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LMI를 이용한 자기부상 시스템의 H_∞ 제어기 설계

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H_∞ Controller Design for Electromagnetic Suspension System using LMIs

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Abstract - In this paper, a fault tolerant control problem is considered for a class of nonlinear system formulated in a gain scheduling form with LMI-based H-inf control technique. Key benefits of this proposed scheme are demonstrated in the simulation of an electromagnetic suspension system with actuator and/or sensor failures, and the method is compared with the convensional state-feedback and output-feedback controller. It is clearly observed that the proposed control scheme shows an improved output performance in comparision with convensional methods

1. Introduction

Methodology used in conventional control system design often presumes that the actuators and sensors of the system are in good working conditions. As a result, a majority of control system designed using conventional techniques may not be able to maintain a satisfactory performance in the presence of actuator and/or sensor failures. In some cases, even the closed-loop system stability may be in jeopardy. To increase the reliability in the presence of such vulnerability, the control system may need to provide some kind of fault tolerance. Here, a control system designed to tolerate actuator and/or sensor failures, while maintaining an acceptable closed-loop system stability and performance, is called fault tolerant control system [1].

In this paper, we investigate the issue of fault tolerant control for an electromagnetic suspension system model, which represents the essential dynamics of magnetically levitated transport system. The system is highly nonlinear with unstable open-loop dynamics. In electromagnetic suspension system, various failures may take place in the actuators and sensors. Since the failure may result in critical performance degradation and even catastrophe, improvement of reliability and safety of the system has been an essential requirement. It is noted that some results for fault tolerant control of linear system are easily found ([2]-[3]) whereas the system including highly nonlinear and open-loop unstable is yet to further investigation for fault tolerant performance. In the present paper, the fault tolerant control methodology is introduced for electromagnetic suspension system and it is demonstrated by using gain scheduling plus LMI-based H_∞ control technique.

2. Problem Formulation

Consider the nonlinear system described as follows:

$$\dot{x} = f(x, u+u_\xi, w),$$
$$z = g(x, u+u_\xi, w),$$
$$y = h(x, w) + y_\xi, \tag{1}$$

where, $x \in R^n$ denote state vector, $u \in R^p$ us the control input, $w \in R^m$ denotes the scheduling parameter as piecewise constant, $z \in R^r$ and $y \in R^q$ indicate the controlled output. The fuctions f, g and h are assumed to be smooth. Here, $u_\xi \neq 0$ represents the susceptible actuator failuire and $y_\xi \neq 0$ represents the susceptible sensor failure. We assume that there exists an open neighborhood Γ of the origin in R^m and smooth functions $x_0(w)$ and $u_0(w)$ for each $w \in \Gamma$ such that

$$0 = f(x_o(w), u_o(w), w),$$
$$r_d = g(x_o(w), u_o(w), w),$$
$$y_o(w) = h(x_o(w), w), \tag{2}$$

where, r_d is the reference input. Now, for each w, let the nominal control law be given as follows[13];

$$u = u_o(w) + K(w)(x - x_o(w)) \tag{3}$$

where, $K(w)$ is determined so that the eigenvalues of closed-loop system have specified values with negative real parts for each w. Let the following deviation variables be defined as;

$$x_\delta = x - x_o(w),$$
$$u_\delta = u - u_o(w),$$
$$z_\delta = z - r_d,$$
$$y_\delta = y - y_o(w). \tag{4}$$

For each fixed w, the correponding linearinzed system can be written in the following form:

$$\dot{x}_\delta = f(x_o(w) + x_\delta, u_o(w) + u_\delta + u_\xi, w) - \dot{x}_o(w),$$
$$z_\delta = g(x_o(w) + x_\delta, u_o(w) + u_\delta + u_\xi, w) - r_d,$$
$$y_\delta = h(x_o(w) + x_\delta, w) - y_o(w) + y_\xi, \tag{5}$$

