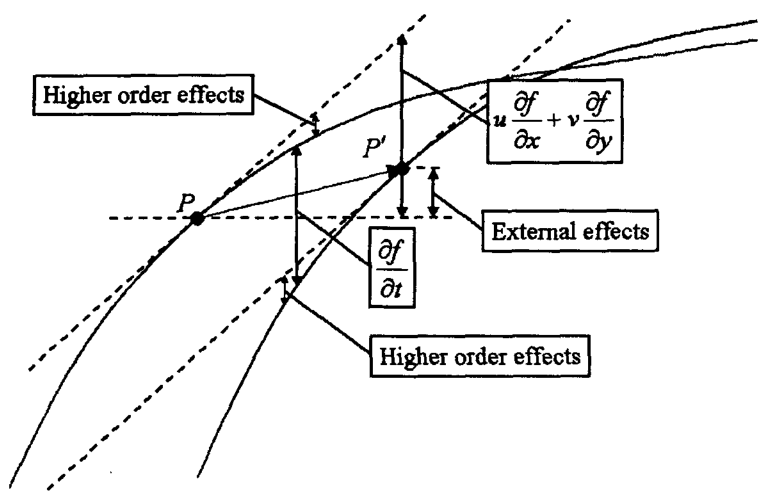


Figure 2 Effects of higher order terms of Taylor's expansion.

Substituting Eqs.(5),(6) into (4) and giving a zero limit to the time interval Δt, following governing equation of measurement in differential equation form is obtained.

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} = \frac{Dg(x,y,t)}{Dt}, \tag{7}$$

where Dg / Dt shows the total luminance differentiation along the transportation direction generated by the external effects such as the illumination, background image and noises. When the effects of background and variance of luminance are regarded as negligible, the governing equation shows the fact that the particle image moves with flow velocity (u,v) without changing its luminance distribution. Equation (7) usually used as the governing equation of gradient method, and the flow velocity (u,v) is obtained by assuming the uniformity of velocity field in local spatio-temporal domain.

Present procedure of the governing-equation definition shows the occurrence of residual error clearly. The higher order terms of governing equation in finite difference form consists of the non-linear components of luminance function change shown by Eqs.(2),(3), and the non-uniformity of velocity field shown by Eqs.(5),(6). The solution of Eq.(7) is usually obtained by the optimum solution of simultaneous equations in local area. Then the higher order effect of Eqs.(2),(3) appeared in the residual error of governing equation itself, and the one of Eqs.(5),(6) affects the solution in finite area that can be regarded as the effects of deformation of image. These higher order considerations can be used for the analysis of uncertainties in measurement.

Correlation method approached from differential equation

The measurement principle of correlation method based on the comparison of two PIV image that is expressed by the correlation function as an example is shown in Eq.(8).

$$C_S = \frac{\int_S f_A \cdot f_B \, dS}{\int_S (f_A^2 + f_B^2) dS}, \tag{8}$$

where f_A, f_B show the luminance of first and second PIV image, and S the so-called correlation area as shown in Fig.3. The correlation peak in search area may give the position of most similar pattern in second image, and it is usually regarded as the transferred position of first particle image. Present measurement principle originated to the optical method analyzing speckle patter by interference fringe, and the same process has been realized in the digital image processing by Fourier transformation. (Willert, C.E. et at, 1990, Keane, R.D. et al, 1992) Equation (8) shows the same process in real space (Kimura et al, 1987), and the way that executes the convolution integral in real space is generally called the direct-correlation method.

As the present method started from the optical method analyzing double exposed image, the basis of the measurement principle is the comparison of two images obtained in a finite time interval. Then the intermediate transportation process along the particle path line has not been considered. The constraint of image is expressed by Eq.(7) in the strict form, and it enables to define the principle of correlation method considering all aspects of image transfer. Equation (7) consists of Lagrange's derivatives of luminance function and the external effects on the image such as the non-uniformity of illumination. It can be used as a cost function evaluating the change of image, and the path line, which satisfy Eq.(7) everywhere on it, gives the correct route of particle transportation in spatio-temporal domain. When path line C is given, a penalty index can be expressed as follows.

