Combined Traffic Signal Control and Traffic Assignment : Algorithms, Implementation and Numerical Results

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ABSTRACT

Traffic signal setting policies and traffic assignment procedures are mutually dependent. The combined signal control and traffic assignment problem deals with this interaction. With the total travel time minimization objective, gradient based local search methods are implemented. Deterministic user equilibrium is the selected user route choice rule, Webster's delay curve is the link performance function, and green time per cycle ratios are decision variables. Three implemented solution codes resulting in six variations include intersections operating under multiphase operation with overlapping traffic movements. For reference, the iterative approach is also coded and all codes are tested in four example networks at five demand levels. The results show the numerical gradient estimation procedure performs best although the simplified local searches show reducing the large network computational burden. Demand level as well as network size affects the relative performance of the local and iterative approaches. As demand level becomes higher, (1) in the small network, the local search tends to outperform the iterative search and (2) in the large network, *vice versa.*

This paper is based on Chungwon Lee's Ph.D. dissertation.

INTRODUCTION

From the transportation planning perspective, traffic assignment models are used to forecast network flow patterns, commonly assuming that capacities decided by network supply parameters such as signal settings are fixed during a short time period with a given particular origin-destination matrix (OD). On the other hand, from the transportation engineering perspective, network flow patterns are commonly assumed fixed during a short time period and control parameters are optimized in order to improve some performance index for prevailing flow patterns. The input flow patterns must either be observed or forecasted through traffic assignment.

The two processes, traffic assignment and signal optimization, are usually dealt with separately; however, the processes mutually influence each other. This mutual interaction can be explicitly considered by effective integration of these two processes, producing the so-called combined control and assignment problem. When drivers follow Wardrop's first principle (*1*), i.e., user equilibrium (UE) flow, the problem is called the equilibrium network traffic signal setting problem, which is normally nonconvex and obtaining explicit gradient information for any gradient based algorithm application is difficult.

Signal setting parameters consist of several decision variables such as green time per cycle ratios, cycle length, phase sequence and offset. Most models optimize a subset of these decision variables assuming the others and traffic demand to be fixed. Changing signal settings may stimulate drivers to adjust route choices; however, changing flow may suggest re-setting signals. Allsop (*2*) first noted the necessity of combining signal calculations and traffic assignment by pointing out that network traffic routing according to Wardrop's first principle is dependent on signal timings and should ideally be regarded simultaneously with signal timing. Gartner (*3*) supported the same point. Allsop suggested an iterative procedure to solve such a problem, which decomposed the problem into two well-researched subproblems as in Figure 1. The assignment uses link performance functions derived by the signal optimization subproblem. Signal optimization is performed with flow patterns provided through the assignment subproblem. In the literature, this is called the iterative optimization and assignment (IOA) procedure, or simply iterative approach (*4,5*). The procedure continues until it converges to a solution, which is called mutually consistent because the flow is at UE and the signal setting is optimal. Allsop's conceptual algorithm was extended by Allsop and Charlesworth (*6*), in which the signal optimization is solved by TRANSYT (*7*). Allsop and Charlesworth carried out the procedure on a six-intersection network. Quite distinct mutually consistent solutions, i.e., different flow and green time patterns, were found but indicated similar total network travel times. Tan et al. (*8*) expressed UE flow pattern as a set of constraints and suggested an Augmented Lagrangian Method for solutions. However the method is inappropriate for large networks because of path enumeration.

Dickson (*9*) and Smith (*10*) noted that the above iterative method is not guaranteed to converge even to a local optimum. Sheffi and Powell (*11*) suggested a local search methodology and solved a small network problem with a simplified link performance function. Smith (*12*) proposed a new signal control policy P_O with a capacity maximizing property, which is different from conventional delay minimization or Webster's equisaturation policy (*13*). Smith and Van Vuren (*14*) analyzed the convergence and uniqueness of solutions by the iterative approach for a large collection of control policies. They showed that the link performance functions and the control policies affect the convergence and uniqueness of mutually consistent solutions. Van Vuren and Van Vliet (*15*) performed a comprehensive experimental study. Cantarella et al. (*16*) described the behavior of the iterative procedure graphically. Cantarella and Sforza (*17*) included an offset optimizer in the iterative procedure. Gartner and Al-Malik (*18*) introduced a simultaneous approach for both route choice behavior and optimal signal setting by representing signal control variables as equivalent flow variables at two phase operating intersections. Yang and Yagar (*19*) developed a gradient descent algorithm to utilize the sensitivity analysis procedure. Cascetta et al. (*20*) implemented gradient based algorithms with stochastic equilibrium assignment.