By using Taylor series expansion, the following equations are given by

$$\dot{x}_\delta = A(w)x_\delta + B(w)u_\delta + \begin{bmatrix} B(w) & I \end{bmatrix} \begin{bmatrix} u_\xi \\ o_f \end{bmatrix},$$

$$z_\delta = C_1(w)x_\delta + D(w)u_\delta + \begin{bmatrix} D(w) & I \end{bmatrix} \begin{bmatrix} u_\xi \\ o_g \end{bmatrix},$$

$$y_\delta = C_2(w)x_\delta + y_\xi + o_h,$$

where,

$$A(w) = \frac{\partial f}{\partial x}(x_o(w), u_o(w), w),\ B(w) = \frac{\partial f}{\partial u}(x_o(w), u_o(w), w),$$

$$C_1(w)=\frac{\partial g}{\partial x}(x_o(w),u_o(w),w), C_2(w)=\frac{\partial h}{\partial x}(x_o(w),u_o(w),w),$$

$$D(w)=\frac{\partial g}{\partial u}(x_o(w),u_o(w),w)$$

Let the following two classes of linearized system (5) be described by state-space models. One is the linearized system with susceptible actuator failure that the entire state is directly avaliable for feedback as follows:

$$\dot{x}_\delta = A(w)x_\delta + B(w)u_\delta + \bar{B}(w)\bar{u}_{\zeta 1},$$
$$z_\delta = C_1(w)x_\delta + D(w)u_\delta + \bar{D}(w)\bar{u}_{\zeta 2}. \quad (5a)$$

The other is the linearized system with susceptible actuator and sensor failures that not all state are directly measurable for feedback as follows:

$$\dot{x}_\delta = A(w)x_\delta + B(w)u_\delta + \bar{B}(w)\bar{u}_{\zeta 1},$$
$$z_\delta = C_1(w)x_\delta + D(w)u_\delta + \bar{D}(w)\bar{u}_{\zeta 2},$$
$$y_\delta = C_2(w)x_\delta + \bar{y}_\zeta, \quad (5b)$$

Note that, unless the actuator and sensor failures are taken into consideration in the controller design state, the closed -loop system may exhibit unsatisfactory H_∞-norm bounded in some cases, can be unstable in the event of some actuator and/or sensor failures. Following Lemma helps turning on the H_∞ constraints to a matrix inequality.

Lemma 1 (bounded real lemma)[4] consider a continuous-time transfer function $T(s)$ of realization $T(s)=D+C(sI-A)^{-1}B$. Then, the following statements are equivalent:

(1) $\|T(s)\|_\infty < \gamma$ and A is stable in the continuous-time sensor ($Re(\lambda i(A)) < 0$).

(2) There exist a symmetric positive definite solution P to the Riccati ineuqality:

$$A^TP+PA+(PB+C^TD)(\gamma^2I-DD^T)^{-1}(B^TP+D^TC)+C^TC<0 \quad (6)$$

where $\gamma^2I-DD^T>0$.

(3) There exists a symmetric positive definite solution P to the matrix inequalities:

$$\begin{pmatrix} A^TP+PA & PB & C^T \\ B^TP & -\gamma I & D^T \\ C & D & -\gamma I \end{pmatrix} < 0 \quad (7)$$

The fault tolerant control problem is to design a controller such that the closed-loop system is robustly stable in spite of any susceptible actuator and/or sensor failures. First, we consider the linearized system (5a). Our goal is to compute a state-feedback controller K_s such that meet H_∞-norm bound γ against any susceptible actuator failure. If a state-feedback control law $u_\zeta = K_s x_\zeta$ is used in the closed-loop system, then the system (5a) also changes to

$$\dot{x}_\delta = (A(w)+B(w)K_s)x_\delta + \bar{B}(w)\bar{u}_{\zeta 1},$$
$$z_\delta = (C_1(w)+D(w)K_s)x_\delta + \bar{D}(w)\bar{u}_{\zeta 2}. \quad (8)$$

Since the failed actuator is considered as a disturbance $\bar{u}_{\zeta 1}$, $\bar{u}_{\zeta 2}$, closed-loop system with a state-feedback can be depicted as figure 1 with transfer functions T_{af1} and T_{af2} given by

$$T_{af1} = C_{1cl}(w)(sI-A_{cl}(w))^{-1}B_{cl}(w)$$
$$T_{af2} = D_{cl}(w)(sI-A_{cl}(w))^{-1} \quad (9)$$

where,

$$A_{cl}(w)=A(w)+B(w)K_s, B_{cl}(w)=\bar{B}(w),$$
$$C_{1cl}(w)=C_1(w)+D(w)K_s, D_{cl}(w)=\bar{D}(w).$$

Here, we will be considered with the following notion of stabilizability for the linearized system (5a)

