Nonlinear Waves of a Two-Layer Compressible Fluid over a Bump

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Abstract

Two-dimensional steady flow of two immiscible, compressible fluids are considered when the temperature of each layer is constant. Both upper and lower fluids are bounded by two horizontal rigid boundaries with symmetric obstruction of compact support at the lower boundary. By using asymptotic method, we derive the forced K-dV equation governing interfacial wave. Various solutions and numerical results are presented.

1. Introduction

Recently, there have been growing interests in studying interfacial waves in an incompressible fluid. Many interesting wave patterns have been found and new mathematical methods have also been developed. Numerical studies of steady flow of a two-layer incompressible fluid over a bump or a step bounded by a free or rigid upper boundary were carried out by Forbes [1], Belward and Forbes [2], Sha and Vanden-Broeck [3], and Moni and King [4], among others, and an asymptotic approach for the case of a rigid upper boundary or a free surface with surface tension were studied by Choi, Sun and Shen [5],[6]. Some formal results and a rigorous asymptotic theory of interfacial solitary waves in a compressible fluid were carried out by Shen [7] and Shen and Sun [8], respectively. In this paper, we consider a two-layer medium of immiscible, inviscid, and compressible fluids. Let the constant temperatures in the upper and lower layers be T^+ and T^-, respectively and $T^- < T^+$. The medium is bounded above by a horizontal rigid boundary and below by a horizontal rigid boundary with a small bump (Fig 1). We shall develop an asymptotic method to derive, so-called, forced KdV equation(FK-dV) for the interfacial wave in the following form:

$$\lambda C_1 \eta_x + C_2 \eta \eta_x + C_3 \eta_{xxx} + C_4 b_x = 0.$$

where $y = b(x)$ is the equation of the obstruction. We investigate the solutions of the FKdV, which represent possible interfacial wave forms.

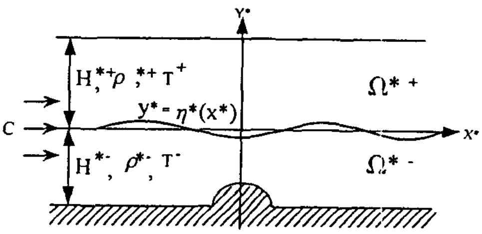

Fig. 1. Fluid Domain

2. Formulation and Model equation

We consider a compressible medium consisting of two layers bounded by two rigid boundaries. At

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equilibrium the lower layer is at temperature T^- with height H^{*+}; the upper layer is at T^+ with height H^{*-}, separated by a contact interface, and the densities at the sides of interface are ρ_s^{*+}, ρ_s^{*-} for the upper and lower layers, respectively. Note that $T^- < T^+$ implies $\rho_s^{*-} > \rho_s^{*+}$. Since a two-dimensional object is moving with a constant speed c along the lower boundary , we choose a coordinate system moving with the object so that, in reference to the coordinate system, the object is stationary and the fluid flow becomes steady. Then the governing equations and boundary conditions are as follows: In $\Omega^{*\pm}$,

$$\begin{gathered}(\rho^{*\pm}u^{*\pm})_{x^*} + (\rho^{*\pm}v^{*\pm})_{y^*} = 0,\\ \rho^{*\pm}(u^{*\pm}u^{*\pm}_{x^*} + v^{*\pm}u^{*\pm}_{y^*}) = -p^{*\pm}_{x^*},\\ \rho^{*\pm}(u^{*\pm}v^{*\pm}_{x*} + v^{*\pm}v^{*\pm}_{y*}) = -p^{*\pm}_{y*} - \rho^{*\pm}g,\\ p^{*\pm}/p_s^* = \rho^{*\pm}/\rho_s^{*\pm};\end{gathered}$$

at the interface, $y^* = \eta^*$,

$$\begin{gathered}u^{*\pm}\eta^*_{x*} - v^{*\pm} = 0,\\ p^{*+} - p^{*-} = 0;\end{gathered}$$

at the rigid boundaries, $y^* = H^{*\pm} + b^*(x^*)$,

$$v^{*\pm} - u^{*\pm}b^*_{x*} = 0\,,$$

where $(u^{*\pm}, v^{*\pm})$ are velocities, $p^{*\pm}$ are pressures, $\rho^{*\pm}$ are densities, g is the gravitational acceleration constant, and p_s^* is the pressure at the interface.