The equilibrium network traffic signal setting problem is known to be not necessarily convex and therefore it may have multiple local solutions. Thus, there is a possibility that some local and mutually consistent solutions show very poor performance compared to global solutions. Recently, the authors implemented two stochastic global optimizations, simulated annealing and a genetic algorithm, and presented preliminary test results for exploring whether local and iterative approaches severely impair solution quality (*21, 22*).

NOTATION

Symbol	*Definition*
CL	cycle length,
λ_a	green time ratio for intersection link or movement a,
s_a	saturation flow rate for intersection link or movement a,
t_a	travel time on link or movement a,
t_o	free flow travel time of nonintersection link,
x_a	flow on link or movement a.

OBJECTIVES, ASSUMPTIONS AND FORMULATION

Objectives

The main objectives of this study are to implement and extend Sheffi and Powell's gradient based local searches to be able to solve complex signal schemes in large networks. Additionally, the objectives include comparing the local searches with the iterative approach at different demand levels and in different size networks regarding solution quality and convergence pattern, and testing the simplified gradient estimation procedure to see if it can reduce the computational burden without losing solution quality.

Assumptions

To keep the problem manageable, the following assumptions are made.

- The control policy for signal settings is total travel time minimization.
- Traffic assignments are steady-state with fixed OD. The driver route choice rule is minimum time path selection so that drivers follow deterministic UE. The Frank-Wolfe algorithm (*23*) is used to solve the UE flow problem.
- The following Webster's two-term delay function is selected for intersection delay modeling and link cruise time is assumed constant.

$$t = \frac{CL \cdot s(1-\lambda)^2}{2(s-x)} + \frac{x}{2s \cdot \lambda(s \cdot \lambda - x)} \tag{1}$$

  Webster's delay function is incompatible with the Frank-Wolfe algorithm when flow exceeds capacity since the algorithm contains a series of all-or-nothing assignments, which may cause flows on some links to be more than their capacities during the iterations. Thus, link costs must be defined throughout the whole flow region. The linear adaptation to combine deterministic queueing and Webster's curve at some flow level where the two curves show the same slope as in Figure 2 is chosen to resolve this problem.
- Only networks with isolated signal control are investigated since including the offset effect *analytically* into the theoretical relationships between flow and control parameters has not yet been properly resolved. Green time is the design

signal parameter and cycle length is assumed fixed. All phases are protected and phase sequences are exogenous.

Formulation

The policy to minimize the total travel time induces the following equilibrium network traffic signal setting problem, P1.

P1:

$$z = \sum_{a} t_a(x_a, \lambda_a) \cdot x_a \tag{2a}$$

subject to:

$$\lambda^{min} \leq \lambda \leq \lambda^{max} \tag{2b}$$

$$x \sim UE \tag{2c}$$

$$x \geq 0 \text{ and } \lambda \geq 0 \tag{2d}$$

Since there is a unique feasible equilibrium flow vector x^* for any feasible λ, i.e., x^* is uniquely decided by λ, P1 can be transformed to P2.

P2:

$$z = \sum_{a} t_a\left(x_a^*(\lambda), \lambda_a\right) \cdot x_a^*(\lambda) \tag{3a}$$

subject to:

$$\lambda^{min} \leq \lambda \leq \lambda^{max} \tag{3b}$$

$$x \geq 0 \text{ and } \lambda \geq 0 \tag{3c}$$

For simplicity, x^* will be denoted by x. Two difficulties in solving P1 or P2 have been frequently mentioned (*24*). First, due to the problem nonconvexity, z may have many local minima. Thus, any gradient-based search will find only a local minimum. Second, z requires knowledge of the OD pattern, which is not easily developed for large realistic networks. The iterative approach has been the most practical alternative strategy.