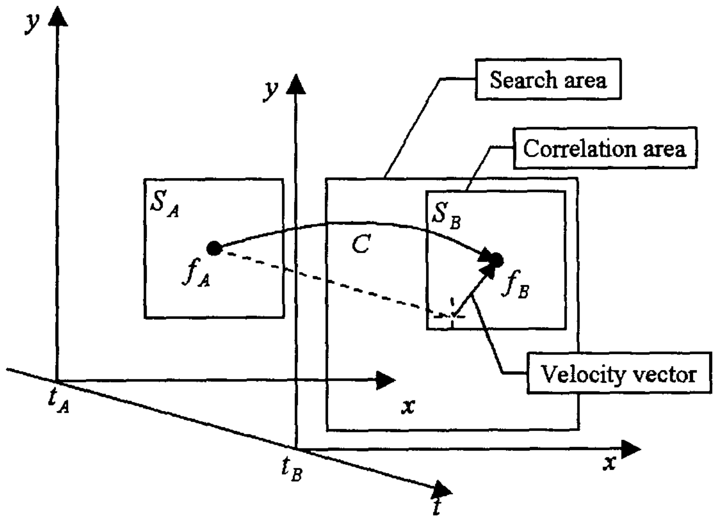


Figure 3 Correlation of PIV image

$$R_i = \left\{ \int_C \left(\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} \right) ds \right\}^2 = \left\{ \int_C \frac{Dg}{Dt} ds \right\}^2 . \tag{9}$$

When the terminal points of path line are expressed by A, B, the integral of constraint equation can be expressed simply by the luminance f_A, f_B and external effects g_A, g_B as follows.

$$\int_C \left(\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} \right) ds = f_B - f_A . \tag{10}$$

$$\int_C \frac{Dg}{Dt} ds = g_B - g_A . \tag{11}$$

Substituting Eqs.(10), (11) into Eq.(9), the penalty index can be expressed as follows.

$$R_i = \{f_B - f_A\}^2 = \{g_B - g_A\}^2 . \tag{12}$$

Equation (11) shows the important fact that the change of the luminance along the streamline is summarized by the differentiation of luminance at the terminals of path line, and it is caused by the external factors. The accumulation of illumination change expressed by Eq.(9) does not depend on the integral path C because the core function can be regarded as a holomorphic function in spatio-temporal domain. This process gives the PIV measurement robustness. On the other hand, if we want to consider the intermediate process in the image time interval, the accumulation process such as Eq.(13) should be considered.

$$R_i' = \int_C \left(\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} \right)^2 ds = \int_C \left(\frac{Dg}{Dt} \right)^2 ds . \tag{13}$$

The accumulation of penalty index over the correlation area gives the correlation coefficient when it is normalized by the norm of luminance.

$$R_S = \int_S \left\{ \int_C \left(\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} \right) ds \right\}^2 dS = \int_S \{f_B - f_A\}^2 dS\,. \tag{14}$$

$$C_S' = \frac{R_S}{\int_S (f_A^2 + f_B^2) dS} = 1 - 2\cdot \frac{\int_S f_A \cdot f_B dS}{\int_S (f_A^2 + f_B^2) dS} = 1 - 2\cdot C_S\,. \tag{15}$$

Then the present approach staring from differential-form constraint equation gives the similar index for image correlation. The conventional definition of correlation C_S gives the correlation peak with its maximum value, and the present one gives it at its minimum value, which has been used in the fast algorithm of correlation method (Kaga et al., 1992).

3. Accumulation in correlation area and the residual errors

Equations (14), (15) shows the main procedures for definition of correlation. When we consider the transportation at single point, the change of the luminance is caused only by the external factors as shown in Eq.(12). However the accumulation processes over correlation area give an excuse to have a blunt peak caused by the internal factors such as the deformation and rotation of image. The integral over correlation area in Eq.(14) is usually executed using parallel path line C, i.e., the assumption of constant velocity, which is called the frozen condition. When the deformation and rotation of image cannot be ignored, the path line difference from the correct one causes additional factors for the bluntness of correlation peak.

