Analysis of Rarefied Nozzle Flow by Generalized Hydrodynamic Equations
GH 방정식을 이용한 희박 노즐의 해석

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Abstract

This paper presents the analysis of flowfield inside a low-density nozzle and its plume into near vacuum. The generalized hydrodynamics equations are numerically solved for the purpose with the help of modern computational fluid dynamic methods. The results taken along the nozzle are compared with those of Navier-Stokes equations and available experimental data. The plume outside the nozzle is also analyzed in order to examine the adverse effects of its impingements.

1 Introduction

The various electrothermal propulsion systems such as resistojet and arcjet are widely used for station keeping, attitude control, and orbital transfer of satellites and spacecrafts. These missions are achieved by heating propellant gas with a electric source and expanding it rapidly through a convergent-divergent nozzle into vacuum. Since a typical nozzle adopted for the purpose is very small in size and the mass flow rate is low, the viscous effects are dominant, which leads to a severe performance loss. The flow physics is more complicated by its experience of varying degrees of rarefraction effects from continuum to free molecular regime. Therefore, not only the analysis of the flowfield, but also its understanding is often limited. Even inside the nozzle near a exit plane, the flow already invokes the nonequilibrium effect invalidating the Navier-Stokes equations based on the continuum assumption and the classical non-slip wall boundary condition[1].

The exhaust plume from the nozzle during satellite operations is also one of the major concerns. Under the vacuum condition, the flow can turn around a nozzle lip beyond the continuum limit leading to mass flux that cannot be ignored in the back flow region. The impingement of back flow can exert significant, often deleterious, effects on the sensitive surface of onboard systems such as solar arrays and scientific equipments in forms of thermal loading, electrical charging and gradual contaminations. In addition, the thrust loss and disturbance torques are unwanted effects that can be caused by these back flows[2].

The accurate prediction for flowfields inside the nozzle and the interactions of exhaust plume and space systems is, thus, crucial in designing satellites and spacecrafts, which can be utilized in order to ensure the successful functioning of satellites and to possibly extend their lifetime. Of numerous methods for investigating this type of flow, the direct simulation Monte Carlo(DSMC) method of Bird[3] is widely used. The DSMC method is a numerical simulation technique in which the representative particles are moved deterministically and collided probabilistically, providing the solution of the Boltzmann equation through a large number of samplings. Several researchers including Boyd et al.[4], Chung et al.[5], Ivanov et al.[6] studied nozzle flows and plumes by the DSMC method. Since the computation is still prohibitively expensive in near continuum region, a hybrid method[7] which applies the Navier-Stokes equations and the DSMC method respectively to different computational domains depending on the continuum breakdown parameter[3] were proposed. It is believed, though, that there exists an ambiguity in dividing the domains, and it seems that these methods are still expensive to be routinely used in the designing stage. Less accurate, but quite useful methods in plume

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analysis are the Simons model[8] and the Method of Characteristics(MOC), which are, despite their own limitations, still in use for performing parametric studies.

The current works are based on Eu's Generalized Hydrodynamic(GH) equations[9] that can be classified as the 13 moment equations. It is turned out that GH equations are the most thermodynamically consistent macroscopic equations and that they recover the correct behaviors at continuum and free molecular limit. In this respect, the GH equations seem to be particularly adequate for our study of the rarefied nozzle and plume.

2 Governing Equations and Numerical Methods

2.1 Navier-Stokes and generalized hydrodynamics equations

The axisymmetric Navier-Stokes equations in conservative forms can be written as

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \frac{1}{y}\mathbf{H} + \frac{\partial \mathbf{E}_v}{\partial x} + \frac{\partial \mathbf{F}_v}{\partial y} + \frac{1}{y}\mathbf{H}_v = 0 \tag{1}$$

where y is a distance from the center line and

$$\begin{aligned}
\mathbf{Q} &= (\rho, \rho u, \rho v, \rho e_t)^T \\
\mathbf{E} &= (\rho u, \rho u^2 + p, \rho uv, \rho(e_t + p)u)^T \\
\mathbf{F} &= (\rho v, \rho uv, \rho v^2 + p, \rho(e_t + p)v)^T \\
\mathbf{H} &= (\rho v, \rho uv, \rho v^2, \rho(e_t + p)v)^T \\
\mathbf{E}_v &= (0, \Pi_{xx}, \Pi_{xy}, \Pi_{xx}u + \Pi_{xy}v + Q_{hx})^T \\
\mathbf{F}_v &= (0, \Pi_{xy}, \Pi_{yy}, \Pi_{xy}u + \Pi_{yy}v + Q_{hy})^T \\
\mathbf{H}_v &= (0, \Pi_{xy}, \Pi_{yy} - \Pi_{\theta\theta})^T
\end{aligned} \tag{2}$$