LOCAL SEARCH

Analytical Method

When performing a local search for P2, computing the gradient of z must be considered first. Typical terms of the gradient of z are given by

$$\frac{\partial z(\lambda)}{\partial \lambda_k} = \sum_a \left\{ \left[t_a(x_a(\lambda),\lambda_a) + x_a(\lambda)\frac{\partial t_a(x_a,\lambda_a)}{\partial x_a} \right] \frac{\partial x_a(\lambda)}{\partial \lambda_k} \right\} + x_k(\lambda)\cdot\frac{\partial t_k(x_k,\lambda_k)}{\partial \lambda_k} \qquad (4)$$

The term $\frac{\partial x_a}{\partial \lambda_k}$ is the partial derivative of the equilibrium flow on a link with respect to the green split on another. Because it is not possible to derive this term analytically, Equation 4 is very difficult to use directly. Sheffi and Powell suggested an approximate method as

$$\frac{\partial x_a(\lambda)}{\partial \lambda_k} \cong \frac{x_a(\ldots,\lambda_k+\Delta_k,\ldots) - x_a(\ldots,\lambda_k,\ldots)}{\Delta_k} \qquad (5)$$

Equation 5 numerically approximates $\frac{\partial x_a}{\partial \lambda_k}$ by the relative equilibrium flow change on link a to the green split change on link k. In Equation 4, one more noticeable term is $\frac{\partial t_k}{\partial \lambda_k}$. It could affect $\frac{\partial z}{\partial \lambda_k}$ severely and give very different values depending on the λ location due to the linear adaptation of Webster's curve. Here two different methods can be devised:

- Derive and use the analytical form of $\frac{\partial t_k}{\partial \lambda_k}$ (6a)
- Use a numerically approximate method: $\frac{\partial t_k}{\partial \lambda_k} \cong \frac{t_k(\cdot,\lambda_k+\Delta) - t_k(\cdot,\lambda_k)}{\Delta}$ (6b)

Numerical Method

The above expressions contain both analytical and numerical terms. Thus it is also natural to use a full version of numerical gradient estimation to avoid bothersome differentiation of some complex delay functions as,

$$\frac{\partial z}{\partial \lambda_k} \cong \frac{z(\ldots,\lambda_k+\Delta,\ldots)-z(\ldots,\lambda_k,\ldots)}{\Delta} \tag{7}$$

Simplified Method

To obtain gradient information, the above two methods need at least one equilibrium assignment iteration for every stage, which is inefficient as the network size grows. Sheffi and Powell suggested, although not tested in their work, a simplified gradient approximation for a large network by assuming that the main gradient portion of stage k comes from stage k itself as follows:

$$\left|\sum_{a\neq k}\left(t_a(\cdot,\cdot)+x_a\cdot\frac{\partial t_a(\cdot,\cdot)}{\partial x_a}\right)\cdot\frac{\partial x_a(\cdot)}{\partial \lambda_k}\right| << \left|\left(t_k(\cdot,\cdot)+x_k\cdot\frac{\partial t_k(\cdot,\cdot)}{\partial x_k}\right)\cdot\frac{\partial x_k(\cdot)}{\partial \lambda_k}\right| \tag{8}$$

As Sheffi and Powell explained, this conjecture may be accurate because of two reasons: (1) $\frac{\partial x_k(\cdot)}{\partial \lambda_k} >> \frac{\partial x_a(\cdot)}{\partial \lambda_k}$ for $a \neq k$, since the network structure may tend to absorb changes; and (2) the terms in the sum in the left-hand side of Equation 8 may be of undetermined signs and may, therefore, cancel each other. Based on this idea, Sheffi and Powell suggested a simplified procedure to estimate all the gradient information by one new equilibrium assignment as follows:

- STEP 1: Perturb λ so all the splits have been increased by Δ,
  $\lambda' = (\lambda_a+\Delta, \lambda_b+\Delta, \lambda_c+\Delta, \ldots)$
- STEP 2: Perform one new equilibrium assignment with λ'.
- STEP 3: Compute partial derivatives by

$$\frac{\partial z}{\partial \lambda_k} \cong \frac{\partial z_k(\lambda)}{\partial \lambda_k} = \left[t_k + x_k\frac{\partial t_k}{\partial x_k}\right]\frac{\partial x_k}{\partial \lambda_k} + x_k\frac{\partial t_k}{\partial \lambda_k} \tag{9}$$

where $\frac{\partial x_k}{\partial \lambda_k} = \frac{x_k(\lambda')-x_k(\lambda)}{\Delta}$. $\frac{\partial t_k}{\partial \lambda_k}$ is either in the analytical or numerical form as in Equation 6. Additionally $\frac{\partial z}{\partial \lambda_k}$ can be numerically estimated as

$$\frac{\partial z}{\partial \lambda_k} \cong \frac{\partial z_k}{\partial \lambda_k} = \frac{z_k(\lambda') - z_k(\lambda)}{\Delta} \tag{10}$$

