제2종 퍼지집합과 그 응용

Type-2 fuzzy sets and their applications

장이채
건국대학교 자연과학대학 전산수학부 응용수학전공

Lee-Chae Jang
Department of Applied Mathematics, KonKuk University
Choongju, Choongbuk 380-701, Korea. Email : leechae.jang@kku.edu

Abstract : In this paper, we are interested in counting the number of elements of a type two fuzzy set. Using concepts of type-two fuzzy sets, we can obtain some properties of these concepts and some results of possibility of type-two fuzzy sets.

1 Preliminaries and definitions

Let R be the real numbers and $[0, 1]$ the unit interval in R. We denote the set of all fuzzy numbers on R by $\mathcal{F}(R)$. In 1965(Zadeh [8]), a fuzzy set was defined by a function $f : X \longrightarrow [0, 1]$, where X is an ordinary set. Zadeh [8], A.L. Ralescu [4], D. Ralescu [5], Wygralak [7], Dubois and Prade [1] investigated concepts of cardinality of a fuzzy set and obtained some properties of these new concepts. A statement such as " most students are smart" has a truth value between 0 and 1 ; such a statement is of the general form " Qx's are A " where Q is a fuzzy quantifier and A is a fuzzy subset of a (finite) universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$.

D. Ralescu[6] and A.L. Ralescu[5] discussed some properties of card A, the cardinality of fuzzy set A. This should be a fuzzy subset of $\{0, 1, \cdots, n\}$, with card$A(k)$ being interpreted as the possibility that A has exactly k elements $(0 \leq k \leq n)$. Also, A. L. Ralescu[5] investigated that card$A$ was k to the extent to which exactly k elements of X belong to A while the other $(n - k)$ elements do not belong to A and obtained the following explicit formular for the fuzzy cardinality:

$$\text{card}A(k) = \mu_{(k)} \wedge (1 - \mu_{(k+1)}), \quad k = 0, 1, \cdots, n$$

where $\mu_{(1)}, \mu_{(2)}, \cdots, \mu_{(n)}$ are the values of $\mu_A(x_1), \cdots, \mu_A(x_n)$ arranged in decreasing order of magnitude, and $\mu_{(0)} = 1, \mu_{(n+1)} = 0$.

We introduce a definition of f-card F(see [2,3]), that is, the cardinality of a type-two fuzzy set F with $m_F : X \longrightarrow \mathcal{F}_0(R)$ by $m_F(x_i) = \mu_{a_{(i)},c_{(i)},d_{(i)}}$ for $i = 1, 2, \cdots, n$, where $\mathcal{F}_0(R)$ is the collection of all functions $\mu_{a_i,c_i,d_i}(z)$, for $i = 1, 2, \cdots, n$, defined by

$$\mu_{a_i,c_i,d_i}(z) = \begin{cases} 1 & z \in [r_i, s_i] \neq \emptyset \\ L_i(z) & z < r_i \\ R_i(z) & z > s_i \end{cases}$$

where $a_i(i = 1, 2, \cdots, n)$ are some real numbers, $r_i = \frac{m-1}{m}a_i$ and $s_i = \frac{m+1}{m}a_i$, the functions L_i are straight lines through points $(1, r_i), (c_i, 0)$ and the functions R_i are straight lines through points $(1, s_i), (d_i, 0)$, for some numbers c_i, d_i and $i = 1, 2, \cdots, n$. It is easily to show that $\mathcal{F}_0(R)$ is a subcollection of $\mathcal{F}(R)$. And also, we denote the set of such type-two fuzzy sets by $\mathcal{F}_2(X)$.

Definition 1.1 The type-two fuzzy cardinality of $F \in \mathcal{F}_2(X)$ is a function f-card F : $\{0, 1, \cdots, n\} \longrightarrow \mathcal{F}_0(R)$ defined by

$$\text{f-card}F(k) = \mu_{a_{(k)},c_{(k)},d_{(k)}} \wedge \mu_{1-a_{(k+1)},1-d_{(k+1)},1-c_{(k+1)}} \quad \text{for each } k = 0, 1, 2, \cdots, n$$

where $a_{(0)} \geq a_{(1)} \geq \cdots \geq a_{(n)}$ are the ordered values of the a_i's, $a_{(0)} = 1$ and $a_{(n+1)} = 0$.

In the above definition, it is not necessary that $\{c_{(i)}\}$ and $\{d_{(i)}\}$ be non-increasing and the operation $\wedge$ is defined by

$$\mu_{a,c,d} \wedge \mu_{b,e,f} = \mu_{a \wedge b, c \vee e, d \wedge f}$$

By the definition of f-card F, it is easily to show that f-card F is an element of $\mathcal{F}_2(\{0, 1, 2, \cdots, n\})$ which is the set of type-two fuzzy sets F with their membership functions defined by m_F : $\{0, 1, \cdots, n\} \longrightarrow \mathcal{F}_0(R)$.

In references[2,3], we defined generalized concepts of cardinality of type-two fuzzy sets. Using concepts of cardinality of type-two fuzzy sets, we can obtain some properties of these concepts. And also, we discuss some results of possibility of type-two fuzzy sets.