where $\mathbf{\Pi}$ and $\mathbf{Q}_h$ are stress tensor and heat flux vectors, respectively, and given by Navier-Stokes-Fourier law. If the system consists of a single component monatomic gas, the GH constitutive relation equations for stress and heat flux along with Eq.(1), after Eu's closure and adiabatic approximation [9], are written in the spirit of 13 moment equations as

$$-2[\mathbf{\Pi} \cdot \nabla \mathbf{u}]^{(2)} - \frac{p}{\eta}\{\mathbf{\Pi} q(k) + 2\eta[\nabla \mathbf{u}]^{(2)}\} = 0 \tag{3}$$

$$-\mathbf{\Pi} \cdot C_p \nabla T - (\mathbf{u} \cdot \nabla \mathbf{u}) \cdot \mathbf{\Pi} - \mathbf{Q}_h \cdot \nabla \mathbf{u} - \frac{pC_pT}{\lambda}\{\mathbf{Q}_h q(k) + \lambda \nabla \ln T\} = 0 \tag{4}$$

where

$$q(k) = \sinh(k)/k \tag{5}$$

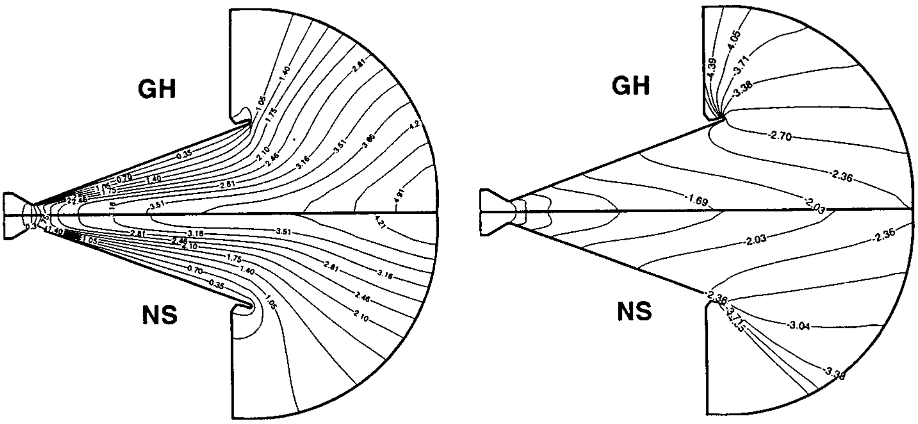
with

$$k = \frac{(mk_BT)^{1/4}}{\sqrt{2}dp}[\frac{\mathbf{\Pi} : \mathbf{\Pi}}{2\eta} + \frac{\mathbf{Q}_h \cdot \mathbf{Q}_h}{\lambda}]^{1/2} \tag{6}$$

where the d and k_B are a diameter of a particle and a Boltzmann constant, respectively. The colon means the double scalar product. The symbol $[\mathbf{x}]^{(2)}$ is traceless symmetric part of $\mathbf{x}$. η is a Chapman-Enskog viscosity and λ is a heat conduction coefficient multiplied by temperature.

2.2 Numerical Methods

While the Navier-Stokes constitutive relations can be computed by central difference, the GH constitutive relations in Eq.(3) and (4) cannot be readily solved, since they are systems of five nonlinear



(a) Mach number contour

(b) Density(log ρ/ρ_0) contour

Figure 1: Various contours from NS and GH equations

algebraic equations. Myong[10] proposed rather simplified GH computational model which can be efficiently solved by iterative methods by observing that Eq.(3) and (4) in two dimensions can be approximated by one dimensional compression-rarefraction wave problem and one dimensional Couette problem. However, in current paper, we fully solved Eq.(3) and (4) by Powell's hybrid method in MINPACK found at www.netlib.org. without any simplification. The inviscid flux is discretized by van Leer's flux vector splitting scheme and and is extended to a higher order by MUSCL approach. The time integrations is performed by ADI method. The calculation is continued until an error residual based on density becomes lower than 8 order from initial error, considering a near vacuum environment outside the nozzle. In rarefied flows, a slip boundary condition is commonly applied to a wall. A slip boundary condition of Maxwell-Smoluchowski with full accommodation is employed with adiabatic wall condition.

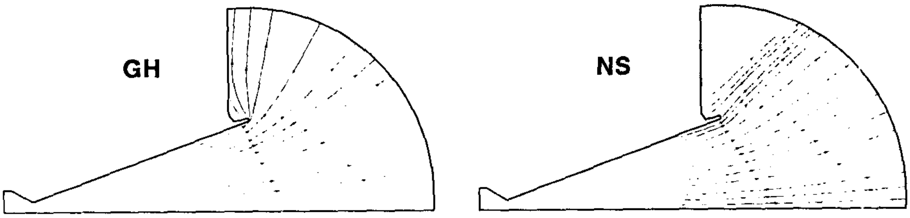
The various definition of Reynolds number exist for nozzle problems. Since the reservoir conditions are usually specified, the definition of

$$Re_t = \frac{\rho_0\sqrt{2h_0}r_t}{\eta_0} \tag{7}$$

is adopted and variables in equations are nondimensionalized accordingly. In Eq.(7), the subscript 0 means reservoir conditions and $\sqrt{2h_0}$ denotes an ideal maximum speed expanded from a reservoir. r_t is a throat radius.

