The Impact of Coordination on Stocking and Promotional Markdown Policies for a Supply Chain

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Abstract

Results of a study of the coordination effect in stocking and promotional markdown policies for a supply chain consisting of a retailer and a discount outlet (DCO) are reported here. We assume that the products are sold in two consecutive periods: Normal Sales Period (NSP) and subsequent Promotional Sales Period (PSP). When managers in the two periods coordinate, they share information on the demand forecast and jointly decide the stocking quantity, markdown time schedule, and markdown price to maximize mutual profit. In the absence of coordination, decisions are decentralized to optimize the individual party's objective function. Optimal coordination policy for the retailer/DCO problem setting is analyzed, and the coordination policy is compared with the uncoordinated policy to explore factors that make coordination an effective approach.

1. Introduction

For many years members of the supply chain have been separated by organization and philosophy. Interactions between them have often been adversarial, with each trying to gain at the other's expense. Today, this long-established pattern is rapidly giving way to coordination (e.g. Magretta [1998], and Buzzel and Ortmeyer [1995]). The advocates of coordination argue that all of the subsystems of a supply chain are connected. The outputs from one system are the inputs of the other systems; therefore, trying to find the best set of tradeoffs for any one system is not sufficient. Integration of the complete scope of the supply chain from the supplier through the manufacturer to the retailer needs to be considered so that fully transparent information is shared freely among members, and collective strategies can be designed to optimize the system's joint objectives. This viewpoint has recently been highlighted by several research works. For example, Parlar and Weng (1997) describe coordination between a firm's manufacturing and supply departments; Kouvelis and Gutierrez (1997) study an internationally coordinated Newsboy problem; Weng (1995, 1997) analyzes channel coordination in pricing and stocking operations; Lee (2000) considers a coordinated supply chain pricing, stocking, and return policy. Our study also emphasizes exploring coordination in the supply chains. In particular, we focus on the important implications of coordination in designing supply chain stocking and promotion policy in a fairly general problem setting.

To address the research issue, we study a simple supply chain consisting of a retailer and a discount outlet (DCO) selling a "short life cycle good" (e.g., personal computers, consumer electronics, fashion items) to possible consumers. These types of products have recently been studied extensively under the classic Newsboy inventory control problem and its extensions (e.g., Emmons and Gilbert [1998], Eppen and Iyer [1997], Fisher, Hammond, Obermeyer, and Raman [1995], and Parlar and Weng [1997].) The sale history of the good is assumed to consist of two well-defined, non-overlapping selling periods. In the first period, referred to as the Normal Sales Period (NSP), the product is sold by a retailer, where the price is set at the highest level. After the NSP, the leftovers are then moved to a discount outlet (DCO) for initiating a Promotional Sales Period (PSP), and the price is discounted permanently. We propose a coordinated decision-making process that considers two periods as a single unit, and have designed a coordination strategy to maximize the joint profit. In particular, when managers of the retailer and the DCO coordinate, they share information on the demand forecast, possibly in terms of demand distribution, and jointly decide the stocking quantity, timing of the promotional sale, and markdown price to maximize mutual profit.

We would expect that coordination between two parties should help; however, in the real-world application designing a sophisticated coordination policy between different organizational units will very likely require enormous managerial efforts, and might even prevent managers from making a prompt decision (see, for example, Souder [1981] and Shapiro [1977]). Therefore, commonly, managers in the different organizational units make a series of decentralized decisions that will only solve individual problems. In this way the complexity of the decisions is reduced since each party is treated independently of the others. Nevertheless, ignoring these decision-making units' dependencies can have costly consequences. The trade-off is between the cost of operating a complicated coordination policy versus the costly consequence of making and implementing a simple decentralized decision. Thus, when is it worthwhile to coordinate, and how should coordination proceed? In this work, we try to provide answers to these questions.

