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Computational modeling of the coronary circulation for the assessment of the coronary artery bypass through left ventricle

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좌심실을 통한 관상동맥 우회술의 평가를 위한 관상순환계의 수치적 모델

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Key Words : Coronary circulation, Coronary bypass surgery, Lumped parameter model

Abstract

In this study we propose the computational model for the coronary circulation. The bypass from left ventricle is also considered. Lumped parameter model with three compartments in the coronary circulation is implemented in this study. We connected the coronary artery compartment with left ventricle to explain the bypass procedure from left ventricle. The asymmetric resistance is assumed in the bypass line from left ventricle. The present numerical method is tested for normal coronary circulation and the results are compared with the existing computational work. The bypass simulation is conducted and the flow pattern is delineated. The effect of shunt resistance and coronary compliance to circulation is investigated for the better design of the bypass shunt.

1. Introduction

About one third of all deaths in the developed world result from coronary artery disease, and most elderly individuals have at least some impairment of the coronary artery circulation[1]. For this reason, the normal and pathological physiology of the coronary circulation is one of the most important subjects in medicine.

Figure 1 shows the heart with its coronary blood supply. The main coronary arteries lie on the surface of the heart and smaller arteries penetrate from the surface into cardiac muscle mass. It is almost entirely through these arteries that the heart receives its nutritive blood supply. The left coronary artery supplies mainly the anterior and lateral portions of left ventricle, whereas the right coronary artery supplies most of the right ventricle as well as the posterior part of the left ventricle in 80 to 90 percent of people. Most of the venous blood flow from the left ventricular capillaries leaves by way of the coronary sinus; this constitutes about 75 percent of total coronary perfusion. Most of the venous blood flow from the right ventricle flows through the small anterior cardiac veins directly into the right atrium. A small amount of coronary blood flows back into the heart through minute thebesian veins, which empty directly into all chambers of the heart.

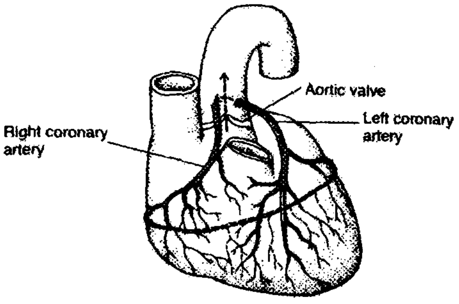

Figure 1 Coronary vessels.

The most common cause of death in the Western world is ischemic heart disease resulting from insufficient coronary blood flow. The most frequent cause of diminished coronary blood flow is

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atherosclerosis. Individuals with a genetic predisposition to atherosclerosis or whose diets include excessive quantities of cholesterol and other fats, are susceptible to the gradual deposition of large quantities of cholesterol beneath the arterial endothelium at particular sites throughout the body. Later, these areas of deposit are invaded by fibrous tissue and frequently become calcified. The net result is the development of atherosclerotic plaques that protrude into the vessel lumen causing progressive blockage of blood flow.

There have been two popular surgical methods to treat the coronary heart disease. One is aortic-coronary bypass Surgery and the other is coronary angioplasty. However, there are still many problems related to these surgical methods. One alternative of the surgical ways is the bypass from left ventricle. This idea has been derived from the reptile's physiology. Reptile has no coronary artery and so they don't suffer from coronary artery disease[2]. However, only a few investigations for the method are available for now.

In this study we present the computational model to simulate the hemodynamics of the surgical way bypssing through left ventricle. Lumped parameter model is used to assess the effects of the shunt design. First we compared our results for normal case with the existing data. We also provide the hemodymnamic results according to the variations of the constriction in the stenosed artery.

2. Computational model of the bypass surgery through left ventricle

The proposed surgical method for coronary artery stenosis introduces a direct connection between the left ventricle and the distal segment of the obstructed artery. For this purpose, a shunt connecting the left ventricle to the coronary artery is inserted in the heart muscle. The schematic diagram of the method is represented in Figure 2.