We nondimensionalize the above equations and boundary conditions making use of the following variables;
$\varepsilon = H/L << 1,\ \ \eta = \varepsilon^{-1}\eta^*/|H^{*-}|$,
$p^\pm = p^{*\pm}/g|H^{*-}|\rho_s^{*-},\ \ p_s = p_s^*/g|H^{*-}|\rho_s^{*-}$,
$(x,y) = (\varepsilon^{1/2}x^*, y^*)/|H^{*-}|$,
$(u^\pm, v^\pm) = (g|H^{*-}|)^{-1/2}(u^{*\pm}, \varepsilon^{-1/2}v^{*\pm})$,
$\rho^\pm = \rho^{*\pm}/\rho_s^{*-},\ \ \rho_s^+ = \rho_s^{*+}/\rho_s^{*-} < 1$,
$\rho_s^- = \rho_s^{*-}/\rho_s^{*-} = 1,\ U = c/(g|H^{*-}|)^{1/2}$,
$h^+ = H^{*+}/|H^{*-}|,\ h^- = H^{*-}/|H^{*-}| = -1$,
$b(x) = b^*(x^*)(|H^{*-}|\varepsilon^2)^{-1}$,

where L is horizontal scale, H is the vertical scale. We let $H^\pm = h^\pm + \varepsilon^2 b(x)$.
We assume that $u^\pm(-\infty) = u_0 + \lambda\varepsilon$, where u_0 and λ are constants, $u^\pm$, $v^\pm$, $\rho^\pm$, and $p^\pm$ are functions of x and y near the equilibrium state $u^\pm = u_0, v^\pm = 0$, $p^\pm = p_s\exp(-y/c_s^\pm)$, and $\rho^\pm = \rho_s\exp(-y/c_s^\pm)$, where $c_s^\pm = p_s/\rho_s^\pm$ is the square of the sonic speed, and possess asymptotic expansions:

$$\begin{aligned}(u^\pm, v^\pm, \rho^\pm, p^\pm) &= (u_0, 0, \rho_s e^{(-y/c_s^\pm)}, p_s e^{(-y/c_s^\pm)})\\ &+ \varepsilon(u_1^\pm, v_1^\pm, \rho_1^\pm, p_1^\pm)\\ &+ \varepsilon^2(u_2^\pm, v_2^\pm, \rho_2^\pm, p_2^\pm) + O(\epsilon^3).\end{aligned}$$

Substituting the asymptotic expansions of $u^\pm$, $v^\pm$, $p^\pm$ and $\rho^\pm$ into the nondimensionalized equations, and comparing orders of ε, we can obtain a sequence of equations and boundary conditions for the successive approximations. The first order approximation is:
in $h^- \le y \le 0$ and $0 \le y \le h^+$,

$$\rho_{1x}^\pm u_0^\pm + \rho_0^\pm u_{1x}^\pm + \rho_{0y}^\pm v_1^\pm + \rho_0^\pm v_{1y}^\pm = 0, \qquad (1)$$

$$\rho_0^\pm u_0^\pm u_{1x}^\pm = -p_{1x}^\pm, \qquad (2)$$

$$-p_{1y}^\pm = \rho_1^\pm, \qquad (3)$$

$$\rho_s^\pm p_1^\pm = \rho_1^\pm p_s; \qquad (4)$$

at $y = 0$,

$$v_1^+ = v_1^-, \qquad (5)$$

$$\eta p_{0y}^+ + p_1^+ = \eta p_{0y}^- + p_1^-; \qquad (6)$$

at $y = h^\pm$,

$$v_1^\pm = 0. \qquad (7)$$

By using (1) to (7), and the fact that $p_1^\pm|_{x=-\infty} = \eta|_{x=-\infty} = 0$, $u_1^\pm|_{x=-\infty} = \lambda$, we obtain

$$p_1^\pm = \exp(-y/c_s^\pm)a_1^\pm\eta, \qquad (8)$$

$$\rho_1^\pm = (1/c_s^\pm)\exp(-y/c_s^\pm)a_1^\pm\eta, \qquad (9)$$

$$u_1^\pm = -a_1^\pm\eta/u_0\rho_s^\pm + \lambda, \qquad (10)$$

$$\begin{aligned}v_1^\pm \ = \ & c_s^\pm((u_0)^2\rho_s^\pm - p_s)(1 - \exp((y - h^\pm)/c_s^\pm))\\ & a_1^\pm\eta_x(x)/(u_0\rho_s^\pm p_s). \qquad (11)\end{aligned}$$