Definition 1 [5][16] Let a constants $\gamma > 0$ be given. The linearized system (5a) is said to be quadratically stabilizable with an H_∞-norm bound γ if there eiist a state-feebdback control law $u_\zeta = K_s x_\zeta$ and a symmetric positive definite P for continuous-time such that the inequality.

$$A_{cl}^T(w)P+PA_{cl}(w)+(PB_{cl}(w)+C_{1cl}^T(w)D_{cl}(w))(\gamma^2I-D_{cl}(w)D_{cl}^T(w))^{-1}$$
$$(B_{cl}^T(w)P+D_{cl}^T(w)C_{1cl}(w))+C_{1cl}^T(w)C_{1cl}(w)<0,$$

with $\gamma^2I-D_{cl}(w)D_{cl}^T(w)>0$.

Lemma 2 [5] Suppose the linearized system (5a) is quadratically stabilized with an H_∞-norm bound $\gamma>0$ via state-feedback. then the closed-loop system us asymptotically stable.

At this time, the main interest is to design a state feedback controller in order not only to robustly stabilize the system against any susceptible actuator failure but also to satisfy H_∞ constraints $\|T_{af1}\|_\infty < \gamma$ and $\|T_{af2}\|_\infty < \gamma$. Using Lemma 1 as measns of H_∞ constraints into LMIs, considering the closed-loop system (8), one can determine the controller K_s by following Theorem 1.

Theorem 1 Consider the linearized system (5a) with susceptible actuator failure and assumption that $(A(w), B(w))$ is a stabilizable pair. The system is robustly stabilizable against any susceptible actuator failure, and also the H_∞ constraints $\|T_{af1}\|_\infty < \gamma$ and $\|T_{af2}\|_\infty < \gamma$ are satisfied if there exist a symmetric matrix X and V such that the following LMIs are all satisfied.

$$\begin{pmatrix} A(w)X+XA^T(w)+B(w)V+V^TB^T(w) & XC_1^T(w)+V^TD^T(w) & [B(w) \quad \bar{B}(w)] \\ C_1(w)X+D(w)V & -\gamma I & [D(w) \quad \bar{D}(w)] \\ [B(w) \quad \bar{B}(w)]^T & [D(w) \quad \bar{D}(w)]^T & -\gamma I \end{pmatrix} < 0$$

$$X>0 \quad (10)(11)$$

Moreover, The control input is given by

$$u_\delta = K_s x_\delta, \; K_s X = V \quad (12)$$

Proof: *skip*

At this time, the main interest is to design a proper output feedback controller in order not only to robustly stabilize the system against any susceptible actuator and sensor failures but also to satisfy H_∞ constraint $\|T_{asf}\|_\infty < \gamma$. Using the lemma 1 as means of H_∞ constraints into LMIs, one can determine the controller K_o by the following Theorem 2.

Theorem 2 Consider the linearized system (5b) with susceptible actuator and sensor failures, and assumption that $(A(w), B(w), C_2(w))$ is stabilizable and detectable. The system is robustly stabilizable against any susceptible actuator and sensor failures, and also the H_∞ constraints $\|T_{asf}\|_\infty < \gamma$ is satisfied if there exist a symmetric matrix X and Y suh that the following LMIs are all satisfied

$$\begin{pmatrix} A'_{cl}(w)X+XA'^T_{cl}(w) & XC'^T_{1cl}(w) & B'_{cl}(w) \\ C'_{1cl}(w)X & -\gamma I & D'_{cl}(w) \\ B'^T_{cl}(w) & D'^T_{cl}(w) & -\gamma I \end{pmatrix} < 0 \quad (13)$$

$$\begin{pmatrix} A'^T_{cl}(w)Y+YA'_{cl}(w) & YB'_{cl}(w) & C'^T_{2cl}(w) \\ B'^T_{cl}(w)Y & -\gamma I & D'^T_{cl}(w) \\ C'_{2cl}(w) & D'_{cl}(w) & -\gamma I \end{pmatrix} < 0, \quad (14)$$