The image displacement vector $\mathbf{s}$ is given by Eq.(16).

$$\mathbf{s} = \int_C ds = \int_A^B \mathbf{u}\, dt\,, \tag{16}$$

The velocity vector inside the correlation area can be expressed using Taylor's expansion.

$$\mathbf{u} = \mathbf{u}_0 + \frac{\partial \mathbf{u}}{\partial x}\Delta x + \frac{\partial \mathbf{u}}{\partial y}\Delta y + O(\delta^2)\,, \tag{17}$$

where $\mathbf{u}_0 = (u_0, v_0)$ shows the velocity at the central point of correlation area $\mathbf{x}_0 = (x_0, y_0)$, $\Delta\mathbf{x} = (\Delta x, \Delta y)$ the displacement of off-center position from $\mathbf{x}_0$. The temporal flow field change has been eliminated here for the simplicity of discussion. Then the image displacement vector is given as follows.

$$\begin{aligned} \mathbf{s} &= \int_A^B \left(\mathbf{u}_0 + \frac{\partial \mathbf{u}}{\partial x}\Delta x + \frac{\partial \mathbf{u}}{\partial y}\Delta y + O(\delta^2) \right) dt \\ &= \int_A^B \mathbf{u}_0\, dt + \int_A^B \begin{pmatrix} \varepsilon_x & \gamma_{xy} - \Omega \\ \gamma_{xy} + \Omega & \varepsilon_y \end{pmatrix} \Delta\mathbf{x}\, dt + O(\delta^2), \end{aligned} \tag{18}$$

where $(\varepsilon_x, \varepsilon_y) = \left(\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}\right)$, $\gamma_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$, $\Omega = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$.

On the other hand, the core function of integral in Eq.(14) can be expressed as follows using Eq.(17).

$$
\begin{aligned}
&\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} \\
&= \frac{\partial f}{\partial t} + \left(u_0 + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y\right)\frac{\partial f}{\partial x} + \left(v_0 + \frac{\partial v}{\partial x}\Delta x + \frac{\partial v}{\partial y}\Delta y\right)\frac{\partial f}{\partial y} + O(\delta^2) \\
&= \left(\frac{\partial f}{\partial t} + u_0\frac{\partial f}{\partial x} + v_0\frac{\partial f}{\partial y}\right) + \left(\varepsilon_x\frac{\partial f}{\partial x} + \gamma_{xy}\frac{\partial f}{\partial y} + \Omega\frac{\partial f}{\partial y}\right)\Delta x + \left(\varepsilon_y\frac{\partial f}{\partial y} + \gamma_{xy}\frac{\partial f}{\partial x} - \Omega\frac{\partial f}{\partial x}\right)\Delta y + O(\delta^2),
\end{aligned}
\tag{19}
$$

Substituting Eqs.(18),(19) into Eq.(10), the effect of the internal factors can be expressed clearly as follows.

$$
\begin{aligned}
&\int_C \left(\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y}\right) ds \\
&= \int_{C_0} \left(\frac{\partial f}{\partial t} + u_0\frac{\partial f}{\partial x} + v_0\frac{\partial f}{\partial y}\right) ds + \int_{\Delta C} \left(\frac{\partial f}{\partial t} + u_0\frac{\partial f}{\partial x} + v_0\frac{\partial f}{\partial y}\right) ds \\
&+ \int_{C_0} \left\{\left(\varepsilon_x\frac{\partial f}{\partial x} + \gamma_{xy}\frac{\partial f}{\partial y} + \Omega\frac{\partial f}{\partial y}\right)\Delta x + \left(\varepsilon_y\frac{\partial f}{\partial y} + \gamma_{xy}\frac{\partial f}{\partial x} - \Omega\frac{\partial f}{\partial x}\right)\Delta y\right\} ds + O(\delta^2),
\end{aligned}
\tag{20}
$$

where

$$
\mathbf{s}_0 = \int_{C_0} ds = \int_A^B \mathbf{u}_0 \, dt, \tag{21}
$$

$$
\Delta \mathbf{s} = \int_{\Delta C} ds = \int_A^B \begin{pmatrix} \varepsilon_x & \gamma_{xy} - \Omega \\ \gamma_{xy} + \Omega & \varepsilon_y \end{pmatrix} \Delta \mathbf{x} \, dt. \tag{22}
$$

When the external effects such as the non-uniformity of illumination can be ignored, the accumulated value of Eq.(19) tends to zero. However, the conventional procedure of correlation method uses only the zero-order terms of Eq.(19). Then the internal effects on the correlation peak calculation can be expressed as shown in Eq.(23).