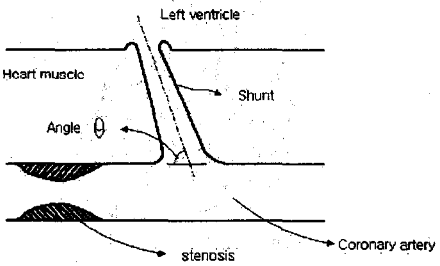


Figure 2. Schematic of the proposed surgical procedure.

To delineate the hemodynamic effect of the bypass surgery we implemented a computational code that can simulate the system dynamics of the cardiovascular system after the coronary bypass surgery. Many critical cases are predicted by this code and we compared these with available experimental data.

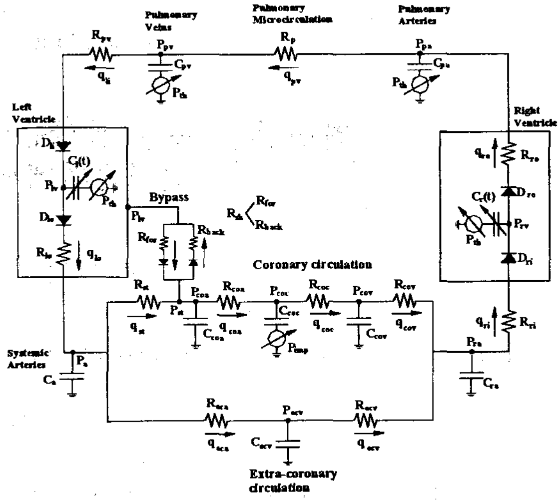


Figure 3. Schematic of a lumped parameter model of the total circulation emphasizing the coronary circulation. Note that the diode arrangement in the bypass circuit allows for a different resistance depending on the direction of flow.

In the present study, we inserted the coronary circulation into an existing model of the entire cardiovascular (CV) system. The complete CV code developed by our group is based on the work of Davis [3]. It represents the arterial, venous and pulmonary circulations and simulates autoregulation functions such as the baroreceptor reflex for short term control of blood pressure and the cardiopulmonary reflex for control of blood volume. One reason for implementing the coronary circulation in a complete CV model is that we can then utilize the baroreflex and cardiopulmonary reflex capabilities to examine a variety of realistic conditions that might be experienced by patients before and after surgical intervention. In addition, it allows us to examine how the surgically-altered system will perform in conjunction with the rest of the circulation. The schematic of the hemodynamic system is represented in Figure 3. The coronary circulation consists of three compartments: the coronary arteries, the coronary capillaries and the coronary veins. The effects of myocardial muscle contraction or relaxation is produced by temporal variations in the bias pressure $P_{imp}(t)$. The flow rates between the respective compartments are thus.

$$q_{li} = \begin{cases} (P_{pv} - P_{lv}) / R_{pv} & if\ P_{pv} > P_{lv} \\ 0 & otherwise \end{cases} \tag{1}$$

$$q_{lo} = \begin{cases} (P_{lv} - P_a)/R_{lo} & if\ P_{lv} > P_a \\ 0 & otherwise \end{cases} \quad (2)$$

$$q_{st} = (P_a - P_{st})/R_{st} \quad (3)$$

$$q_{sh} = (P_{lv} - P_{st})/R_{sh} \quad \begin{cases} R_{sh} = R_{for} & if\ P_{lv} > P_{st} \\ R_{sh} = R_{back} & if\ P_{lv} < P_{st} \end{cases} \quad (4)$$

$$q_{coa} = \begin{cases} (P_{coa} - P_{coc})/R_{coa} & if\ P_{coa} > P_{coc} \\ \dfrac{(P_{coa} - P_{coc})}{(R_{coa} + \beta / V_{coc}^2)} & otherwise \end{cases} \quad (5)$$