where $c_s^{\pm} = p_s/\rho_s^{\pm}$,

$$
\begin{aligned}
a_1^+ &= (((u_0)^2\rho_s^- - p_s)(\rho_s^- - \rho_s^+)\rho_s^+ c_s^- A^-) \\
&\quad /(((u_0)^2\rho_s^+ - p_s)\rho_s^- c_s^+ A^+ \\
&\quad -((u_0)^2\rho_s^- - p_s)\rho_s^+ c_s^- A^-), \\
a_1^- &= a_1^+ + (\rho_s^- - \rho_s^+),
\end{aligned}
$$

where

$$
\begin{aligned}
A^+ &= 1 - \exp(-h^+/c_s^+), \\
A^- &= 1 - \exp(-h^-/c_s^-).
\end{aligned}
$$

By using a similar method, we can find the expressions for $v_2^{\pm}$.
If we suppose, at $y = \varepsilon\eta$,

$$\varepsilon u^- \eta_x^- - v^- = 0, \tag{12}$$

then from the asymptotic expansion of u^-, v^-, we have at $y = 0$,

$$v_1^- - \eta_x u_0 + \varepsilon(\eta v_{1y}^- + v_2^- - \eta_x u_1^-) + O(\varepsilon^2) = 0. \tag{13}$$

We call u_0 the critical speed if the zeroth order term of (13), $v_1^- - \eta_x u_0$, vanishes. Thus

$$u_0^2 = (p_s(f_1 + f_2) \pm p_s\sqrt{\Gamma})/(2\rho_s^+\rho_s^-(g_1 + g_2)),$$

where

$$\Gamma = (f_1 + f_2)^2 - 4g_1\rho_s^+\rho_s^-(g_1 + g_2),$$

where
$$
\begin{aligned}
f_1 &= A^+A^-({\rho_s^-}^2 - {\rho_s^+}^2), \\
f_2 &= -(\rho_s^-)^2A^+ + (\rho_s^+)^2A^-), \\
g_1 &= (\rho_s^- - \rho_s^+)A^+A^-, \\
g_2 &= -\rho_s^-A^+ + \rho_s^+A^-.
\end{aligned}
$$

Then put u_i^-, v_i^-, $i = 1, 2$ into (13) to have the following equation for $\eta(x)$,

$$
\begin{aligned}
&(\lambda - \lambda a_1^- c_s^- m_1^- A^- - \lambda B_1^- c_s^- k^- A^-)\eta_x \\
&+(a_1^- k^- e^- - a_1^-/u_0\rho_s^- + (a_1^-)^2 m_2^- c_s^- A^- \\
&-c_s^- k^- A^- B_2^-)\eta\eta_x \\
&+(c_s^- a_1^- k^-(c_s^- A^- - 2h^- e^-)(u_0^2 - c_s^-) \\
&-c_s^- k^- A^- B_3^-)\eta_{xxx} \\
&-(c_s^- A^- k^- B_4^- e^- + e^- u_0)b_x^- \\
&= O(\varepsilon),
\end{aligned} \tag{14}
$$

where

$$
\begin{aligned}
k^{\pm} &= ({u_0}^2\rho_s^{\pm} - p_s)/(u_0\rho_s^{\pm}p_s,, \\
e^{\pm} &= \exp(-h^{\pm}/c_s^{\pm}), \\
m_1^{\pm} &= 1/p_s + 1/(\rho_s^{\pm}(u_0)^2), \\
m_2^{\pm} &= u_0/(p_s)^2 + 1/((\rho_s^{\pm})^2(u_0)^3), \\
B_1^- &= \alpha^{-1}(a_1^- c_s^- m_1^- A^- - a_1^+ c_s^+ m_1^+ A^+),
\end{aligned}
$$

$$
\begin{aligned}
B_2^- = {} & (2\alpha)^{-1}((e^+a_1^+k^+ - e^-a_1^-k^-) \\
& +(a_1^+)^2 m_2^+ c_s^+ A^+ - (a_1^-)^2 m_2^- c_s^- A^- \\
& +(p_s/(c_s^+)^2 - p_s/(c_s^-)^2 - 2a_1^+/c_s^+ \\
& +2a_1^-/c_s^-)k^- c_s^- A^- \\
& +a_1^-/(u_0\rho_s^-) - a_1^+/(u_0\rho_s^+)) \\
& +p_s/2(c_s^+)^2 - p_s/2(c_s^-)^2 - a_1^+/c_s^+ + a_1^-/c_s^-,
\end{aligned}
$$