$$
\begin{aligned}
&\int_{C_0} \left(\frac{\partial f}{\partial t} + u_0\frac{\partial f}{\partial x} + v_0\frac{\partial f}{\partial y}\right) ds \\
&= -\int_{\Delta C} \left(\frac{\partial f}{\partial t} + u_0\frac{\partial f}{\partial x} + v_0\frac{\partial f}{\partial y}\right) ds \\
&- \int_{C_0} \left\{\left(\varepsilon_x\frac{\partial f}{\partial x} + \gamma_{xy}\frac{\partial f}{\partial y} + \Omega\frac{\partial f}{\partial y}\right)\Delta x + \left(\varepsilon_y\frac{\partial f}{\partial y} + \gamma_{xy}\frac{\partial f}{\partial x} - \Omega\frac{\partial f}{\partial x}\right)\Delta y\right\} ds + O(\delta^2),
\end{aligned}
\tag{23}
$$

The internal effects of the correlation method consist of the difference of integral path line ΔC and the effects in core function. Eq.(23) shows the first order of the internal effects, and it can be expressed using the image deformation factors $(\varepsilon_x, \varepsilon_y)$, γ_{xy} Ω.

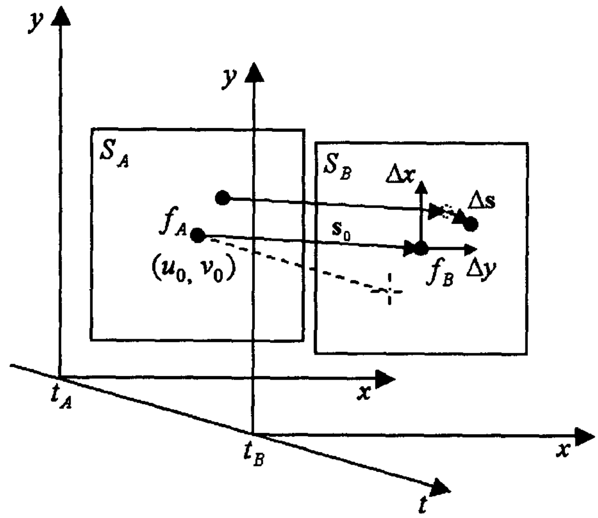


Figure 4 Analytic understand of the PIV image correlation

Following the process of present analytical definition of measurement principles, the higher order terms of correlation method can be summarized to external and internal factors. The external factor $g(x,y,t)$ in Eq.(13) is generated by such as the effects of non-uniformity of illumination, characteristics of light scattering and noises. These are not evaluated in advance usually, and it causes bluntness of correlation peak and uncertainties. The internal factor has been expressed using the basic fluid motion of deformation and rotation; $(\varepsilon_x, \varepsilon_y)$, γ_{xy} Ω. These internal factors also make the correlation peak blunt and the uncertainty band wider. The internal factors have been shown clearly with the connection of fluid motion here, and it will be the basis of the higher performance of PIV measurement algorithm.

4. Applications of mathematical approach

Differential-integral method

The general explanation, which based on the image constraint equation, has been made, and it covers over both the gradient method and the conventional correlation method. It enables us to give the higher performances on PIV measurement.

The differential and integral approaches of PIV measurement can be combined, which enables to utilize the advantages of both technique, and to cancel the disadvantages. (Sugii et at, 2000) The integral approach, the conventional correlation method with offset correlation area, gives the large measurement dynamic range and robustness of its measurement, but needs additional sub-pixel process to obtain reasonable velocity resolution. On the other hand, the differential approach, the so-called spatio-temporal derivative method, enables to get high spatial and vector resolution, but has small dynamic range with its Euler type approach. The authors have proposed a combined algorithm of differential and integral approaches. It based on the constraint equation of Eq.(7) whole through the analyzing process, and has the conventional dynamic range, the high spatial resolution and the extreme sub-pixel accuracy.

Figure 5 shows the image-analysis procedure of present method. The integral approach is basically the same with the conventional correlation method. The direct correlation method has been adopted for the consistency and the efficiency of the algorithm. The correlation coefficients are calculated using the shifted correlation area, and the correlation peak is examined inside the search area. The search area is usually given symmetrically around the center of first image correlation area, but it sometimes became a main factor of error vector occurrence when the image displacement is large. The search area is offset and reduced its size by the adaptive technique in recursive searching process, and it makes efficient applying the differential approach by its low error vector rate. The differential approach is applied as the final process of this algorithm. The gradient method enables to obtain the local solution of constraint equation with fine sub-pixel accuracy. When the correct displacement is obtained in the integral approach, the correlation area size can be reduced, which gives the high spatial resolution performance to the measurement.