For the shunt resistance in Eq. 4, the resistance values of the shunt can be changed according to the direction of flow. The forward and backward resistances are shown in Fig. 3. In Eq. 5, the flow rate to capillaries may be either forward (i.e. $q_{coa} > 0$) or retrograde, depending on the sign of the pressure gradient. However, reverse flow ceases as the capillary volume approaches zero since nothing then remains to be squeezed out. Moreover, as the capillary vessels are compressed, their resistance increases and they will throttle the flow. Accordingly, for a negative pressure gradient Eq. 5 reduces backward flow to zero as the capillary volume approaches zero. For flow into the veins and right atrium, a similar approach can be applied, producing the following two equations.

$$q_{coc} = \begin{cases} \dfrac{(P_{coc} - P_{cov})}{(R_{coc} + \beta / V_{coc}^2)} & if\ P_{coc} > P_{cov} \\ \dfrac{(P_{coc} - P_{cov})}{(R_{coc} + \beta / V_{cov}^2)} & otherwise \end{cases} \quad (6)$$

$$q_{cov} = \begin{cases} \dfrac{(P_{cov} - P_{ra})}{(R_{cov} + \beta / V_{cov}^2)} & if\ P_{cov} > P_{ra} \\ \dfrac{(P_{cov} - P_{ra})}{R_{cov}} & otherwise \end{cases} \quad (7)$$

$$q_{eca} = (P_a - P_{ecv})/R_{eca} \quad (8)$$

$$q_{ecv} = (P_{ecv} - P_{ra})/R_{ecv} \quad (9)$$

$$q_{ri} = \begin{cases} (P_{ra} - P_{rv})/R_{ri} & if\ P_{ra} > P_{rv} \\ 0 & otherwise \end{cases} \quad (10)$$

$$q_{ro} = \begin{cases} (P_{rv} - P_{pa})/R_{ro} & if\ P_{rv} > P_{pa} \\ 0 & otherwise \end{cases} \quad (11)$$

$$q_{pv} = (P_{pa} - P_{pv})/R_p \quad (12)$$

The state form of the node equations can be written in terms of these flow rates.

Conservation of mass at the left ventricular node yields:

$$q_{li} = q_{lo} + q_{lv}' \quad (13)$$

where $q_{lv}' = \dfrac{d}{dt}\left[C_l(P_{lv} - P_{th})\right] = C_l \dfrac{dP_{lv}}{dt} + (P_{lv} - P_{th}$ (14)

$$\therefore \quad \frac{dP_{lv}}{dt} = \frac{q_{li} - q_{lo} - (P_{lv} - P_{th})\, dC_l(t)/dt}{C_l(t)} \quad (15)$$

At the systemic arteries node :

$$q_{lo} = q_a + q_a' \quad (16)$$

where $q_a' = C_a \dfrac{dP_a}{dt}$ (17)

$$\therefore \quad \frac{dP_a}{dt} = \frac{q_{lo} - (q_{st} + q_{eca})}{C_a} \quad (18)$$

At the coronary artery node :

$$q_a + q_{sh} = q_{coa} + q_{coa}' \quad (19)$$

where $q_{coa}' = C_{coa} \dfrac{dP_{coa}}{dt}$ (20)

$$\therefore \quad \frac{dP_{coa}}{dt} = \frac{(q_{st} + q_{sh}) - q_{coa}}{C_{coa}} \quad (21)$$

At the coronary capillaries node :

$$q_{coa} = q_{coc} + q_{coc}' \quad (22)$$

where $q_{coc}' = C_{coc} \dfrac{d(P_{coc} - P_{imp})}{dt}$ (23)

$$\therefore \quad \frac{dP_{coc}}{dt} = \frac{q_{coa} - q_{coc}}{C_{coc}} + \frac{dP_{imp}}{dt} \quad (24)$$

Capillary volume can be obtained by the relation.