3 Numerical Results

In this section, we analyzed Rothe's experimental nozzle[11] which are widely used as a validation for rarefied nozzle flows. The nozzle geometry and reservoir conditions are given in Ref.[11]. The propellant gas is N_2 and Reynolds number in Eq.(7) is 590. The reservoir Knudsen number is 0.00524. Even though the theory of GH equations are based on monatomic molecules, they are applied to



(a) GH equations (b) NS equations

Figure 2: Streamline patterns

this case with viscosity index and specific heat ratio set to those of N_2. The viscosity index is 0.72 and specific heat ratio is 1.4. It should be mentioned that the rotational temperature modeling is necessary as Larsen-Borgnakke model of DSMC method, since the present equations alone cannot predict the rotational temperature. However, the GH equations always produce improved results over those from NS equations, as will be shown later. It is a common practice to generate grid system only inside the nozzle and apply extrapolation as exit boundary condition even in boundary layer. Results from such a computational domain show a large deviation especially in boundary layer from those of DSMC method which includes the outside computational domain. Therefore, we include the outside domain in order to avoid ambiguities arising from boundary condition. We can be assured that a simple extrapolation is valid, since outside region is supersonic everywhere.

A Mach number contour both from NS and GH equations is shown in Fig.1, where a slightly faster expansion is observed by the GH equations inside the nozzle. While a boundary layer steadily grows till nozzle lip is reached with NS calculation, it ends with a sonic line at nozzle lip with GH calculation, as Bird previously observed with his DSMC results[12]. Fig.1 (b) shows a log scaled density contour. The NS results indicates a steep gradient aligned with the nozzle lip. However, contour lines in radial direction are observed near a nozzle lip, which is in qualitative agreement with those of DSMC[5]. Finite back flows can be seen in streamlines by GH calculations in Fig.2, while the NS calculation do not exhibit such behaviors. The degree of contamination induced by this backflows can be assessed through studies of mass flux at a place where we are interested in. The detailed structures of flows inside the nozzle can be seen from Fig.3. The density along the nozzle centerline shows an excellent agreement with experimental data, compared with NS density. In order to investigate pure characteristics of adopted equations, non-slip boundary condition is applied to both of equations. As shown in figure, GH equations exhibit improved capability of analyzing rarefied flows over NS equations in the sense that the GH density gives closer centerline density to experimental data. It seems that flows of this case can be still analyzed by NS equations with appropriate slip boundary condition, given relatively low Knudsen number in reservoir. It is interesting to note that the density with slip boundary condition follows that with non slip boundary condition from $z/r_t = 30$, indicating that wall boundary condition inside the nozzle affects little on plume structure. Fig.4 shows transverse density profiles at a few nozzle positions. Also in this case, there is an improvement with the GH equations.

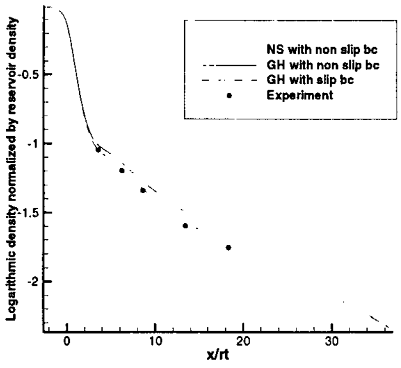


Figure 3: Density($\log \rho/\rho_0$) along nozzle centerline

4 Conclusions

In this paper, rarefied nozzle flows are analyzed by the GH equations. Inside the nozzle there exists quantitative difference between NS and GH results, where the GH equations always produce more accurate results than NS equations, compared with experimental data. The NS equations are proved to exhibit incompetence for analyzing plumes outside the nozzle showing no backflows at nozzle lip, while the GH equations show qualitatively conforming results to DSMC data.

In order to analyze the diatomic or polyatomic gas more accurately, a rotational temperature modeling is necessary. A temperature relaxation equation[13] which is similar to vibrational temperature model commonly used in hypersonic flows is thought be a plausible candidate for the purpose.

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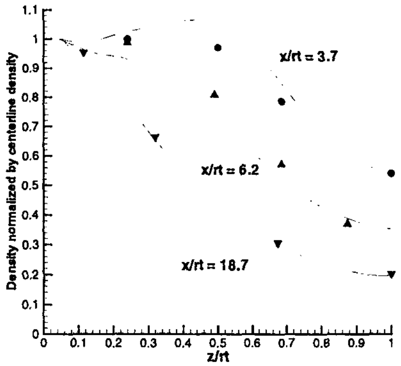

Figure 4: Transverse density profiles(ρ/ρ_c), solid lines:GH, dotted lines:NS, symbols:experiment

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