$$
\begin{aligned}
B_3^- = {} & \alpha^{-1}(c_s^+a_1^+k^+({u_0}^2 - c_s^+)(c_s^+A^+ - 2h^+e^+) \\
& -c_s^-a_1^-k^-({u_0}^2 - c_s^-)(c_s^-A^- - 2h^-e^-) \\
& +(e^+c_s^+a_1^+({u_0}^2 - c_s^+) - e^-c_s^-c_1^-({u_0}^2 - c_s^-)) \\
& k^-c_s^-A^-) + e^+c_s^+a_1^+({u_0}^2 - c_s^+) \\
& -e^-c_s^-a_1^-({u_0}^2 - c_s^-),
\end{aligned}
$$

$$B_4^- = \alpha^{-1}u_0,$$

where

$$
\begin{aligned}
\alpha = {} & (((u_0)^2\rho_s^+ - p_s)\rho_s^- c_s^+ A^+ - ((u_0)^2\rho_s^- - p_s) \\
& \rho_s^+ c_s^- A^-)/(\rho_s^+\rho_s^- u_0 p_s).
\end{aligned}
$$

3.Forced K-dV equation

From (14), we obtain the following forced K-dV equation;

$$\lambda C_1\eta_x + C_2\eta\eta_x + C_3\eta_{xxx} + C_4 b_x = 0. \tag{15}$$

where

$$
\begin{aligned}
C_1 &= (1 - a_1^- c_s^- m_1^- A^- - B_1^- c_s^- k^- A^-), \\
C_2 &= a_1^- k^- e^- - a_1^-/u_0\rho_s^- + (a_1^-)^2 m_2^- c_s^- A^- \\
&\quad -2c_s^- k^- A^- B_2^-),
\end{aligned}
$$

$$C_3 = (c_s^- a_1^- k^- (u_0{}^2 - c_s^-)(c_s^- A^- - 2h^- e^-) - B_3^- c_s^- k^- A^-),$$
$$C_4 = -(B_4^- c_s^- k^- e^- A^- + e^- u_0).$$

We integrate (15) with respect to x and obtain

$$\eta_{xx} = -\lambda D_1 \eta - D_2 \eta^2/2 - D_3 b(x). \quad (16)$$

where $D_1 = C_1/C_3$, $D_2 = C_2/C_3$, $D_3 = C_4/C_3$. Since the sign of the coefficient play an important role in solving this equation, we must investigate the sign of $D_i, i = 1,2,3$. We find numerically that $-D_1$ and $-D_3$ are positive. Since the sign of $-D_2$ determines the shapes of the solutions of (16), we show in Fig.2 a relation between h and ρ_s^+ for the sign of $-D_2$ when $p_s = 1.0$.

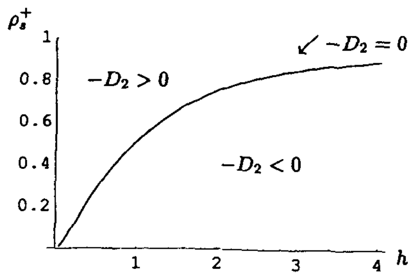

Fig. 2. The sign of $-D_2$ under $p_s = 1.0$.

If $-D_2 > 0$, the same analysis as in Ref.[9] can be carried out and we consider the case $-D_2 < 0$.

Assume $b(x) = \sqrt{1-x^2}$ for $[-1,1]$ and $b(x) = 0$ for $x > 1, x < -1$. When $b(x) = 0$ and $\lambda > 0$, the equation (16) has the following either bounded or unbounded wave solutions

$$\eta(x) = -k\ \mathrm{sech}^2(\sqrt{-\lambda D_1}(x - x_0)/2), \quad (17)$$

$$\eta(x) = k\ \coth^2(\sqrt{-\lambda D_1}(x - x_0)/2) - k, \quad (18)$$

where $k = 3\lambda D_1/D_2$, and x_0 is a phase shift. For $\lambda < 0$, there is no solitary wave solution. We use (17), (18) and $\eta(x) = 0$ for $(-\infty, -1)$. Next we find the periodic solution of (16) for $x > 1$.

By multiplying $\eta(x)$ to (16) , and integrating it from 1 to $x > 1$, it follows that

$$(\eta_x(x))^2 = -D_2\eta^3/3 - \lambda D_1 \eta^2 + c, \quad (19)$$

where
$c = D_2\alpha^3/3 + \lambda D_1 \alpha^2 + \beta^2$,
$\eta(1) = \alpha$, $\eta_x(1) = \beta$,
$\eta_{xx}(1) = -D_2\alpha^2/2 - \lambda D_1 \alpha$.