This procedure would therefore greatly reduce the computational burden of gradient estimation for large networks. Naturally, however, such a procedure may not be guaranteed to converge to any minimum. Sheffi and Powell hypothesized that in large networks the results may be close to the true local minimum. The experiment to justify this argument has not yet been reported in the literature.

In summary, three different local searches depending on gradient calculation method are implemented: "analytical", "numerical" and "simplified." In addition, $\frac{\partial t_k}{\partial \lambda_k}$ is evaluated in two ways: analytically or numerically. Therefore, three different estimations and two different $\frac{\partial t_k}{\partial \lambda_k}$ evaluation techniques constitute six combinations as in Table 1.

Local Search Algorithm

The local search algorithm is summarized as follows:

- STEP 0: (Initialization) Obtain feasible splits, and set n=0.
- STEP 1: (Updating) Calculate travel time for given splits and perform UE assignment.
- STEP 2: (Gradient) Calculate $\frac{\partial z}{\partial \lambda_a}$.
- STEP 3: (Descent direction) Decide descent direction and maximum step size.
- STEP 4: (Step size determination) Find an optimum step size and update splits.
- STEP 5: (Convergence test) Return to STEP 1 until stopping criterion is met.

An initial signal setting is used to find initial UE flow. Then, gradient is calculated, and descent direction (d) and an optimum step size are determined with the gradient projection method to maintain feasibility. The search stops when d is close to zero, n becomes the maximum iteration, or change of green splits is minimal.

Chain Rule for Complex Multiphase Operations

The gradient $\frac{\partial z}{\partial \lambda_a}$ is the rate of total travel time change with respect to the green time ratio change of movement a. In signal control, however, green time can be changed stage by stage, rather than movement by movement. Thus, when the signal control is complex, such that one movement receives green during more than one stage, a stage gradient is actually required. By the chain rule, the stage gradient can be calculated with Equation 11.

$$d_t = \sum_{a \in S_t} \frac{\partial z}{\partial \lambda_a} \tag{11}$$

where d_t is the gradient of stage t and S_t is a set of the movements that receive green during stage t. To save space, the detailed proof is omitted. Finally, when an intersection has N stages, at least one stage green time ratio should be decided by the other N-1 stages. The former and the latter will be called dependent and independent stage(s), respectively.

Relation between Local and Mutually Consistent Solutions

It is useful to clarify why the local and mutually consistent solutions are different before the numerical results are given. Imagine a network with two independent variables: x and λ. Now, denote the system objective function $z(\lambda, x)$ as contours in the (λ, x) space in Figure 3. Then, two imaginary curves can be defined:

- Curve P: the signal optimization with an input flow x such as $\lambda = P(x)$.
- Curve E: the UE assignment with a fixed green split λ such as $x_{UE} = E(\lambda)$.

In Figure 3 (a), z^* is the optimum with UE flow, and z_C is a so-called mutually consistent point: here x_C is UE when the signal is fixed at λ_C and λ_C is optimal when flow is fixed at x_C. z^* and z_C can be identical but naturally will be different, and usually z^* cannot be worse than z_C. This relation may not be true, however, if there are multiple valleys, caused by the nonconvexity problem, as in Figure 3 (b). When z^* and z_C are located in different valleys, z^* can be worse than z_C. For a large network, obtaining a global optimum among many local optima is not an easy task.

EXPERIMENT

Network Representation

To explicitly model directional movements, intersections are coded as a set of links, representing all possible movements from the upstream approaches as shown in Figure 4. Meneguzzer (*25*) used this representation. In the figure, links serving intersection turning movements are called "intersection links" (see "a") while others (see "b") are called "nonintersection links."

