LMI를 이용한 다중 선형 시스템의 디지탈 재설계

Digital Redesign of Multiple Linear Systems by Using LMIs

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Abstract

A new digital redesign method which can construct a digital controller for multiple linear systems is developed. The proposed method utilized the recently developed LMI theory to obtain a single digital controller which provide good state matching properties with multiple linear systems. A numerical example is provided to evaluate the feasibility of the proposed approach.

1 Introduction

Digital redesign technique is the design technique to construct a digital controller by using the predesigned analogue controller with the properties of the analogue one maintained. This technique is very useful since the most plant dynamics and design specifications are described in continuous-time. The digital redesign technique, however, has been hardly applied to the simultaneous stabilization problem. Simultaneous stabilization is an important problem in the control field. It is the problem of determining a single controller which will simultaneously stabilize a finite collection of plants. It may apply to apply to linear plants characterized by difference modes of operations or to the stabilization of nonlinear plants linearized at several equilibria. In this paper, we develop a new digital redesign method which can be applied to the stabilization of multiple linear systems.

2 Problem formulation

Consider the continuous-time linear time invariant system as shown in equation (1).

$$\begin{aligned} \dot{x}_c(t) &= A_i x_c(t) + B_i u_c(t) \\ y_c(t) &= C_i x_c(t) \end{aligned} \tag{1}$$

where, $x_c \in R^{n\times 1}$ is the state, $u_c \in R^{m\times 1}$ is the control input, and $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$, $C_i \in R^{p\times n}$ are constant matrices.

The control input applied to the system (1) is

$$u_c(t) = -K_c y_c(t) = -K_c C_i x_c(t) \tag{2}$$

where $K_c \in R^{m\times p}$.

The digital counterpart of the continuous-time controller (2) is

$$\begin{aligned} u_d(t) &= -K_d y_d(kT) = -K_d C_i x_d(kT) \\ &\text{for } kT \le t < kT + T \end{aligned} \tag{3}$$

where,

$$\begin{aligned} \dot{x}_d(t) &= A_i x_d(t) + B_i u_d(t) \\ y_d(t) &= C_i x_d(t) \end{aligned} \tag{4}$$

and $K_d \in R^{n\times p}$.

The design problem is to determine the digital controller gain K_d such that the closed-loop response of the system (1) and (3) closely matches that of the system (1) and (2)

3 Digital redesign

Consider the following augmented system.

$$\dot{z}(t) = \bar{A}_i z(t) - \bar{B}_i \bar{K} z(kT) \tag{5}$$

where

$$z(t) = \begin{bmatrix} x_c \\ x_d \end{bmatrix},$$
$$\bar{A}_i = \begin{bmatrix} A - BK_cC & 0_{n\times n} \\ 0_{n\times n} & G \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} 0_{n\times m} \\ B \end{bmatrix}$$
$$\bar{K} = \begin{bmatrix} 0_{m\times n} & K_dC \end{bmatrix},$$

By using the block-pulse function method in [1], we also obtain

$$x_c(kT+T) = (I + \frac{1}{2}HK_cC)^{-1}(G - \frac{1}{2}HK_cC)x_c(kT) \tag{6}$$

Using (6), discrete system of the hybrid system (5) can be represented as

$$z(kT+T) = (\bar{G} - \bar{H}\bar{K})z(kT) \tag{7}$$

where

$$z(kT) = \begin{bmatrix} x_c(kT) \\ x_d(kT) \end{bmatrix}$$
$$\bar{G} = \begin{bmatrix} (I + \frac{1}{2}HK_c)^{-1}(G - \frac{1}{2}HK_c) & 0_{n\times n} \\ 0_{n\times n} & G \end{bmatrix}$$
$$\bar{H} = \begin{bmatrix} 0_{n\times m} \\ H \end{bmatrix}$$
$$\bar{K} = \begin{bmatrix} 0_{m\times n} & K_dC \end{bmatrix}$$

The corresponding cost function is defined as

$$J_c = \sum_{k=0}^{\infty} J(kT) \tag{8}$$

where [2]

$$\begin{aligned} J(kT) &= \tfrac{1}{2}(x_c(kT) - x_d(kT))^\top Q(x_c(kT) - x_d(kT)) \\ &= \begin{bmatrix} x_c(kT)^\top & x_d(kT)^\top \end{bmatrix} \begin{bmatrix} Q & -Q \\ -Q & Q \end{bmatrix} \\ &\quad \times \begin{bmatrix} x_c(kT) \\ x_d(kT) \end{bmatrix} \end{aligned}$$

Therefore, the digital redesign problem is converted to the standard discrete-time controller design problem. The following is the main result of this paper.

