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Review of Internal Waves in Ocean
해양에서의 내부파

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1. INTRODUCTION

Internal waves, including all kinds of wave phenomena inside a stratified fluid system, have been a subject of interest for a long time in geophysics, coastal and ocean engineering, applied mathematics and hydrodynamics. The vast oceans on the earth are a complex stratified fluid system widely distributed with pycnoclines, which are horizontally layered regions with a large density variation, commonly located about 50-150 meters below the ocean surface. With the development of oceanography and ocean engineering, more and more researches on internal waves have been reported.

It is now well known that ocean-going crafts off the coast of Norway suddenly found themselves unable to maintain their accustomed speed as they moved past the mouth of a fjord. This "dead water" phenomenon is related to the internal waves as explained by the Swedish oceanographer Ekman. At certain speeds, much of the power from the ship's engines is spent, wastefully, on the creation of internal waves. Internal waves are also responsible for the flexing of drilling risers of offshore drilling rigs in the deep ocean of the Andaman Sea (Osborne and Burch, 1980). The loss of the U.S.S. Thresher including the crew of 129 men, a nuclear-powered submarine, which occurred 350km off the coast of Massachusetts on April 1963, may have been related to internal waves (Thurman, 1988).

Osborne and Burch (1980) reported field observations in the deep waters of the Andaman Sea off Thailand and Farmer and Smith (1980) in the Western Coast Area of Canada. Other field investigations have been reported by Liu et al. (1985) and Apel et al. (1985) in the Sulu Sea near the Philippines and Liu (1988) in the New York Bight. A significant feature of these field observations is the discovery of how large the amplitude of these internal waves occurring in nature can be. Internal solitary waves as high as 60m in amplitude have been reported (Osborne and Burch, 1980) although the top surface waves that accompany such giant internal waves are invariably very small.

Important experimental studies on internal waves have been performed by many researchers. Among them are Maxworthy (1979), Hammack (1980), Koop and Butler (1981), Segur and Hammack (1982) and Lansing and Maxworthy (1984). Internal waves have been investigated theoretically by Benjamin (1966, 1967, 1992), Davis and Acrivos (1967), Koop and Butler (1981) and Baines (1984) for two-layer fluid systems, and Benjamin (1966, 1967), Kao et al. (1985), Koop and McGee (1986) and Smyth (1988) for continuously stratified fluid systems.

In this paper, an attempt has been made to review the current research works related to internal waves in ocean. The derivation of governing equation is presented in section 2. Internal solitary waves, of the kind observed in the Andaman Sea, are also shown in section 2. In section 3 the generation mechanism of internal waves is briefly discussed. The propagation and dissipation of internal waves are described in section 4. Finally, some concluding remarks are made in section 5.

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2. GOVERNING EQUATIONS

For the purposes of analysis, internal waves have been divided into two types: small amplitude periodic waves and long water waves. In the former case the equations of motion are linearized by assuming that the wave amplitude is small in comparison with the wavelength, and the subsequent development yields an infinite number of modes of travelling periodic waves (Davis and Acrivos, 1967).

On the other hand, the analysis of long water internal waves, which has been discussed in detail by Benjamin (1966, 1992) and Segur and Hammack (1982) is based on the simplifying assumption that the horizontal scale of the motion is long compared with the fluid depth, and yields, in addition to periodic waves, an infinite number of travelling wave modes.

Since the governing equation for small amplitude waves is relatively simple and discussed in class, we here briefly outline the derivation of governing equation for long internal waves. We consider a two-fluid configuration, in which a layer of lighter fluid overlies a layer of heavier fluid, resting on a horizontal impermeable bed in a constant gravitational field as shown in Fig. 1. This is the simplest configuration that supports internal gravity waves, and it is adequate to model the internal waves observed by Osborne and Bunch (1980) and by Liu et al. (1985).