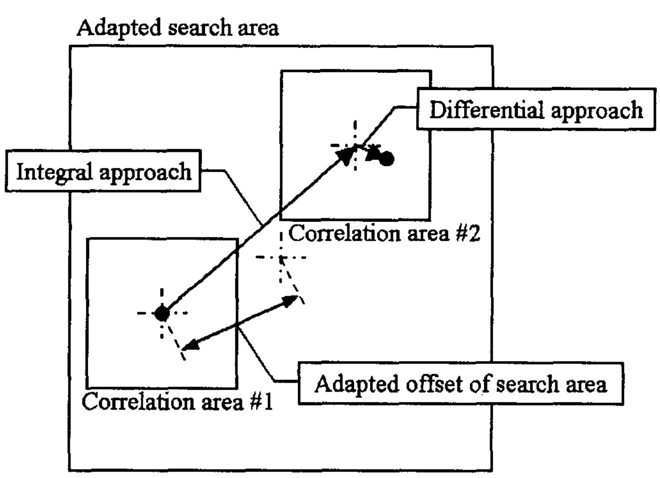


Figure 5 Differential-integral approach

Measurement around waving wing

Two-dimensional wing model was used for the analysis of basic propulsion performance of fish like motion. The NACA0018 wing section was adopted for the present model. Figure 6 shows the side view of a dolphin and present wing section. The slenderness ratio of the wing section, 0.18, is close to it of the major marine animals such as dolphins and whales. The wing section is supported at three quarter of the chord length, and sway and yaw motions are driven around the point. Present wing model consists of five parts connected with four joints. The maximum bending angle of each joint is 30-degree.

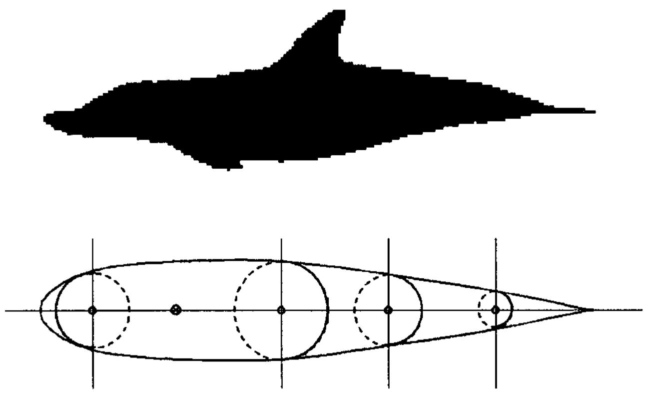


Figure 6 Side view of a dolphin and wing section of the waving wing model

The flow visualization and measurement was carried out at a small circulating water channel whose observation section is 1000mm length, 300mm width and 200mm draft. Figure 7 shows the schematic view of the experimental set-up. The wing model was connected to a linear way, which is driven by two servomotors for yaw and sway motion. The servomotors, which are installed on the wing model, drive the bending motion. Figure 3 shows the total view of the model. The cord length and span of the model are 150mm and 300mm respectively. The servomotors for bending motion should be light with respect its torque because it restricts the motion speed.

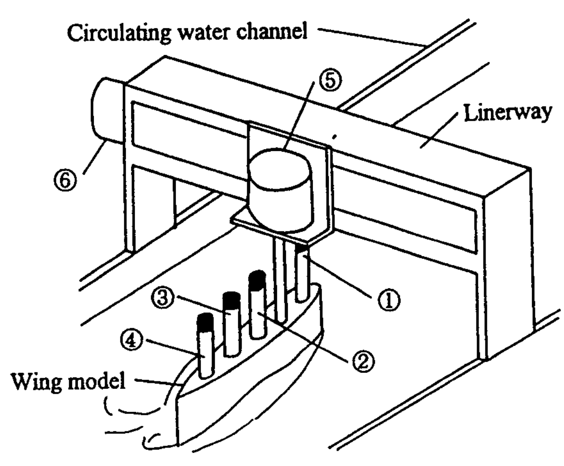


Figure 7 Schematic view of experimental set-up.