Experimental Scheme

To compare with the local solutions, the IOA procedure is implemented. Signal setting is optimized utilizing the *pressure* concept in Smith et al. (*26*). Different control optimization policies will find different mutually consistent points. Smith (*12*) devised the so-called P_0 policy and its variations. According to Van Vuren and Van Vliet (*15*), no superior policy was found over different OD levels. In this study, total time minimization is the selected control policy of the IOA.

To test the IOA and six variations of three local searches, four example networks in Figure 5, 6 and 7 were chosen. Figure 5 contains two simple networks denoted by "VV", and "2x1." Figure 6 is a medium size network denoted by "MED", having 10 zone centroids and 11 signalized intersections with multiphase operation and overlapping movements, 40 total phases, 108 intersection links and 158 total links. Figure 7 is a network located in Austin, Texas, USA, denoted by "AST", consisting of 21 zone centroids and 27 signalized intersections with multiphase operation and overlapping movements, 83 total phases, 267 intersection links and 323 total links.

Network congestion can affect the relative performance of the algorithms. Five different OD demand levels from 1 to 5 are selected so that their network wide volume per capacity ratios show 0.1, 0.3, 0.5, 0.7 and 0.9, respectively. Initial control setting is an important factor of solution quality because of the nonconvexity. Thirty-two different initial settings are selected. Simplified local searches are not applied for the two simple networks since they are originally devised for bigger networks. Therefore the experiment has 3520 total cases:

- VV: $640 = 4(\text{codes}) \cdot 5(\text{demandlevels}) \cdot 32(\text{initialsettings})$
- 2x1: $640 = 4(\text{codes}) \cdot 5(\text{demandlevels}) \cdot 32(\text{initialsettings})$
- MED: $1120 = 7(\text{codes}) \cdot 5(\text{demandlevels}) \cdot 32(\text{initialsettings})$
- AST: $1120 = 7(\text{codes}) \cdot 5(\text{demandlevels}) \cdot 32(\text{initialsettings})$
- Total cases: $3520 = 640 + 640 + 1120 + 1120$

When any numerical approximation is involved for gradient calculation, Δ is set to 0.05, which means 3 seconds when the cycle length is 60 seconds. If Δ is too small, the derivative estimate is subject to roundoff noise; whereas if Δ is too large, it no longer measures the local gradient. According to the test, 3 seconds was an appropriate Δ value.

Experiment Results

Means of total travel times are summarized in Table 2 and Figure 8. For the VV and 2x1 networks, the numerical local search is overall superior or comparable to the two analytical local searches and IOA at all demand levels. For the VV network, at high demand levels, the numerical local search outperformed the IOA, which was a bit weaker in the 2x1 network. For the MED and AST networks, again the numerical local search performed better than other local searches but was outperformed by the IOA at high demand levels. According to the paired t-test, the solutions of the IOA and local searches were significantly different at high demand levels (4 and 5) showing p-values less than 0.005. This relative performance that the IOA is superior at high demand in a large network while local searches are superior at high demand in a small network is illustrated in Figure 9 and can be ascribed to the following:

> When the network is small, there may be very few distinct local solutions, which can be found by local searches. Although the mutually consistent solution is intrinsically suboptimal, it is quite similar to the local solutions when demand level is low, and as demand grows, the difference grows. On the other hand, for the large network, there may be many local or quasi-local solutions. Thus, any local search can be easily trapped to worse solutions if the initial solution is not in a good domain neighborhood. Since the IOA includes a signal optimization procedure, it finds a good solution showing small total travel time, which may not be mutually consistent until convergence, whether the initial solution is in a good domain neighborhood or not. Then the search drifts to find a mutually consistent point. When the network is large and demand is high, there may be many mutually consistent points so finding one around the signal optimized point is likely.

The simplified local searches do not impair the solution quality much as shown in Table 2 for the MED and AST networks, and do not significantly increase iterations as shown in Table 3. Moreover, they reduce total UE repetitions compared with their regular local searches, (i.e., simplified-A vs. analytical-A, simplified-B vs. analytical-B and simplified-C vs. numerical). Therefore, when fast computing is required, this method can be tested as an alternative. Generally searches using numerical gradient estimation work better than those containing analytical components.