$$Y := X^{-1} > 0 \qquad (15)$$

Moreover, The control inout is given by

$$u_\delta = K_o y_\delta, \qquad (16)$$

Proof: *skip*

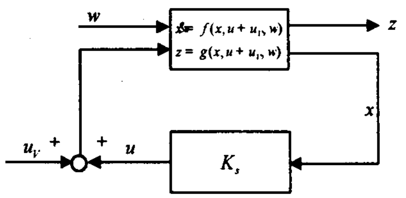

Figure 1 System with susceptible actuator failure

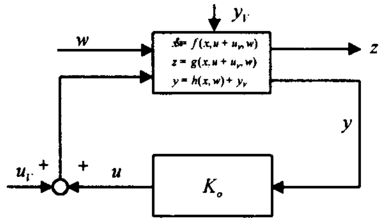

Figure 2 System with susceptible actuator and sensor failures

III. Design of a fault tolerant magnetic suspension system

Problems of magnetically levitated transport systems are to cope with mass variation, actuator and/or sensor failures. The primary causes of its performance deteration are the mass variation. The masss variation results from taking passengers, loading flight, or the failure of any one of the controlled suspension magnet group. Furthermore, the actuator and sensor failures in the railway system often cause instability of the magnetically levitated vehicle and tend to increase highly the regulation air-gap error at the levitated state. Note that disturbances such as lifting force variation due to the actuator and/or sensor failures are a dominant sources which deterioate the system performance. It is therefore very important to decrease the regulation air-gap error to guarantee safety and enhance ride quality of the railway systems.

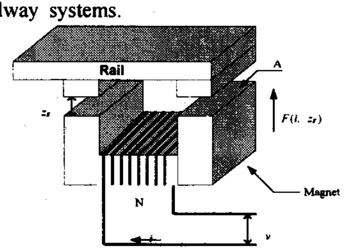

Figure 3. Electromagnet-track configuration

As a possible application, we apply the proposed control method to an electromagnetic suspension system. The single magnet suspension system in Figure 3 is described by the following nonlinear dynamic equation [6].

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = -\frac{\mu_0 N^2 A}{4m}\left(\frac{x_3}{x_1}\right)^2 + G$$

$$\dot{x}_3 = \frac{x_2}{x_1}x_3 - \frac{2R}{\mu_0 N^2 A}x_1 x_3 + \frac{2x_1}{\mu_0 N^2 A}u,$$

$$G(x)G(x)^T = G(x)(G(x)$$

where, x_1, x_2 and x_3 are the vertical air-gap z_gm the verical velocity $\dot{z}_g$ and the magnet current i respectively, while u denotes the magnet coltage v and y is the palnt output z_g. And m is the total suspension mass, N is the numbet of turns of the coil wrapped arround the magnet, A is the effective magnet pole area, μ_0 us the permeability of free space, G is the gravity constant and R is the coil resistance. Here, we assume that the above nonlinear dynamics if exactly modelled for the electromagnetic susepension system.

In order to consider the mass m as scheduling parmeter, we can obtain the estimated mass $\tilde{m}$ from the second equation of (29) as follows;

$$\tilde{m} = \frac{\mu_0 N^2 A}{4(G - \dot{x}_2)}\left(\frac{x_3}{x_1}\right)^2 \qquad (17)$$

And the nominal state values and input values with a constant $\tilde{m}$ can be given by

$$x_o(\tilde{m}) = \begin{bmatrix} r_d \\ 0 \\ \frac{2r_d}{N}\sqrt{\frac{G\tilde{m}}{\mu_0 A}} \end{bmatrix}, \quad (18) \qquad u_o(\tilde{m}) = \frac{2r_d R}{N}\sqrt{\frac{G\tilde{m}}{\mu_0 A}} \quad (19)$$

Then, for each fixed $\tilde{m}$, the corresponding linearized system is given by

$$\begin{aligned} \dot{x}_\delta &= A(\tilde{m})x_\delta + B(\tilde{m})u_\delta + \bar{B}(\tilde{m})\bar{u}_{\varsigma 1}, \\ z_\delta &= C_1(\tilde{m})x_\delta + D(\tilde{m})u_\delta + \bar{D}(\tilde{m})\bar{u}_{\varsigma 2}, \\ y_\delta &= C_2(\tilde{m})x_\delta + \bar{y}_\varsigma, \end{aligned} \qquad (20)$$