$$V_{coc} = C_{coc}(P_{coc} - P_{imp}) \quad (25)$$

Mass conservation at the coronary veins node :

$$q_{coc} = q_{cov} + q_{cov}' \quad (26)$$

where $q_{cov}' = C_{cov} \dfrac{dP_{cov}}{dt}$ (27)

$$\therefore \quad \frac{dP_{cov}}{dt} = \frac{q_{coc} - q_{cov}}{C_{cov}} \quad (28)$$

For the capillary veins, venous pressure is calculated from the pressure-volume relation:

$$P_{cov} = V_{coc} C^{0}_{cov} e^{\sigma(V_{cov} - V^{0}_{cov})} \tag{29}$$

A volume-dependent compliance is defined as the derivative of V_{cov} with respect to P_{cov} :

$$C_{cov}(V_{cov}) = \frac{dV_{cov}}{dP_{cov}} = C^{0}_{cov}(1 + \sigma V_{cov})^{-1} e^{-\sigma(V_{cov} - V^{0}_{cov})} \tag{30}$$

Here, V^{0}_{cov} , C^{0}_{cov} are the reference venous volume and compliance, respectively, and σ is the slope of the change in compliance.

Mass conservation at the extra-coronary veins node :

$$q_{eca} = q_{ecv} + q'_{ecv} \tag{31}$$

where $$q'_{ecv} = C_{ecv} \frac{dP_{ecv}}{dt} \tag{32}$$

$$\therefore \quad \frac{dP_{ecv}}{dt} = \frac{q_{eca} - q_{ecv}}{C_{ecv}} \tag{33}$$

At the right atrium node :

$$q_{cov} + q_{ecv} = q_{ri} + q'_{ra} \tag{34}$$

where $$q'_{ra} = C_{ra} \frac{dP_{ra}}{dt} \tag{35}$$

$$\therefore \quad \frac{dP_{ra}}{dt} = \frac{q_{cov} + q_{ecv} - q_{ri}}{C_{ra}} \tag{36}$$

At the right ventricle node :

By the same procedure with Eq. 15, we can get the equation for the right ventricle node.

$$\frac{dP_{rv}}{dt} = \frac{q_{ri} - q_{ro} - (P_{rv} - P_{th})\, dC_r(t)/dt}{C_r(t)} \tag{37}$$

At the pulmonary arteries node :

$$\frac{dP_{pa}}{dt} = \frac{q_{ro} - q_{pv}}{C_{pa}} \tag{38}$$

At the pulmonary veins node :

$$\frac{dP_{pv}}{dt} = \frac{q_{pv} - q_{li}}{C_{pv}} \tag{39}$$

The lumped parameter hemodynamic model leads to a matrix equation of the form:

$$dp/dt = Ap + b \tag{40}$$

Here $\boldsymbol{p}$ is the vector of compartmental pressures, A represents the time constants for exchange between compartments, and $\boldsymbol{b}$ is the input to the system.

In Figure 3, diodes ensure unidirectional flow. Each compartment is characterized by an inflow resistance R with units of peripheral resistance units (1 PRU = 1 mmHg·s/ml), a compliance or, in the electrical analogy, a capacitance C with units of ml/mmHg, a volume at zero transmural pressure V_0 (zero pressure filling volume, ZPFV) with units of ml, and an outflow resistance. Transmural pressure across the pulmonary capacitance varies according to intra-thoracic pressure. We also assumed different resistance values of the shunt according to the flow direction in order to examine the potential benefit of such an asymmetry.

3. Numerical results and discussion

Since blood flow through the coronary circulation is small relative to that for the total circulation, the coronary circulation has little influence on overall circulation. In contrast, the aortic and left ventricular pressures determined from the overall circulation are the main determinants of coronary blood flow. Figure 4 shows computed pressures in the aorta and left ventricle.