2 Main Results.

In references [2,3], we introduced the concept of complement (denoted by $\overline{F}$) of F in $\mathcal{F}_2(X)$(see [2,3]), that is, the membership $\overline{\mu}$ of $\overline{F}$ is defined by

$$\overline{\mu}_{a_{(i)},c_{(i)},d_{(i)}} = \mu_{1-a_{(n-i+1)},1-d_{(n-i+1)},1-c_{(n-i+1)}}$$

for $i = 1, 2, \cdots, n$.

Proposition 2.1 f-card $F(k) = \mu_{1,1,1}$ if and only if F is a nonfuzzy set with k elements.

We note that the membership function of a non-fuzzy set F with k-element is represented by

$$m_F = \begin{pmatrix} x_1 & \cdots & x_k & x_{k+1} & \cdots & x_n \\ \mu_{1,1,1} & \cdots & \mu_{1,1,1} & \mu_{0,0,0} & \cdots & \mu_{0,0,0} \end{pmatrix}$$

When we discuss the possibility of a type-two fuzzy statement, we need the following definitions and propositions.

Definition 2.2 Let $\mu_{a,c,d},\ \ \mu_{b,e,f} \in \mathcal{F}_0(R)$. Then, the maximum of $\mu_{a,c,d}$ and $\mu_{b,e,f}$ is defined by

$$\mu_{a,c,d} \vee \mu_{b,e,f} = \mu_{a\vee b, c\vee e, d\vee f}$$

Proposition 2.3([2,3]) Let $F \in \mathcal{F}_2(X)$. Then we have f-card $\bar{F}(i) =$ f-card $F(n-i)$

Now, we discuss a representation of f-card$F$ as in references[2,3,6]. Since

$$a_{(k)} \wedge [1 - a_{(k+1)}] = \begin{cases} a_{(k)}\ , & \text{if } a_{(k)} + a_{(k+1)} \leq 1 \\ 1 - a_{(k+1)}, & \text{otherwise,} \end{cases}$$

there is a unique value j such that the following inequalities hold :

$$1 + a_{(0)} \geq a_{(1)} + a_{(2)} \geq \cdots \geq a_{(j-1)} + a_{(j)} > 1 \geq a_{(j)} + a_{(j+1)} \geq \cdots \geq a_{(n-1)} + a_{(n)} \geq a_{(n)} \geq 0$$

Thus the definition 2.1 gives

$$\text{f-card}F(k) = \begin{cases} \mu_{1-a_{(k-1)}, f_{(k)}, g_{(k)}} & \text{for } 0 \leq k \leq j-1 \\ \mu_{a_{(k)}, f_{(k)}, g_{(k)}} & \text{for } j \leq k \leq n \end{cases}$$

and we obtain the following representation of f-card F. It is easily to show that f-card F lies in $\mathcal{F}_0(\{0, 1, \cdots, n\})$, that is,

$$\begin{pmatrix} 0 & 1 & \cdots & j-1 & j & j+1 & \cdots & n \\ \mu_{1-a_{(1)}, f_{(0)}, g_{(0)}} & \mu_{1-a_{(2)}, f_{(1)}, g_{(1)}} & \cdots & \mu_{1-a_{(j)}, f_{(j-1)}, g_{(j-1)}} & \mu_{a_{(j)}, f_{(j)}, g_{(j)}} & \mu_{a_{(j-1)}, f_{(j-1)}, g_{(j-1)}} & \cdots & \mu_{a_{(n)}, f_{(n)}, g_{(n)}} \end{pmatrix} \quad (2.1)$$

where $f_{(k)} = c_{(k)} \wedge [1 - d_{(k+1)}]$ and $g_{(k)} = d_{(k)} \wedge [1 - c_{(k+1)}]$ for $k = 0, 1, \cdots, n$, and $a_{(0)} \geq a_{(1)} \geq \cdots \geq a_{(n)}$. But, it is not necessary that $\{f_{(i)}\}$ and $\{g_{(i)}\}$ are non-increasing.

Using the national number j and (2.1), we can define the following possibilty that F has exactly k elements.

Definition 2.4 The possibility $\mathrm{Poss}\{\mathrm{f}-\mathrm{card}F = k\}$ that F has exactly k elements is defined by

$$\mathrm{Poss}\{\mathrm{f}-\mathrm{card}F=k\} = \begin{cases} \mu_{a_{(k)},f_{(k)},g_{(k)}} \ , & \text{if } k \geq j \\ \mu_{1-a_{(j)},f_{(j-1)},g_{(j-1)}} \vee \mu_{a_{(j)},f_{(j)},g_{(j)}} & \text{if } k < j \end{cases}$$

Proposition 2.5 Let $F \in \mathcal{F}_2(X)$. Then we have

$$\mathrm{Poss}\{\mathrm{f}-\mathrm{card}F \geq k\} = \begin{cases} \mu_{a_{(k)},p_{(k)},q_{(k)}} \ , & \text{if } k \geq j \\ \mu_{(1-a_{(j)})\vee a_{(j)},p_{(j-1)},q_{(j-1)}} & \text{if } k < j \end{cases}$$

where j is as defined in (2.1) and $p_{(k)} = f_{(k)} \vee \cdots \vee f_{(n)}$, $q_{(k)} = g_{(k)} \vee \cdots \vee g_{(n)}$

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