Following Segur and Hammack (1982) the velocity and potential in the upper and lower fluids may be written as

$$\Phi_1 \sim (A\sinh kz + B\cosh kz)\ e^{ik(x-ct)} \qquad (1)$$

$$\Phi_2 \sim D\cosh k(z+h_2)\ e^{ik(x-ct)} \qquad (2)$$

in which subscripts 1 and 2 represent the upper and the lower layer, respectively, A, B and D are unknowns to be determined by the proper boundary conditions, and the phase speed c is a function of wave number k and should satisfy the linear dispersion relation given by

$$\left(\frac{kc^2}{g}\right)^2\left[1+(1-\Delta)T_1T_2\right]-\frac{kc^2}{g}\left[T_1+T_2\right] +\Delta T_1T_2=0 \qquad (3)$$

in which

$$\Delta=\frac{\rho_2-\rho_1}{\rho_2},\ T_i=\tanh kh_i,\ i=1,\ 2$$

By assuming Boussinesq approximation, that is $\Delta \to 0$ but $g\Delta$ is finite, we can simplify the dispersion relation (3) as

$$c_s^{\,2}=\frac{g}{k}\tanh k(h_1+h_2)+O(\Delta) \qquad (4)$$

for surface waves, and

$$c_i^2=\frac{g\Delta}{k}\frac{T_1T_2}{T_1+T_2}\left[1+O(\Delta)\right] \qquad (5)$$

for internal waves.

In the Boussinesq limit, the phase speeds given in equations (4) and (5) always are distinct for given k. Henceforth, we will let $\Delta \to 0$, and retain only dominant terms. The Korteweg-de Vries (KdV) limit is obtained by letting $k(h_1+h_2) \to 0$, that is the wavelength is much greater than the total water depth, so that

$$c_s^2=g(h_1+h_2) \qquad (6)$$

for surface waves, and

$$c_i^2=\frac{g\Delta h_1h_2}{h_1+h_2}=c_0^2 \qquad (7)$$

for internal waves.

The KdV equation is a hydrodynamic wave equation with weakly dispersive and weakly nonlinear terms. Aspects of KdV theory for long internal waves have been discussed by many researchers (e.g. Benjamin, 1966, 1992; Segur and Hammack, 1982). To derive KdV equation we assume that

1. The wavelength is much greater than the total water depth, so that $\mu^2=k^2(h_1+h_2)^2 \ll 1$, where $1/k$ represents a characteristic horizontal wavelength scale.
2. The waves are small, so that if a denotes a characteristic wave amplitude then $\varepsilon=a(h_1+h_2)\ll 1$.
3. The two effects are in approximate balance, that is $O(\varepsilon)=O(\mu^2)\ll 1$.
4. Viscous effects are weaker than either of $O(\mu^2)$ or $O(\varepsilon)$.

in which μ^2 and ε are small parameters representing the frequency dispersion and the nonlinearity, respectively. We also assume that the motion is two-dimensional, and that the fluid is incompressible.

At the interface, the normal velocity and pressure should be continuous, while the pressure must vanish at the free surface. In addition, both the free surface and the interface satisfy kinematic conditions. Finally, all motion should vanish as $|x| \to \infty$.

After applying perturbation method and lengthy algebra, the KdV equation is derived as (Segur and Hammack, 1982)

$$\frac{1}{c}\frac{\partial \zeta}{\partial t} + \frac{\partial \zeta}{\partial x} + \frac{3}{2}\left(\frac{1}{h_2} - \frac{1}{h_1}\right)\zeta\frac{\partial \zeta}{\partial x} + \frac{1}{6}h_1 h_2 \frac{\partial^3 \zeta}{\partial x^3} = 0 \qquad (8)$$

in which ζ denotes the free surface displacement and c_o is given in equation (7). Equation (8) is valid for $\Delta \ll 1$. A more detailed mathematical description of KdV equation is given by Benjamin (1992) and physical description by Herman (1992).

To derive a classical form of KdV-type equation we let

$$\chi = \frac{x - c_o t}{(h_1 h_2)^{1/2}}, \quad \tau = \frac{1}{6}\left(\frac{g\Delta}{h_1 + h_2}\right)^{1/2} t, \quad f = \frac{2}{3}\left(\frac{1}{h_2} - \frac{1}{h_1}\right)\zeta \qquad (9)$$

Then $f(\chi, t)$ satisfies

$$f_\tau + 6ff_\chi + f_{\chi\chi\chi} = 0 \qquad (10)$$

Herman (1992) also confirms that internal solitary waves, of kind observed in the Andaman Sea, can be modeled mathematically with the KdV equation given in equation (10).