Figure 8 shows an example of the application of present method. The velocity field around a waving wing was measured with fine spatial resolution. (Nishio et al, 2000) The spatial interval of the velocity vector is 5 pixel with permitting 50% overlap rate. The instantaneous velocity vector field shows the detail of vortex structure, and it enables to consider the mechanism and the performance of fish-like-motion propulsion.

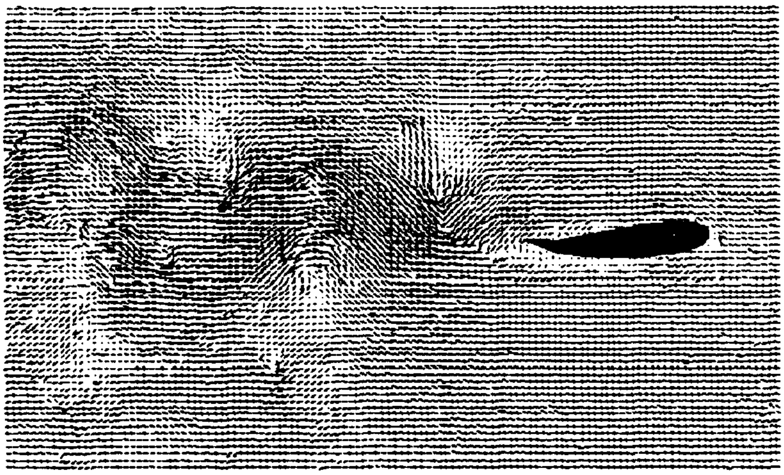
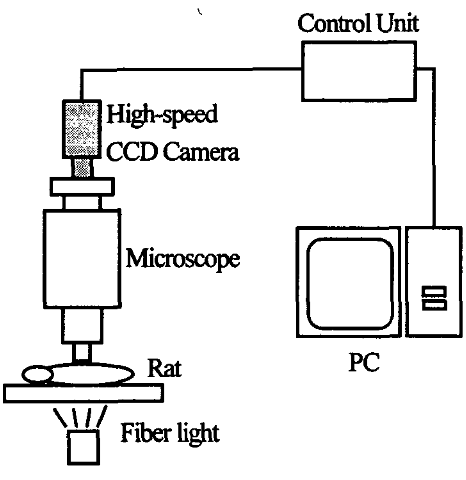


Figure 8 Measured instantaneous flow field around waving wing

Blood flow measurement

The micro scale blood flow of rat is measured using present method. (Sugii et al, 2001) Figure 9 shows the experimental set-up of the blood flow measurement. The arteriole in the rat mesentery was visualized by observing the red blood corpuscles with back light illumination and the intravital-microscope. The visualized image was recorded by the high-speed digital video system, which enables to obtain the appropriate time interval and resolution for the micro scale measurement.



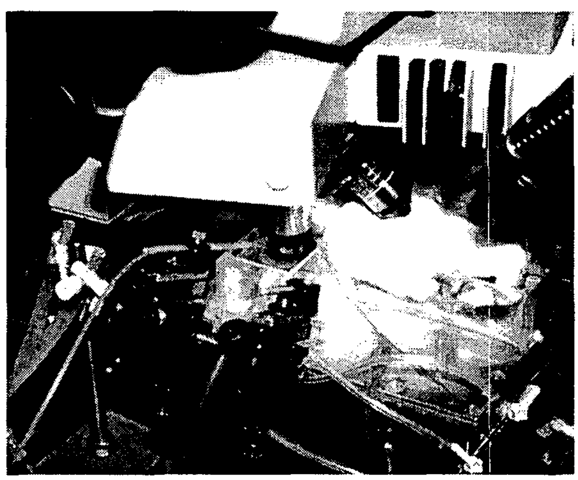


Figure 9 Experimental set-up for blood flow measurement.

Figure 10 shows the original image of blood flow and the velocity vector field measured by the present method. The measured velocity field shows time-averaged field, and the detail of the velocity profile was obtained clearly with its high spatial resolution performance. More than 20 points of velocity data in the cross section was obtained even the diameter was about 30 μm.