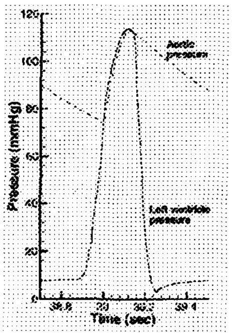
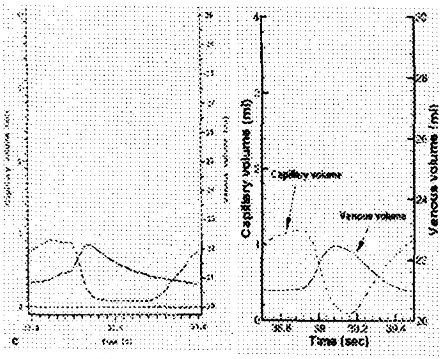


Figure 4 Computed left ventricular and aortic pressures.

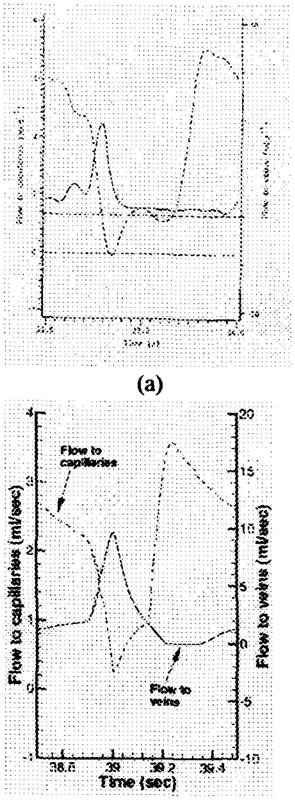
3.1 Verification of the present method

For the purpose of verification we simulated the coronary circulation in a normal state and compared its results with the existing results of Schreiner et al.[4].



(a) (b)

Figure 5. Coronary capillary and venous volume in (a) a previous computation[4] and (b) the present calculation.



(a)

(b)

Figure 6. Flow rate to coronary capillaries and veins in (a) a previous computation[4] and (b) the present calculation.

During systole, the coronary capillary flow rate decreases due to the increased resistance resulting from the contraction of myocardial muscle, whereas flow through the coronary veins increases due to compression of the capillaries (Figure 6). Capillary and venous volumes vary in a manner similar to the flow rate variations shown in Figure 6. In our model, peak systolic pressure is not sustained for as long a time as in Schreiner et al.[4], which results in the narrower band of reduced capillary flow rate during systolic (Figure 3); otherwise the results are quite similar.

3.2 Parametric studies of the simulation following bypass surgery

The reference value of the shunt resistance can be obtained using Poiseuille flow assumption (fully developed, steady and laminar). Flow rate, Q, through the tube under this assumption is expressed as follows:

$$Q = \frac{A^2 \Delta P}{8\pi\mu L} \tag{41}$$

Here, A, ΔP, μ and L represent cross-sectional area, pressure drop, fluid viscosity, and tube length respectively. From this relation we can estimate the resistance of the shunt.

$$R_{sh} = \frac{\Delta P}{Q} = \frac{8\pi\mu L}{A^2} \tag{42}$$

If we use typical values for the shunt (diameter D=2 mm, length L=2 cm, fluid viscosity μ=0.003 kg/(m·s), respectively), then the calculated resistance is 1.147 PRU.

We adjusted the coronary arteriolar resistance R_{coa} and coronary capillary resistance R_{coc} until the baseline flow rate was 0.667 ml/sec in the normal circulation. Since our simulation is modeling total coronary flow, we sought to obtain a normal value of the blood flow through the left anterior descending (LAD) coronary artery, about 1 ml/sec at rest. In this study what we want to simulate is the flow in the distal LAD. The LV-LAD shunt will generally be placed about 2/3 the way down the vessel and the resting flow was therefore estimated to be approximately 0.667 ml/sec.

We also determined the values of R_{coa} and R_{coc} needed to establish a flow rate through an unobstructed LAD of 3.333 ml/sec reflecting an increase over resting flow by a factor of about 5 due to arteriolar dilation. Values to produce this flow rate were R_{coa}=16.5 and R_{coc}=1.65. Since the epicardial coronary arteries of a patient with severe stenosis are likely to be maximally dilated, it is reasonable to use resistance values accommodating a higher value of flow rate for the simulation of the LV-LAD shunt. A preliminary computation has been made to establish the relation between the shunt resistance and flow rate through the shunt. In this case the LAD is fully occluded.