Fig. 2 shows a series of internal solitary waves, of the kind observed in the Andaman Sea, can be modeled mathematically with the KdV equation (Herman, 1992). The wave travels along an interface. It starts as a negative hump travelling into progressively shallower water. As the wave propagates longer distance, internal wave becomes shorter and broader due to frequency dispersion. At the same time, some water is forced backward, creating an elevated hump behind the wave (b). At a given depth-called the turning point-the nonlinear effects become zero (see equation (8)), and the wave is purely dispersive (c). Beyond the turning point, however, the nonlinear term begins to increase, but it has reversed in sign. This creates a new solitary wave, but one that extends upward (d). The original solitary wave disappears through dispersion (e), and the newly created solitary wave races away (f) followed by a series of smaller solitary waves.

3. GENERATION OF INTERNAL WAVES

The generation mechanism of internal waves in a stratified fluid has been a subject of considerable interest (e.g. Cox and Sandstrom, 1962; Bell, 1975; Thorpe, 1975; Garrett and Munk, 1979; Maxworthy, 1979). Among these Thorpe (1975) and Garrett and Munk (1979) have well summarized the generation mechanism of internal waves in deep ocean.

In this section, we describe the generation mechanism of internal waves in brief. In general, the generation mechanism is divided into three groups: surface (atmospheric), interior and bottom generation.

3.1 Surface Generation

Internal waves can be generated by external forcings acting on the free surface such as travelling pressure field and travelling stress. For instance, the internal wave generation by travelling pressure field dates back to Ekman's observations on "dead water" phenomenon, that is increased drag on a slow-moving object in strongly stratified water due to the generation of internal waves.

The internal waves generated by travelling stress is less known about the spectrum of the stress of the wind on the ocean surface than that of the pressure. At first sight it appears highly unlikely that surface waves, with their relatively high frequencies, and wave numbers and large phase speeds, will interact in a coherent way so as to generate internal waves. When, however, we recall that microseisms are generated by the seemingly improbable process of a second-order interaction of a pair of surface waves moving in opposite directions (Thorpe, 1975).

Internal waves can also be generated by other forcings acting on the free surface such as the travelling buoyancy flux, interaction of a pair of surface waves and Ekman layer instability and parametric instability. The detailed description for the surface generation can be found in Thorpe (1975) and Garrett and Munk (1979).

3.2 Interior Generation

The physics and mathematics involved in interior generation of internal waves are not well understood. Internal waves can interact to transfer energy to lower frequencies, but the direction of the net transfer is unknown.

It is likely that internal waves may be generated in the process of decay of large-scale circulations and mesoscale eddies by breaking of baroclinic instability, but no measurements or estimates are available (Garrett and Munk, 1979). The gap that often separates internal gravity waves from waves with periods of 4 or 5 days in the frequency spectra of horizontal kinetic energy suggests that no significant homogeneous local cascade of energy occurs between low-frequency motions and internal waves. The interior generation mechanism is also presented by Thorpe (1975).

3.3 Bottom Generation

The generation of internal waves by steady flows over specific topographic features has received considerable attention in the meteorological literature within the context of the mountain lee wave problem (e.g. Baines, 1987).

In the ocean, however, a significant time dependent components of flow in the basic state should also be considered, especially those of tidal period. The study of generation of internal waves by the interaction of tidal flows with deep ocean topography might be initiated by Cox and Sandstrom (1962). They investigated generation of internal wave modes of tidal frequency by the interaction of the surface or external tide with an isolated patch of bottom roughness.

Thorpe (1975) employed an approach involving localized generation and energy propagation upward through the water column, which is more consistent with the apparent nature of the dissipation than the modal approach of Cox and Sandstrom, although some properties of the internal wave field appear to be equally well predicted by either approaches.

Maxworthy (1979) investigated that the tidal flow over an isolated ridge may also be an important source of internal tidal waves in those areas where such features exist. As the ebb tide proceeds, a lee wave (depression) is formed behind the obstacle. He addressed that the characteristics of this depression depend on the basic density stratification, the shape of bottom topography and the magnitude and direction of the tidal current.

It is quite possible that the bottom, with its rough topography and turbulent boundary layer, may act as an energy sink for internal waves. Internal tsunamis generated by earthquakes have been examined by Hammack (1980) and Segur and Hammack (1982).