Regarding convergence, most local searches take 2 to 4 iterations to converge while the IOA averages 5 to 15 iterations as shown in Table 3. Figure 10 shows the objective value improvement versus the iteration number of the local and IOA procedures. Although the IOA requires a bit more iterations, the UE iterations involved in the optimum step size decision of the local searches are not negligible as shown in Table 3. To reduce this local search computational burden, many search algorithms utilize a streamlined method such as a predetermined step size or a limited maximum number of line searches. This study, however, does not include these. On the other hand, the IOA also involves a line search for optimal green swapping decisions in the control optimization procedure, which can be efficiently performed.

CONCLUSION

The combined control and assignment problem is examined focusing on two different approaches, gradient based and iterative approaches, under a planning (off-line) perspective. Sheffi and Powell's gradient based local search algorithm has been implemented in three different codes. To include different gradient estimation methods for Webster's curve, six cases by the three codes were investigated. The iterative approach was also coded to compare the solution quality.

According to the comprehensive experimental test, a full version of the numerical local search showed relatively superior performance compared to the analytical local searches. When the network was small, the iterative and local searches found good solutions simultaneously, but only for low demand levels. When demand was high, the iterative approach failed to produce good solutions. On the other hand, when the network was large, as demand level became higher, the iterative approach tended to find better solutions.

Simplified gradient estimation local searches showed fairly good performance as well as computational efficiency. The UE repetitions involved in the optimal step size decision of local searches are not negligible and should be further examined to improve efficiency. Because of the nonconvexity, finding a (near) global optimum using any local search is very difficulty in large networks. Thus, further research is in progress investigating the necessity and applicability of the global search with simulated annealing and a genetic algorithm.

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TABLE 2 Means of Total Travel Times (hr) of IOA and Local Searches at the Five Demand Levels

N/W	OD	IOA	analytical local search A	analytical local search B	numerical local search	simplified local search A	simplified local search B	simplified local search C
VV	1	6	6	6	5	N/A	N/A	N/A
	2	21	22	23	21	N/A	N/A	N/A
	3	46	42	40	39	N/A	N/A	N/A
	4	84	56	63	56	N/A	N/A	N/A
	5	172	108	120	80	N/A	N/A	N/A
2x1	1	4	5	5	4	N/A	N/A	N/A
	2	15	15	35	14	N/A	N/A	N/A
	3	25	28	139	25	N/A	N/A	N/A
	4	43	179	324	45	N/A	N/A	N/A
	5	83	332	571	80	N/A	N/A	N/A
MED	1	33	36	36	36	36	36	36
	2	131	146	148	144	145	149	147
	3	274	330	365	324	346	361	329
	4	657	779	921	788	788	962	825
	5	1398	1705	1921	1781	1721	2105	1799
AST	1	147	155	156	155	155	155	155
	2	555	596	610	595	602	612	599
	3	1030	1265	1421	1250	1261	1421	1259
	4	1838	2503	2771	2624	2521	2997	2625
	5	3345	5521	6003	5487	5467	5799	5203

best across algorithms

TABLE 1 Local Search Techniques and Acronyms

Gradient estimation method	$\frac{\partial t_k}{\partial \lambda_k}$ estimation	Acronym	N[a]
analytical calculation by Equation 4	analytical or numerical	→ analytical-A → analytical-B	S
numerical calculation by Equation 7	not needed	→ numerical	S
analytical calculation by Equation 9	analytical or numerical	→ simplified-A → simplified-B	1
or numerical calculation by Equation 10	not needed	→ simplified-C	

[a]Needed equilibrium assignment numbers to obtain an entire gradient where S is the number of independent stages of the network.

TABLE 3 Average Number of Iterations and UE Assignment Repetitions until Convergence[a]

N/W	IOA	analytical local search A	analytical local search B	numerical local search	simplified local search A	simplified local search B	simplified local search C
VV	3.7, 3.7	3.5, 67	2.6, 104	3.1, 93	N/A	N/A	N/A
2x1	4.1, 4.1	3.5, 105	2.1, 75	5.1, 209	N/A	N/A	N/A
MED	8.3, 8.3	2.8, 103	2.0, 82	3.0, 90	2.9, 51	2.0, 53	2.7, 90
AST	14.7, 14.7	3.0, 154	2.0, 104	3.0, 139	3.0, 47	2.0, 50	2.9, 93

[a](average iteration to converge, average UE repetitions)

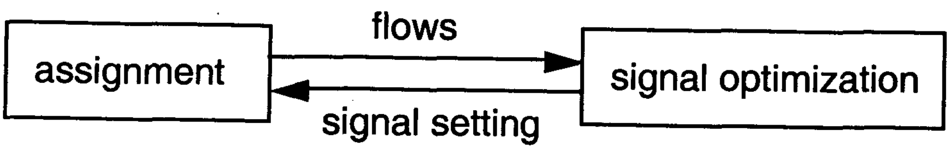

FIGURE 1 Iterative optimization and assignment procedure.