The linearized system coeffinets are given by

$$A(\tilde{m}) = \frac{\partial f(x(\tilde{m}), u(\tilde{m}), \tilde{m})}{\partial x} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2G}{r_d} & 0 & -\frac{N\sqrt{AG\mu_0}}{r_d\sqrt{\tilde{m}}} \\ 0 & \frac{2\sqrt{G\tilde{m}}}{N\sqrt{A\mu_0}} & -\frac{2r_d R}{A\mu_0 N^2} \end{bmatrix}$$

$$B(\tilde{m}) = \frac{\partial f(x(\tilde{m}), u(\tilde{m}), \tilde{m})}{\partial u} = \begin{bmatrix} 0 & 0 & \frac{2r_d R}{A\mu_0 N^2} \end{bmatrix}'$$

$$\bar{B}(\tilde{m}) = \begin{bmatrix} \frac{\partial f(x(\tilde{m}), u(\tilde{m}), \tilde{m})}{\partial u} & I \end{bmatrix}' = \begin{bmatrix} \begin{bmatrix} 0 & 0 & \frac{2r_d R}{A\mu_0 N^2} \end{bmatrix}' & I \end{bmatrix}'$$

$$C_1(\tilde{m}) = \frac{\partial g(x(\tilde{m}))}{\partial x} = [1 \quad 0 \quad 0], \quad C_2(\tilde{m}) = \frac{\partial h(x(\tilde{m}))}{\partial x} = [1 \quad 1 \quad 1]$$

$$D(\tilde{m}) = \begin{bmatrix} [0 \quad 0 \quad 1]' & I \end{bmatrix}', \quad \bar{D}(\tilde{m}) = \begin{bmatrix} [1 \quad 0 \quad 0]' & I \end{bmatrix}'$$

At first, we design a fault tolerant state-feedback controller for susceptible actuator failure only. This can be obtained by solving LMIs from Theorem 1. The controller gain K_s, whose design objective is to minimize $trace(X)$ subject to LMIs (10)-(12) with scheduling parameter $\tilde{m}$ as follows:

$$\begin{pmatrix} A(\tilde{m})X + XA^T(\tilde{m}) + B(\tilde{m})V + V^T B^T(\tilde{m}) & XC_1^T(\tilde{m}) & [B(\tilde{m}) \quad \bar{B}(\tilde{m})] \\ C_1(\tilde{m})X & -\gamma I & [D(\tilde{m}) \quad \bar{D}(\tilde{m})] \\ [B(\tilde{m}) \quad \bar{B}(\tilde{m})]^T & [D(\tilde{m}) \quad \bar{D}(\tilde{m})]^T & -\gamma I \end{pmatrix} < 0$$

$$X > 0$$

From (12) and (16), the control law of the nonlinear system is obtained by

$$u = u_o(\tilde{m}) + K_s x_\delta$$

To show the effectiveness of the proposed method, we compare the proposed method with a conventional State Feedback Controller (SFC) given by

$$u = \frac{2r_d R}{N}\sqrt{\frac{\tilde{m}g}{\mu_0 A}} + k_1(x_1 - r_d) + k_2 x_2 + k_3\left(x_3 - \frac{2r_d}{N}\sqrt{\frac{\tilde{m}g}{\mu_0 A}}\right)$$

where k_1, k_2 and k_3 are the feedback gains.

Next, we design a fault tolerant output-feedback controller for susceptible actuator and sensor failures. The corresponding LMI formulation from Theorem 2 consists of minimizing $trace(X) + trace(Y)$ subject to the LMIs (13)-(15) with scheduling parameter $\tilde{m}$ as follows:

$$\begin{pmatrix} A'_{cl}(\tilde{m})X + XA'^{T}_{cl}(\tilde{m}) & XC'^{T}_{1cl}(\tilde{m}) & B'_{cl}(\tilde{m}) \\ C'_{1cl}(\tilde{m})X & -\gamma I & D'_{cl}(\tilde{m}) \\ B'^{T}_{cl}(\tilde{m}) & D'^{T}_{cl}(\tilde{m}) & -\gamma I \end{pmatrix} < 0$$

$$\begin{pmatrix} A'^{T}_{cl}(\tilde{m})Y + YA'_{cl}(\tilde{m}) & YB'_{cl}(\tilde{m}) & C'^{T}_{2cl}(\tilde{m}) \\ B'^{T}_{cl}(\tilde{m})Y & -\gamma I & D'^{T}_{cl}(\tilde{m}) \\ C'_{2cl}(\tilde{m}) & D'_{cl}(\tilde{m}) & -\gamma I \end{pmatrix} < 0,$$

$$Y := X^{-1} > 0$$

From (16) and (19), the control law of the nonlinear system is obtained by

$$u = u_o(\tilde{m}) + K_o y_\delta,$$

We compare the proposed method with a conventional Output Feedback Controller (OFC) given by

$$u = \frac{2r_d R}{N}\sqrt{\frac{\tilde{m}g}{\mu_0 A}} + (D_k + C_k(sI - A_k)^{-1}B_k)y_\delta$$

where A_k, B_k, C_k and D_k are the conventional output feedback controller matrix.