This paper is structured as follows. In § 2, a problem description, assumptions, and notations are presented. In § 3.1 and § 3.2, objective functions of the retailer and the discount outlet are formulated; then, the optimal policies for the coordination model are developed and analyzed. Finally, a brief discussion in § 4 completes the paper.

2. The Modeling Issues

The system under consideration consists of one retailer and one discount outlet (DCO) operating to meet random demand for one short life-cycle good. The sales history is divided into two well-defined, non-overlapping, consecutive selling periods. In the first period, referred to as the Normal Sales Period (NSP), the product is sold by a retailer and the price is at its highest level. After the first period, leftovers are moved to a discount outlet (DCO) for initiating a Promotional Sales Period (PSP), and the price of the item is discounted permanently.

The problem is formulated as a two-period Newsboy model with the objective of maximizing the expected profit. (See Porteus [1990] for an excellent review of the Newsboy problem.) The supplier initiates the first period (NSP) by offering a wholesale price C, for which he will sell the product to the retailer. In response to the wholesale price (C), the retailer determines an order quantity Q to order from the supplier, and a length of NSP for which he will sell the product to the consumer at price P. The length of NSP (PSP) is formulated as $0 \le \alpha \le 1$ ($1-\alpha$) fraction of the total life cycle τ, i.e., $\alpha\tau$ ($(1-\alpha)\tau$). When the product demand in the NSP exceeds availability (Q), selling opportunities are lost. However, in the complementary case, the retailer initiates the second period (PSP) by offering a wholesale price C_r, for which the retailer will sell the product to the DCO prior to the PSP. In the PSP, the DCO determines a markdown sales quantity (q), and a markdown sales price $P\gamma$, $0 \le \gamma \le 1$. Here, the fraction γ denotes the markdown price ratio (we use *markdown price ratio* to distinguish it from *markdown rate* $\bar{\gamma} = 1-\gamma$). For example, if a product with an initial list price of P=\$10 is later priced at 35% ($\bar{\gamma} = 0.35$) off the original face value, then $P\gamma$=10(0.65)=\$6.50.

The demands of the two periods are probabilistic, depending on the lapse in the sales period and/or markdown price, and assumed to be constituted of two components. The first component, representing the mean (deterministic part) of the random demand, is influenced by the lapse in the sales period and/or the markdown price. The second component, representing the shape (probabilistic part) of the random demand, is independent of the lapse in the sales period and markdown price. This two-component approach was used in the various literatures to formulate random demands due to its simplicity and flexibility. (See, for example, Leland [1972], Young [1978], Lau and Lau [1988], and Emmons and Gilbert [1998] for price-dependent random demand, and Eharhardt and Taube [1987], Gerchak, Vickson, and Parlar [1988], and Henig and

Gerchak [1990] for a random yield model.) Leland (1972), for example, has considered two price-dependent random demand functions--multiplicative and additive. The multiplicative model formulates random demand $d = h(P)\lambda$ as the product of a deterministic component ($h(P)$) and a probabilistic component (λ with $E(\lambda)=1$). Here, $h(P)$, a decreasing function of price P, denotes the mean of the random demand, and λ denotes the uncertainty of the random demand. It implies that the variance in the random demand increases as demand increases (price decreases). This assumption is supported by Lau and Lau (1988), who argue that for a low price (high demand) level beyond the normal operating range, the random demand may have a large variance due to a lack of past experience to draw on. The second demand function takes an additive form: $d = h(P) + \lambda$ and $E(\lambda)=0$. When demand is additively separable, the variance in random demand is assumed to be constant across all possible prices, which may be overly simplistic and not practical. In this work, a general multiplicative demand function is used to formulate the random demand that depends on the lapse in the sales period and/or markdown price. Let $D_N(\alpha)$ denote the mean of the retailer's random demand during the NSP. We assume that $D_N(\alpha)$ is increasing at the decreasing rate with respect to the length of the normal sales period (α), i.e., $dD_N(\alpha)/d\alpha > 0$ and $d^2 D_N(\alpha)/d\alpha^2 < 0$. That is, the increase in sales for each subsequent time period tends to be lower than for the initial periods. Similarly, let $D_P(\alpha, \gamma)$ denote the mean of the DCO's random demand during the promotional sale period (PSP). We assume that demands in PSP are concave decreasing in α, i.e., $\partial D_P(\alpha,\gamma)/\partial\alpha < 0$ and $\partial^2 D_P(\alpha,\gamma)/\partial\alpha^2 < 0$. Furthermore, we assume $\partial D_P(\alpha,\gamma)/d\gamma < 0$, and $\partial^2(\gamma D_P(\alpha,\gamma))/\partial\gamma^2 < 0$. Table 1 lists additional assumptions regarding the means of the two periods.