Theorem 1 *The digital controller K_d is a digitally redesigned controller if there exist a symmetric positive definite matrix Γ and a matrices $\bar{F} = \bar{K}\Gamma$ such* *that the following LMI's are satisfied.*

$$\Gamma > 0 \tag{9}$$

$$\begin{bmatrix} \Gamma & * & * \\ \bar{Q}^{1/2}\Gamma & I & * \\ \bar{G}_i\Gamma - \bar{H}_iF & 0 & \Gamma \end{bmatrix} > 0 \tag{10}$$

If above inequalities has feasible solutions $\Gamma > 0$ and F, then the digital controller K_d is

$$K_d = F(C\Gamma_2)^{-1} \tag{11}$$

where $\Gamma_2 \in R^{n\times 2n}$ and satisfies

$$\Gamma = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix}$$

Proof: The proof is omitted here for page limitation

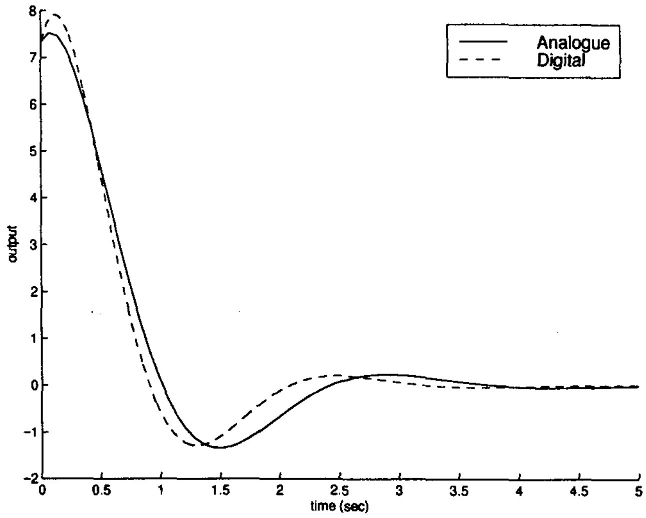

Figure 1: Comparison of steady-state output responses for the 1st model

4 Simulation

Consider a set of following systems

$$\begin{aligned} \dot{x}_c(t) &= A_ix_c(t) + B_iu_c(t) \\ y_c(t) &= Cx_c(t), \quad (i = 1, 2) \end{aligned} \tag{12}$$

where,

$$A_1 = \begin{bmatrix} 6.0199 & 8.7345 \\ 2.5356 & 5.1340 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 7.3265 \\ 4.2223 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 9.6137 & 5.5341 \\ 0.7206 & 2.9198 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 8.5796 \\ 3.3576 \end{bmatrix}$$
$$C = \begin{bmatrix} 6.802 & 0.5344 \end{bmatrix}$$

We choose the sampling period $T = 0.05$ s. Sampling (A_i, B_i, C) with the sampling period T using ZOH, we have

$$\begin{aligned} x_d(kT + T) &= G_ix_d(kT) + H_iu_d(kT) \\ y_d(kT) &= Cx_d(kT), \quad (i = 1, 2) \end{aligned} \tag{13}$$

where,

$$G_1 = \begin{bmatrix} 1.3882 & 0.5826 \\ 0.1691 & 1.3291 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.4875 \\ 0.2712 \end{bmatrix}$$
$$G_2 = \begin{bmatrix} 1.6244 & 0.3809 \\ 0.0496 & 1.1637 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.5805 \\ 0.1907 \end{bmatrix}$$

The stabilizing feedback controller law is found to be $K_c = 0.2612$ and the digitally redesigned gain is $K_d = 0.2548$. The output responses of the designed systems are shown in Fig. 1 - 2 and Figure 3 - 4 show the control inputs.

5 Conclusion

A new digital redesign method which can construct a digital controller for multiple linear systems is developed. A numerical example is provided to evaluate the feasibility of the proposed approach.