To illustrate the proposed control scheme under the mass variation, actuator and/or sensor failures of electromagnetic suspension system, the simulation were performed as the following information;

$$m = 300kg, R = 1\Omega, G = 9.8[m/\sec^2],\ r_d = 10[mm]$$

$$\mu_0 = 4\pi\times10^{-7}[H/m],\ N = 660,\ A = 0.04[m^2]$$

The simulation result is given in Figure 4, which show time response of the proposed controller output (solid line) and the conventional state feedback controller output (dotted line), respectively. Note that, in the figure, the LMI-based H_∞ controller meets the disturbance attenuation bound γ against the additive disturbance including susceptible actuator and sensor failures while the mass variation of plant is engaged in the closed-loop system. Obviously, the simulation result shows that the conventional output-feedback controller hardly reduces the effect of the mass variation, the susceptible actuator and sensor failures.

4. Conclusion

The fault tolerant control of a class of nonlinear system is studied and applied for an electromagnetic suspension model. The method is shown to be effective for the system not only with mass variation, but also with the susceptible actuator and/or sensor failures. The benefits of these proposed controllers are demonstrated via simulation of an electromagnetic suspension system with susceptible actuator and/or sensor failures. In the present paper, the design was restricted to a nonlinear system with the susceptible actuator and/or sensor failures. To be general, however, the method should be extended to an uncertain nonlinear system with susceptible actuator and/or sensor failures, and be tested and validated by means of actual experiments involving a magnetically levitated vehicle. It is remarked that the study in the paper was originally motivated by experiences in a national project on Development of a magnetically levitated vehicle System.

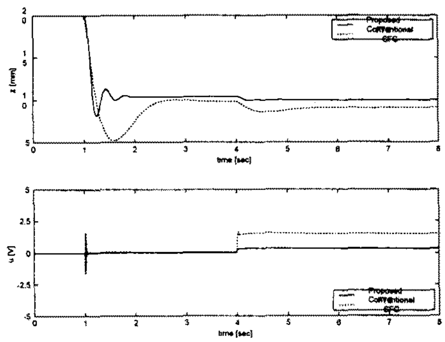

Figure 4 Proposed FTC and conventional SFC performance against actuator failure

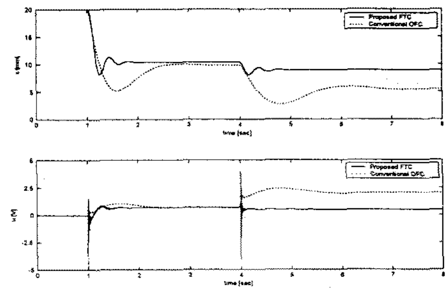

Figure 5 Proposed FTC and conventional OFC performance against susceptible actuator and sensor failures

[Reference]

[1] Srichander, R. qnd B.K. Walker,"Stochastic stability analysis for continuous-time fault tolerant control system", Int. J. Control, 57(2), 433-452, 1993.

[2] Se-Hwa Park and Zeungnam Bien,"Robust and reliable H_∞ control with performance bounded for a class of uncertain linear system", Automatica, 1995[2]

[3] C. J. Seo and B. K. Kim," Robust and reliable H_∞ control for linear systems with parameter uncertainty and actuator failure", Automatica, 32(3), 465-467,1996.

[4] P. Gahinet and P. Aparian," A linear matrix inequality approach to H_∞ control", Int. J.of robust and nonlinear contr., 4:421-448, 1994.

[5] C. J. Seo, A study on robust and reliable H_∞ control for uncertain linear systems, Ph.D. Thesis, KAIST, 1995.

[6] C. Scherer, P. Gahinet, and M. Chilali," Multi-objective output feedback control via lmi optimization", IEEE Trans. Automat. Contr., 42(7), 896-911,1997.[6]