4. PROPAGATION AND DISSIPATION

4.1 Propagation

We here describe briefly the propagation of a group of internal waves generated within the deep ocean. We assume that a change of Brunt-Vaisala frequency, N, is small over one wavelength and that the Richardson number is sufficiently large. Then, the propagation of a group of internal waves can be described by a WKB-type approximation (Garrett and Munk, 1979).

The frequency of internal waves ω relative to the fluid should exist between N and the local inertial frequency, f, and is given locally by

$$\omega^2 = \frac{N^2(l^2+m^2)+f^2n^2}{k^2} \tag{11}$$

where **k** is the wave number vector with l, m and n as its components. The group velocity vector is written as

$$c_g = \left(\frac{\partial\omega}{\partial l}, \frac{\partial\omega}{\partial m}, \frac{\partial\omega}{\partial n}\right) \tag{12}$$

and c_g is orthogonal to the direction **k** of wave-phase propagation, that is $\mathbf{k}\cdot c_g = 0$, so that as the wave group propagates the lines of constant phase move at right angles and toward the horizontal plane if $N > f$.

The propagation direction of the group velocity to the horizontal plane α is given by

$$\tan\alpha = \left[\frac{\omega^2 - f^2}{f^2 - \omega^2}\right]^{1/2} \tag{13}$$

so that near-internal waves propagate almost horizontally and high-frequency waves propagate vertically.

In reflection from a rigid boundary the waves retain their inclination to the horizontal plane. If the waves propagate into a region where N falls to a value less than that of the wave frequency, they will be reflected, their group velocity becoming vertical near the surface where $N = \omega$. The waves may be trapped in a wave guide if N falls below both above and below the group. The more detailed description and explanation for the propagation of internal waves in moving media, the resonant interaction between internal waves and finite amplitude effects are given in Thorpe (1975).

4.2 Dissipation

Next we review the dissipation phenomenon of internal waves. Internal waves can be dissipated by viscous effects, turbulent layers and microstructure, breaking and shear instability. According to Turner (1973), parts of the ocean are known to be horizontally layered, and horizontally elongated turbulent patches occur, perhaps resulting from internal wave breaking, shear instability of the mean or transient flow, of double-diffusive effects.

As the energy in the internal tide propagates up through the ocean, it finds itself in regions of ever increasing Brunt-Vaisala frequency. The wave rays are bent more and more toward the horizontal plane, and the energy density in the wave field builds up to such an extent that nonlinear effects become important. Spectral transport by weak interactions in the internal wave field should tend to transfer energy from the internal tide to other regions of the internal wave spectrum over much of the water column, whereas strong interactions as manifested by localized instabilities should serve to effectively dissipate much of this energy in the upper ocean (Bell, 1975).

LeBlond (1966) investigated the attenuation of internal waves in a fluid of constant N and concluded that internal tides that have a length of about 200km will propagate for 2,000km or more, although standing oscillations in ocean basins are not possible. The deep ocean must be very far from land to avoid the effects of internal tides propagating from continental slopes.

For wave having larger amplitude than a critical amplitude the particle velocity exceeds the phase velocity, and unstable gradients occur. These unstable gradients lead to convective instability in a small volume of fluid. This is not a complete breakdown of the wave, although energy will be extracted and transferred into turbulent kinetic energy and into increasing the mean potential energy of the fluid. A similar breakdown in atmospheric lee waves leads to rotor formation and possibly rotors may also be produced in the ocean (Thorpe, 1975). No estimates have yet been made of the total energy that may be dissipated through these processes in the ocean.

Internal waves may cause local overturning and turbulence and lose energy by shear instability. For this condition to happen the shear generated by the waves, added to any preexisting shear, must become so large that the local Richardson number falls below a certain critical value (which is for many flows) and remains below for a time long enough for shear instability to grow and produce unstable density gradients or billows.

A description of the instability at an interface between two homogeneous layers (Kelvin-Helmholtz instability) in a laboratory experiment has been made by Thorpe (1973, 1984). Internal wave breaking in continuously stratified fluid is also investigated theoretically and experimentally by Kao et al. (1985). The dissipation effect on the resonant flow of a stratified over a bottom topography is investigated by Smyth (1988).

5. CONCLUDING REMARKS

Internal waves reach greater heights from smaller energy input than do the waves resulting from very large energy input observed at the ocean surface. This is because they move along interfaces across which the density difference is less than that which exists between the free surface and the atmosphere. They are believed to move as long waves. These internal waves are important because of wide range of effects such as ocean circulations, salinity structure, biological variability found in the tows, and military affairs.