------------Insert Table 1 About Here----------

Finally, let $f(y)$ and $g(x)$ denote the probability density of Y and X; the random demands are formulated as products of the mean of random demands $D_N(\alpha)$ ($D_P(\alpha,\gamma)$) and probabilistic components Y (X) (which is not contingent on the markdown period or price), i.e., $YD_N(\alpha)$ ($XD_P(\alpha,\gamma)$) with $E(Y)=1$ ($E(X)=1$).

3. The Retailer-DCO Decision Processes

Section 3 develops two coordination models (CM) and an uncoordinated model for both the retailer and the discount outlet. In section 3.1, the retailer and the DCO form an alliance and jointly design an integrated ordering and sale schedule (Q,α) policy, referred to as Coordination Model 1 (CM1), for an arbitrary markdown price ratio (γ). Then, in section 3.2 an integrated ordering and markdown policy (Q,γ) is designed for an arbitrary sale schedule (α). We call the second model Coordination Model 2 (CM2). Finally, in section 3.3 an uncoordinated model (UCM) is developed for both the retailer and the discount outlet when they do not coordinate their policies.

3.1 Jointly Optimal Order Quantity and Selling Time Schedule Model:CM1

In the presence of coordination (CM1), the retailer-DCO alliance shares relevant information on the demand forecast and designs a common system to jointly decide on a promotional sale schedule to maximize the mutual profit. The alliance initially orders Q units from the supplier. If the random demand ($yD_N(\alpha)$) during the sale season (NSP, $\tau\alpha$) exceeds the availability (Q), i.e., $yD_N > Q$, selling opportunities are lost, and the profit equals $Q(P-C)$. On the other hand, if $yD_N \le Q$, the resulting profit is computed by subtracting the purchase cost CQ from the sales revenue PyD_N. Let

$I(Q,y,\alpha)=\max[Q-yD_N,0]$ denote the leftovers at the end of the NSP. The retailer-DCO alliance jointly determines a *target markdown sale quantity* q. If the leftovers I are more than q, i.e., $I(Q,y,\alpha)>q$, the retailer disposes of R units of leftovers and delivers $q=I-R$ units to the DCO. Otherwise, if $I\leq q$ the DCO will receive all of the leftovers I. In both cases the unit leftover wholesale price equals C_r.

In the second period (PSP), the expected profit can be obtained in a similar fashion. Let $D_P(\alpha|\gamma)$ denote the α dependent expected demand of the DCO for a given markdown price ratio γ. If the random demand ($xD_P(\alpha|\gamma)$) exceeds the availability (q or I), selling opportunities are lost and the profit equals $(P\gamma-C_r)$(q or I). For the complementary case, the resulting profit is computed by subtracting the purchasing cost C_r(q or I) from the sales revenue $P\gamma xD_P$. Let Π_{RD}^{CM1} denote the joint expected profit of the retailer and the discount sales outlet (the subscript "RD" denotes the retailer-DCO alliance) for CM1. Maximization of the expected joint profit, a function of order size (Q), the timing of markdown sale (α), and the target markdown sales quantity ($q=I-R$), is formulated as the sum of the two profit functions of the retailer and the DCO, and can be expressed by the following stochastic dynamic program recursion:

$$\max_{Q,\alpha}\Pi(Q,\alpha)_{RD}^{CM1}=\int_0^{\xi_N}PyD_N\,dF+\int_{\xi_N}^{\infty}PQ\,dF-CQ+E_y\{\max_R\Pi(R|I,\alpha)\},\text{ and}$$

$$\Pi(R|I,\alpha)=\int_0^{\xi_P}P\gamma xD_P\,dG+\int_{\xi_P}^{\infty}P\gamma(I-R)dG,\qquad(1)$$

where $\xi_N(Q,\alpha):=Q/D_N(\alpha)$ and $\xi_P(I,R,\alpha):=(I-R)/D_P(\alpha|\gamma)$. As a first step towards the solution of this model, we solve $\partial\Pi(R|I,\alpha)/\partial R=0$, and obtain R^*, satisfying the following policy:

$$\begin{cases} R^* = 0 & if \ \ q(\alpha) \geq I \\ R^* = I - q(\alpha) & if \ \ q(\alpha) < I \end{cases} \qquad \text{where } \ q(\alpha) = D_P(\alpha|\gamma)G^{-1}[1].$$

Assuming now $G^{-1}[1] = \infty$, then $R^* = 0$. The results show that when the retailer and the DCO coordinate, the supply chain operates as if the retailer were operating a common system. Internal transactions that do not contribute to an increase in the supply chain joint profit are eliminated, and the subsystem's efforts are channeled to maximize total-system joint profit. For example, when coordination prevails, the leftover wholesale price (C_r) or, equivalently, the discount sales outlet's unit purchase price, acting as the internal parameter, has no impact on the determination of jointly optimal policies. Therefore, as the overage cost ($C_r = 0$) of leftovers becomes irrelevant to the determination of the target markdown sales quantity (q), the retailer reserves all of the leftovers to the DCO ($R^* = 0$), and maximizes the selling opportunities.

Substituting $R^* = 0$ into the objective function in (1), the expected profit function is given by $E_y\{\Pi(R^*|I,\alpha)\} = \int_0^{\xi_N} \Pi(R = 0|I,\alpha)dF$. Let $\Pi_{RD} := \Pi(Q,\alpha)_{RD}$. Solving $\partial\Pi(Q,\alpha)/\partial\alpha = 0$ results in the following:

$$\int_0^{\xi_N} y(1 - \gamma\overline{G}(\xi_P))\frac{dD_N(\alpha)}{d\alpha}dF + \int_0^{\xi_N}\int_0^{\xi_P} \gamma x \frac{\partial D_P(\alpha|\gamma)}{\partial\alpha}dGdF = 0. \tag{2}$$

Finally, the necessary condition for the optimal order quantity Q^* maximizing (1) is given by the following expression (see Appendix 1):

$$1 - C/P = F(\xi_N) - \int_0^{\xi_N} \gamma\overline{G}(\xi_P)dF. \tag{3}$$

The retailer's centralized decision-making process will simultaneously solve for (Q,α) by Eqs. (2) and (3). Proposition 1 provides the properties of the optimal solutions.

Proposition 1.

The objective function $\Pi(Q,\alpha)_{RD}^{CM1}$ is strictly concave with respect to (Q,α).

In the next section an integrated ordering and markdown policy (Q,γ) is designed for an arbitrary sale time schedule (α) so as to maximize the retailer-DCO joint profit.