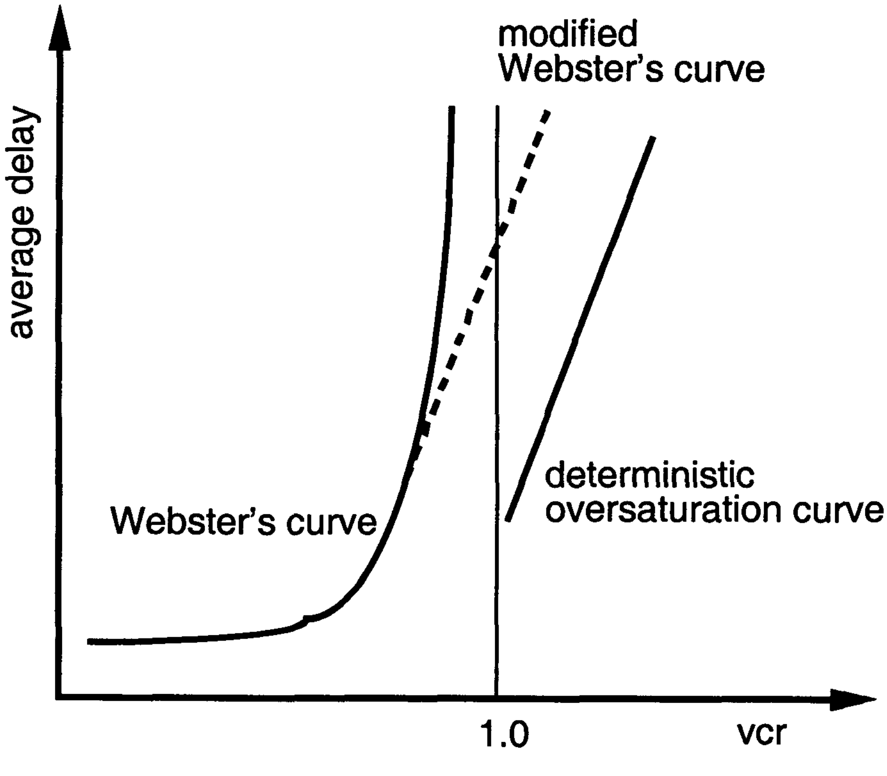

FIGURE 2 The modified Webster's curve.
(Source: Van Vuren and Van Vliet (1992); reprinted with minor amendments)