There is still much to be learned about internal waves, but their existence, generation and propagation well understood during last decades. The generation mechanism of internal waves are well explained except interior generation mechanism which is still wanting more detailed research in both physics and mathematics.

The propagation of internal waves in deep ocean can be modeled, by either small amplitude wave theory or long wave theory. For long internal waves, the governing equation is a KdV-type equation. On the other hand, the dissipation of internal waves still wants more research though a significant number of research works have been carried out. Field observation and measurement are important to get more reliable real data which can be used as the criterion to calibrate the laboratory experimental data and numerical solutions.

As shown by some experimental studies, large breaking waves have greater internal velocities. If such wave breakings occur in deep water, then the forces exerted by them should be considered in designing the offshore structures. In future this issue should be investigated in detail. Numerical method may be used as an efficient tool.

Although there has been no serious study on the internal waves in Korea, it should be studied and investigated significantly in the near future.

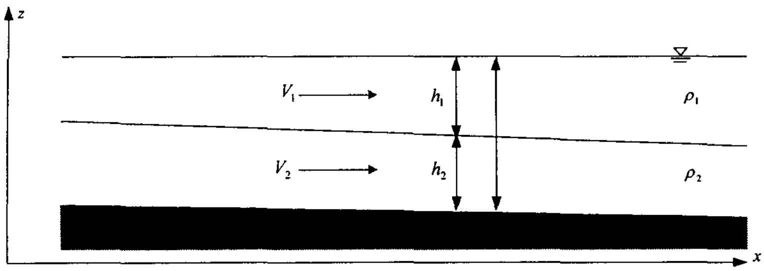

Fig. 1. A typical two-layer fluid system

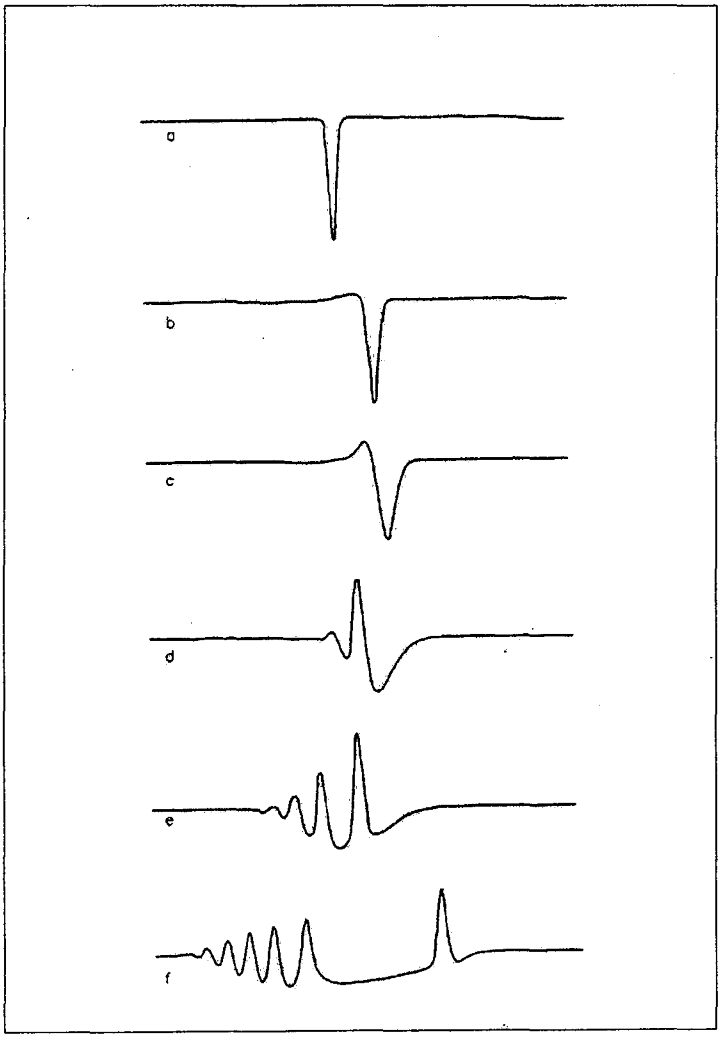

Fig. 2. Series of internal solitary waves

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