We assume that $-D_2 > 0$. In case of $-D_2 < 0$, setting $\bar{\eta}(x) = -\eta(x)$ make the above equation for $\bar{\eta}(x)$ with a positive coefficient of cubic term.

We inspect solutions of (19) according to roots of the equation:

$$-D_2\eta^3/3 - \lambda D_1\eta^2 + c = 0. \quad (20)$$

Let c_1, c_2, and c_3 be three roots of (20)

Case 1: All c_1, c_2, c_3 are real and $c_1 < c_2 = c_3$. Then

$$(\eta_x(x))^2 = (-D_2/3)(\eta - c_1)(\eta - c_3)^2.$$

Letting $t = (\eta - c_1)^{1/2}$, using the separation of variables, we have the solution of the following form:

$$\eta = q\coth^2((-qD_2/3)^{1/2}(x - x_0)/2) + c_1.$$

where $q = c_3 - c_1$ and x_0 is a constant.

Case 2: All c_1, c_2, c_3 are real and distinct. Then,

$$(\eta_x(x))^2 = (-D_2/3)(\eta - c_1)(\eta - c_2)(\eta - c_3), \quad (21)$$

where $c_1 < c_2 < c_3$.

Define z, k, M such that

$$z^2 = (c_3 - c_1)/(\eta - c_1), \quad k^2 = (c_2 - c_1)/(c_3 - c_1),$$
$$M^2 = (c_3 - c_1)/4.$$

This transforms (21) to a form that can be easily solved by using $sn(x,k)$ which is the Elliptic functions of Jacobi defined as the solution of:

$$(dz/dx)^2 = (1 - z^2)(1 - k^2z^2)$$

Thus we have a solution of (21) as the following form:

$$\eta(x) = c_1 + q/\mathrm{sn}^2((-qD_2/3)^{1/2}(x - x_0)/2, k),$$

where $q = c_3 - c_1$, $k^2 = (c_2 - c_1)/(c_3 - c_1)$. Note that since $sn(2nK, k) = 0$, where $2K$ is half the period of sn, this solution diverges if $x - x_0 = 2nK$. So although x_0 is arbitrary, but it must be a complex number in order for this solution to be valid.
We may shift $\eta(x)$ by the relation $sn(u + K'i) = 1/(ksnu)$, where K' is half the period of sn on the imaginary axis, and get

$$\eta(x) = c_1 + k^2 q\ \mathrm{sn}^2((-qD_2/3)^{1/2}(x - x_1)/2, k),$$

for an appropriate x_1.
If the limit is taken in above periodic solution as $c_3 \to c_2$, and the formula $\mathrm{sn}(u, 1) = \tanh u$, and $1 - \tanh^2 u = \mathrm{sech}^2 u$ are used, the above solution reduces to

$$\eta(x) = c_2 + q'\mathrm{sech}^2((-qD_2/3)^{1/2}(x - x_1)/2),$$

where $q' = c_1 - c_2$.
Since x_0 was chosen arbitrary, we may choose an appropriate x_0 to make the resulting x_1 to be a real number.

4. Numerical results

So far we have investigated the behavior of a solutions ahead of and behind the bump. We calculate the numerical solution of (16) over the bump. Then, by using the matching process, we construct a global solution of (16), consisting of two exact solutions ahead of and behind the bump, and numerical solution over the bump. There are two cases in which soliton-like solutions are obtained. One is the case when (16) is numerically solved in $-1 \le x \le 1$ with the initial conditions $\eta(-1) = 0$ and $\eta_x(-1) = 0$. The other is the case when (16) is solved in $-1 \le x \le 1$ with the initial conditions $\eta(-1) = -k\ \mathrm{sech}^2(\sqrt{-\lambda D_1}(-1 - x_0)/2)$, or $\eta(-1) = k\ \coth^2(\sqrt{-\lambda D_1}(-1 - x_0)/2) - k$, and $\eta_x(-1) = \pm\sqrt{-D_2(\eta(-1))^3/3 - \lambda D_1(\eta(-1))^2}$, where $k = 3\lambda D_1/D_2$.