3.2 Jointly Optimal Order Quantity and Markdown Sales Price Model:CM2

Here in CM2, the optimization problem, a function of the order size (Q) and the markdown price ratio (γ) is defined by the following stochastic dynamic program recursion:

$$\max_Q \Pi(Q)_{RD}^{CM2} = \int_0^{\xi_N} P\gamma D_N \, dF + \int_{\xi_N}^{\infty} PQ \, dF - CQ + E_y\{\max_{R,\gamma} \Pi(R,\gamma|I)\}, \quad \text{and}$$

$$\Pi(R,\gamma|I) = \int_0^{\xi_P} P\gamma x D_P \, dG + \int_{\xi_P}^{\infty} P\gamma(I-R)\, dG, \tag{1.1}$$

The optimal return quantity $R^* = 0$, and the optimal order quantity (Q) are given by the necessary condition (3). Solving $\partial\Pi(R^*,\gamma|I)/\partial\gamma = 0$ gives the optimal markdown price ratio $\gamma^*(I)$ (a function of I) satisfying the following expression (see Appendix 2)

$$D_P(\gamma|\alpha) + \gamma \frac{dD_P(\gamma|\alpha)}{d\gamma} = \frac{-I\overline{G}(\xi_P)}{\int_0^{\xi_P} x dG}, \tag{4}$$

where $D_P(\gamma|\alpha)$ denotes γ dependent expected demand of the DCO for a given α. Referring to Fig. 1, the RHS in (2) is negative, and the absolute value decreases in γ and approaches 0 when γ approaches 1. Since by assumption $d^2(\gamma D_P(\gamma|\alpha))/d\gamma^2 < 0$, the LHS changes its sign from positive to negative and continuously decreases thereafter. The two properties give a unique optimal markdown price ratio γ^* that is greater than the risk-less price ratio γ_b, which satisfies $D_P(\gamma|\alpha) + \gamma\, dD_P(\gamma|\alpha)/d\gamma|_{\gamma=\gamma_b} = 0$.

---------Insert Fig. 1 About Here--------------

Proposition 2.

2.1 $\Pi(R^*, \gamma|I)$ is concave with respect to (Q, γ); and

2.2 $\Pi(Q)_{RD}^{CM2} = \theta(Q) + E_v\{\Pi(R^*, \gamma^*|I)\}$ ($\theta(Q) = \int_0^{\xi_N} P\gamma D_N\, dF + \int_{\xi_N}^{\infty} PQ\, dF - CQ$) is concave with respect to Q.

If one of the following two conditions holds

(1) $g(x)$ is an increasing function, i.e., $dg(x)/dx \geq 0$.

(2) $dg(x)/dx < 0$, and $\partial^2\Pi(R^*, \gamma|I)/\partial Q \partial\gamma < 0$. For example, this condition can be met if the demand density is an exponential distribution with $g(x) = e^{-\frac{x}{\bar{x}}}/\bar{x}$, where $\bar{x}$ is by assumption equal to 1.

Proposition 2 reveals that the objective function is concave for a strictly increasing demand density $g(x)$. It also tells us that there may be specific cases where $g(x)$ is not strictly increasing but the objective function is still concave. For example, the exponential distribution is a relatively widely used "long tailed" density that is quite probable for formulating a random demand (see, for example, Parlar and Weng [1997] and Li, Lau, and Lau [1991].)

4. Discussion and Conclusion

In this paper, we designed and tested a model to study the effects of retailer-DCO coordination in supply chain stocking and promotional markdown operations. Our focus so far does not allow us to study the possibility of a situation involving multiple markdown periods. Generally, in a real-world application a markdown operation may consist of more than one discount period. The multiple markdown period problem has been studied by Khouja (1995) on a single company level, but in his model demand in the markdown period is treated independently of markdown rate and markdown timing. Future work on the two-party progressive promotional markdown model could certainly shed further light on the topic. Finally, the problem of jointly deciding normal sales price and markdown discount rates also should receive future research effort.

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Table 1. Additional Assumptions

$$D_P(\alpha = 1) = 0\,,\quad D_P(\gamma = 1) = 0$$

$$\partial D_P/\partial\alpha\big|_{\gamma=1} = 0\,,\quad \partial D_P/\partial\gamma\big|_{\alpha=1} = 0\,,\quad \partial^2 D_P/\partial\alpha\partial\gamma > 0\,,\quad \partial^2(\partial D_P/\partial\alpha)/\partial\gamma^2 > 0\,.$$