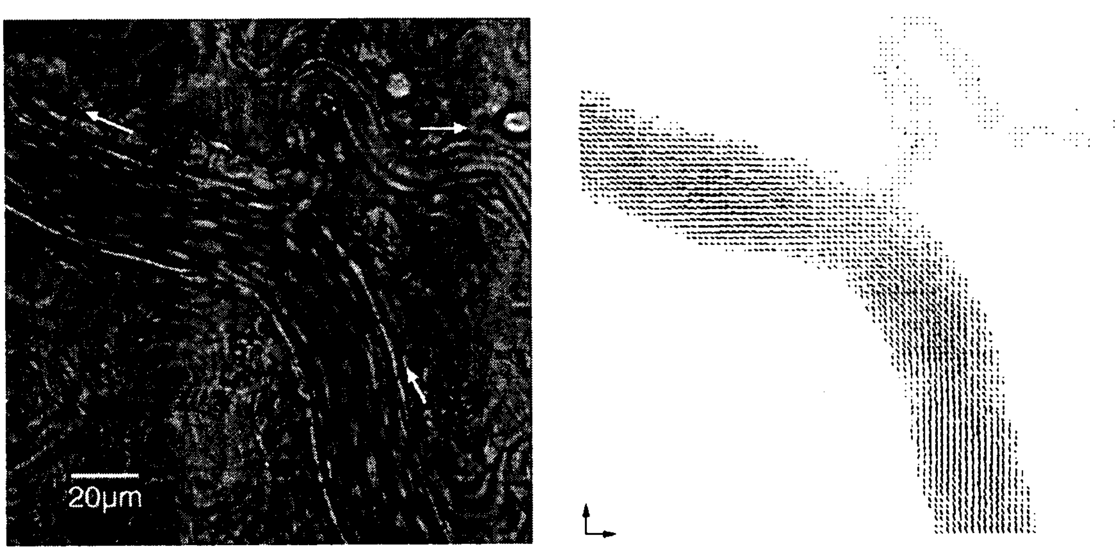


Figure 10 Microscope image and measured velocity field of blood flow

Figure 11 shows the velocity profile of the measured velocity filed. The velocity profile shows the blunt shape compare with the fully developed Newtonian fluid profile. The non-symmetrical profile shows the effects of the centrifugal force.

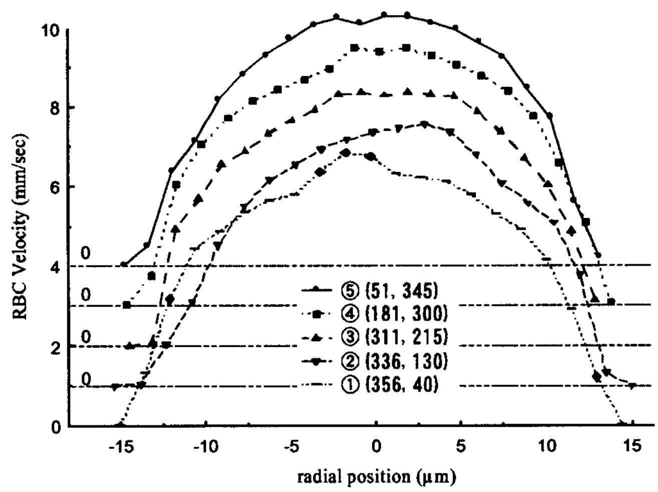


Figure 11 Velocity profile of cross section

5. Conclusions

The principles of PIV measurement were explained by the approach of analytics, and it enables to describe the most of the PIV algorithm with same basis. It is shown that all the behavior of PIV image can be expressed strictly by the constraint equation, which can be expressed by a differential equation. The differential constraint equation has been used as the governing equation of gradient method, and the principle of correlation method was explained using the same basis with considering the process along the particle path theoretically.

The analytical approach shows the background of measurement principle clearly, and it enables to consider the higher order effects, that appeared in the finite differential form of governing equation. These procedures can be used for the consideration of the measurement uncertainties, and be feed back to add the higher performance on the measurement. The fact that the second order of measurement equation can be expressed by the fluid motion such as the deformation and the rotation shows the possibility of the exclusion of its effects from the correlation map.

An application of the present mathematical approach was also shown. The combination of differential approach and integral one produce the high performance of measurement on its accuracy and spatial resolution. The innovative approach enables to analyze the propulsive performance of fish-like motion with fine measured vortex structure and the micro scale blood flow.

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