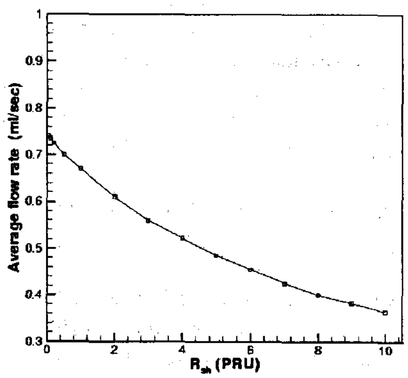


Figure 7. Flow rate variation as a function of shunt resistance with the coronary artery totally occluded.

According to the above plot, the increase of the shunt resistance induces a decrease in the net flow rate through the shunt.

To test the effect of the compliance of the coronary artery we performed calculations for two values of compliance: a normal value (0.005 ml/mmHg), and a higher value (0.05 ml/mmHg).

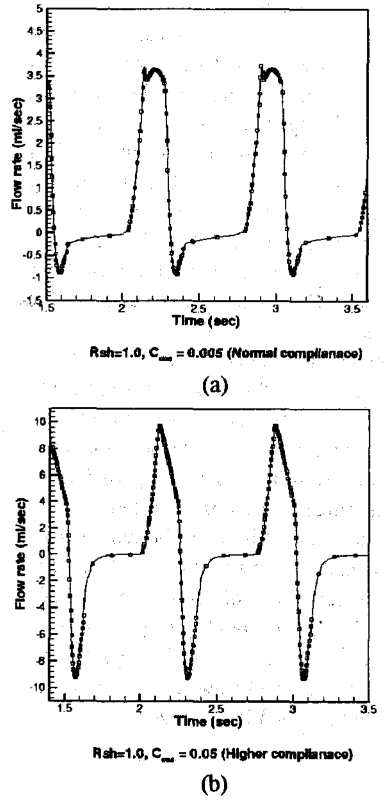


Figure 8. Time varying flow patterns for two different values for arterial compliance.

The peak positive and negative flow rates with the higher epicardial capacitance is larger than that of the lower epicardial capacitance. However, the net (cycle-averaged) flow rate during one cardiac cycle is nearly identical in these two cases. Blood is simply shunted into and out of the arterial capacitance.

4. Conclusions

In this study we presented the computational model to simulate the coronary circulation. The bypass from left ventricle was considered. We employed the lumped parameter model for the homodynamics of the coronary circulation as well as the overall cardiovascular system. Coronary hemodynamics of normal case was simulated to verify the present numerical method. The computational results agreed well with the existing numerical data. To simulate the stenosed coronary artery with the shunt from left ventricle, we assumed the bypass line from left ventricle to coronary artery. Two diodes are inserted along the line to explain the asymmetric characteristic of the shunt. Numerical results on the bypass show that the increase of the shunt resistance induces a decrease in the net flow rate through the shunt.

References

(1) Guiot C, Merletti A, Pagliaro P, Losano G, 2000, “Model-based assessment of pressure and flow-dependent coronary responses following abrupt pressure drops.,” Comput Biol Med, Vol. 30(3), pp. 111-26.

(2) McNulty P, Pfau S, Deckelbaum L, 2000, “Effect of plasma insulin level on myocardial blood flow and its mechanism of action.” Am J Cardiol, Vol. 85(2), pp. 161-5

(3) Davis, T. D., 1991, "Teaching physiology through interactive simulation of hemodynamics," *MIT M. S. Thesis*, Cambridge, MA.

(4) Schreiner, W., Neumann, F., and Mohl, W., 1990, “Simulation of coronary circulation with special regard to the venous bed and coronary sinus occlusion,” *J. Biomed. Eng.*, Vol. 12, 429-443.