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Dynamics of multi-photon resonances in a driven Jaynes-Cummings system

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Fock-state is a highly non-classical radiation-field state. So if one can generate a Fock-state it is possible to study many interesting quantum-mechanical aspects. But in spite of its attraction, it is very difficult to generate a Fock-state experimentally although there have been many theoretical and experimental efforts to do it. Recently Chough et. al.[1] proposed a feasible scheme to achieve quasi number states. The key is to exploit the multi-photon resonances occurring in a driven Jaynes-Cummings system, so it is important to understand the processes at multi-photon resonances. In the present work we study the dynamics of multi-photon resonances in the driven Jaynes-Cummings system.

In our model a two-level atom resonantly coupled with a single-mode cavity is driven by a classical field of arbitrary-frequency as described by a Hamiltonian

$$H=1/2\ \hbar\omega_A\sigma_z+\hbar\omega_c a^+a+i\hbar g(a^+\sigma_- - a\sigma_+)+i\hbar\varepsilon(\sigma_+e^{-i\omega_Lt}-\sigma_-e^{i\omega_Lt}),$$

where $\omega_A, \omega_c, \omega_L$ are the atomic transition, the cavity resonance, the driving-field frequency respectively. And g is the atom-cavity coupling strength, ε the coupling strength of the driving field and the atom proportional to the driving-field amplitude. Without the external driving-field, the atom-cavity system has the energy eigenstates (dressed states) and eigenenergies as follows.

$$|\Psi_n^+\rangle = \cos\theta_n|n-1,e\rangle+i\sin\theta_n|n,g\rangle, \qquad |\Psi_n^-\rangle = \sin\theta_n|n-1,e\rangle+i\sin\theta_n|n,g\rangle,$$

$$E_n^{\pm}=n\hbar\omega_c-\frac{\hbar\delta}{2}\pm\hbar\sqrt{g^2n+\frac{\delta^2}{4}},$$

where $\tan 2\theta_n=-\frac{2g\sqrt{n}}{\delta}$, $\quad \delta=\omega_c-\omega_A$

When the frequency of the driving-fields is such that $N\hbar\omega_L=E_N^{\pm}$, then N-photon resonance appears. Moreover, if we observe the time-evolution of the mean photon-number and the excited-state population of the atom, it is inferred that the atom-cavity system evolves sinusoidally between $|0,g\rangle$ and $|\Psi_N^{\pm}\rangle$.

In the figure below, we draw the dynamics for instance at two-photon resonance as a function of the normalized time gt. It shows the intra-cavity mean photon number and the excited-state population which coincide with each other. The atom-cavity system evolves adiabatically between the two states $|0,g\rangle$ and $|\Psi_2^+\rangle$. Of course, there is no direct coupling between $|0,g\rangle$ and $|\Psi_2^+\rangle$

which can be easily seen in the above Hamiltonian. Therefore, the intermediate states $|\Psi_1^+\rangle$, $|\Psi_1^-\rangle$ should intervene in the interaction.

When the two-photon resonance condition is satisfied ($2\hbar\omega_L = E_2^+$), the foregoing Hamiltonian in the interaction picture with four truncated bases $|0,g\rangle$, $|\Psi_1^+\rangle$, $|\Psi_1^-\rangle$, $|\Psi_2^+\rangle$ is given as

$$H' = \begin{pmatrix} 0 & -iV_1^- & -iV_1^+ & 0 \\ iV_1^- & \Delta_1^- & 0 & V_2^- \\ iV_1^+ & 0 & \Delta_1^+ & -V_2^+ \\ 0 & V_2^- & -V_2^+ & 0 \end{pmatrix}$$

where

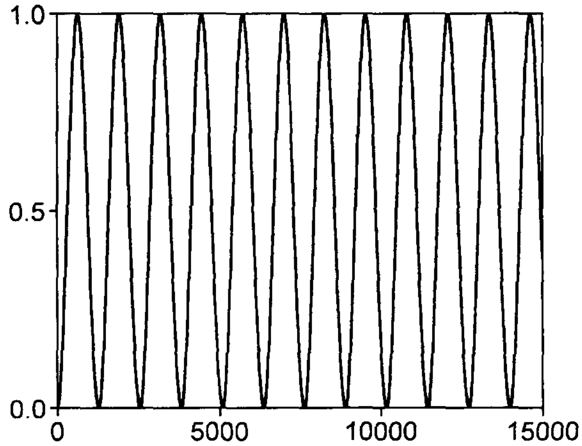
$$\Delta_1^\pm = E_1^\pm/\hbar - \omega_L$$

$$V_1^+ = \varepsilon\cos\theta_1, \qquad V_1^- = \varepsilon\sin\theta_1$$

$$V_2^+ = \varepsilon\cos\theta_2\sin\theta_1, \qquad V_1^- = \varepsilon\cos\theta_2\cos\theta_1$$

If $\Delta_1^\pm$ are rather larger than the off-diagonal elements, then two eigenvalues (λ_1, λ_2) are very small and the corresponding eigenstates are the superposition of nearly only two states $|0,g\rangle$ and $|\Psi_2^+\rangle$. Thus if the initial state is $|0,g\rangle$ then the system evolves adiabatically between $|0,g\rangle$ and $|\Psi_2^+\rangle$ with the periodicity $2\pi/|\lambda_1-\lambda_2|$.

This adiabaticity is very similar to the STIRAP[2] used in the coherent population transfer. But in this case we cannot transfer the system from $|0,g\rangle$ to $|\Psi_2^+\rangle$ since $V_1^+, V_1^-, V_2^+, V_2^-$ are not controllable independently.



We will also discuss the anti-damping effect occurring at two-photon resonance. The mean photon number may be increased even if the damping constants are increased.

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