Supercritical case $\lambda > 0$

There appear three types of soliton-like solutions. The first type is positive symmetric soliton-like solution and the second type is positive unsymmetric soliton-like solution. These appear when the initial condition is
$\eta(-1) = -k\ \mathrm{sech}^2(\sqrt{-\lambda D_1}(-1 - x_0)/2)$, where $k = 3\lambda D_1/D_2$.
The first and second types solutions are given in Fig.3 and Fig.4, respectively. In Fig.4 we can see that the two unsymmetric solution shows symmetricity with respect to the y axis. The third type is negative symmetric soliton-like solution, and it can be obtained by taking the initial condition;
$\eta(-1) = k\ \coth^2(\sqrt{-\lambda D_1}(-1 - x_0)/2) - k$, where $k = 3\lambda D_1/D_2$.
The third type solution is given in Fig 5. In Fig.6, we show the relationship between λ and the phase shift x_0. All numerical computations in Supercritical case are performed under the following conditions; $\rho_s^+ = 0.1, h = 5.0, p_s = 1.3$.

Subcritical case $\lambda < 0$

In this case, we put $\eta(-1) = 0$ as the initial condition. The numerical results of this case are given in Fig.7 to 10. In Fig.7 a typical periodic solution is presented. In Fig.8 a hydraulic fall solution is given, which is a limiting solution of Fig.7. Fig.9 presents a one-crest solution which is another limiting solution of Fig.7. Figure 10 shows two-crest solution which take place as λ decreases. All numerical computations in Subcritical case are performed under the same conditions of Supercritical case.

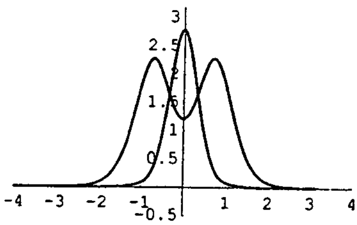

Fig.3 Symmetric solitonlike solutions with $\lambda = 1.0$

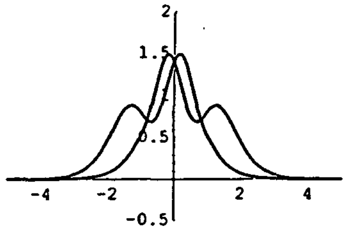

Fig.4 .Unsymmetric solitonlike solutions with $\lambda = 0.4$

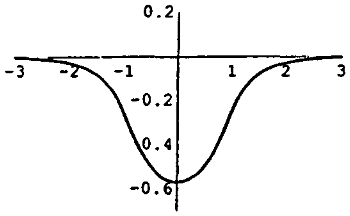

Fig.5 Negative symmetric soliton-like solution with $\lambda = 0.3$

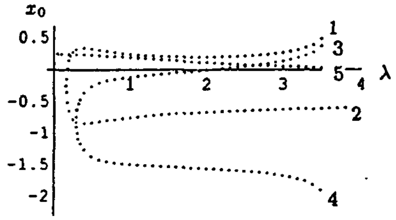

Fig.6 Relationship between λ and the phase shift x_0 of soliton-like solutions. 1. Symmetric solutions with one crest. 2. Symmetric solutions with two crests. 3 and 4. Unsymmetric solutions with two crests. 5. Negative solutions with one crest.

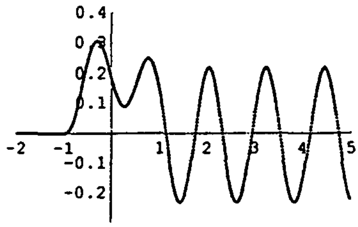

Fig.7 Typical periodic solution with $\lambda = -2.0$

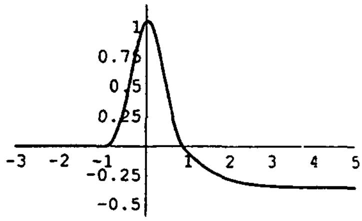

Fig.8 Hydraulic fall solution with $\lambda = -0.236503$

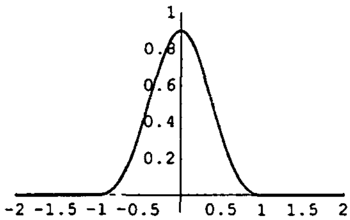

Fig.9 Symmetric solution with one crest with $\lambda = -0.417654$

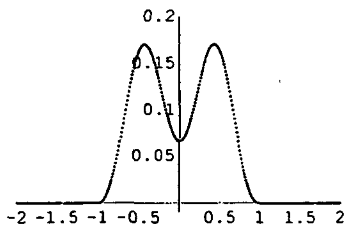

Fig.10 Symmetric solution with two crests with $\lambda = -3.44110$

Acknowledgement

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