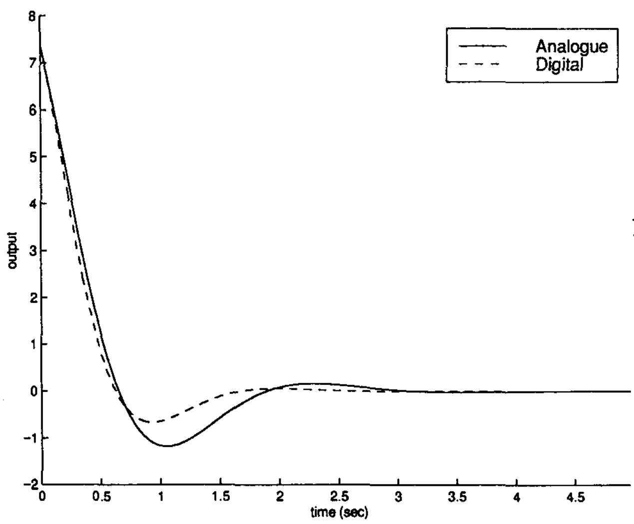

Figure 2: Comparison of steady-state output responses for the 2nd model

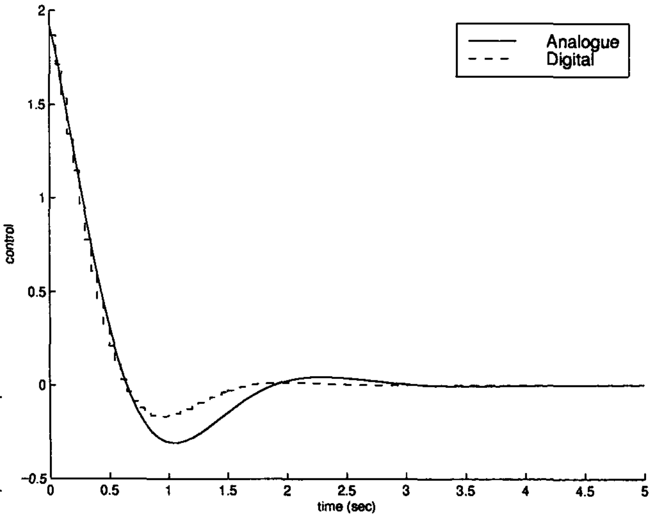

Figure 4: Comparison of steady-state control signals for the 2nd model

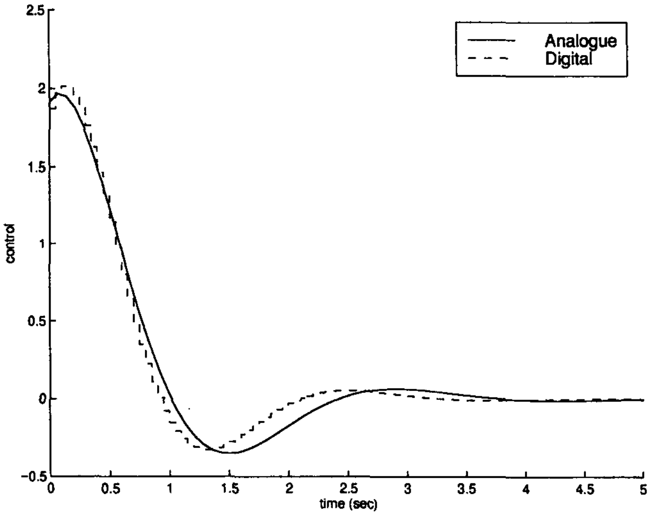

Figure 3: Comparison of steady-state control signals for the 1st model

References

[1] Y. H. Joo, G. Chen, and L. S. Shieh, "Hybrid state-space fuzzy model-based controller with dual-rate sampling for digital control of chaotic systems," *IEEE Transactions on Fuzzy Systems*, vol. 7, pp. 394–408, Aug. 1999.

[2] L. E. Sheen, J. S. H. Tsai, and L. S. Shieh, "Optimal digital redesign of continuous-time systems with input time delay and/or asynchronous sampling," *Journal of Frankling Institute*, vol. 335B, no. 4, pp. 605–616, 1996.