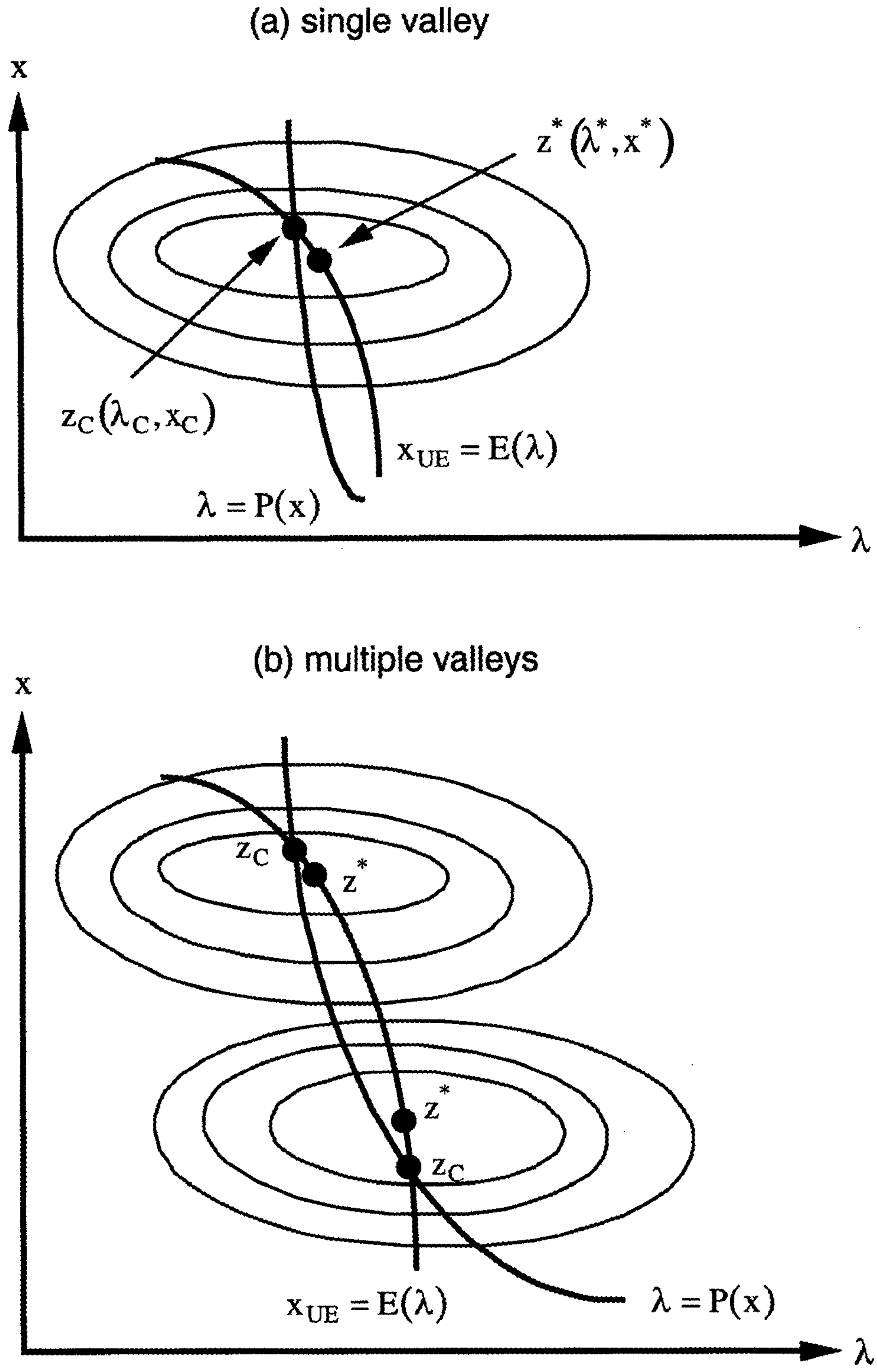

FIGURE 3 Local (z^*) and mutually consistent point (z_C).

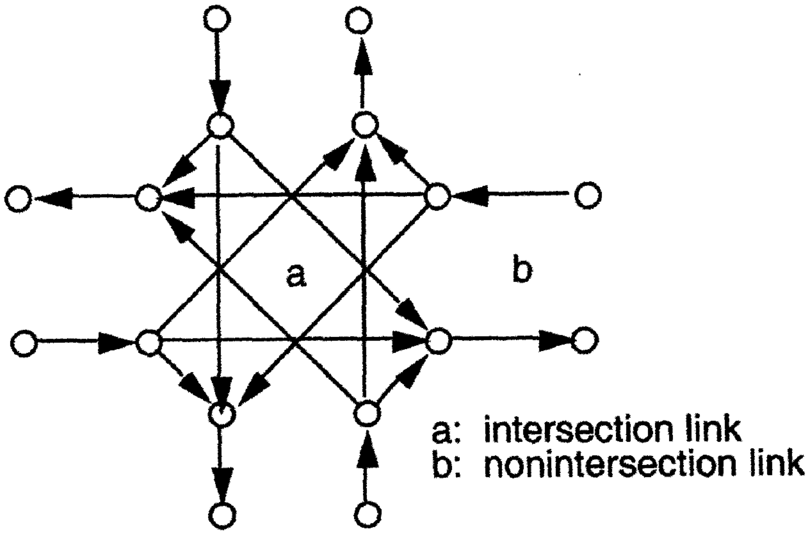

FIGURE 4 Detailed intersection representation.

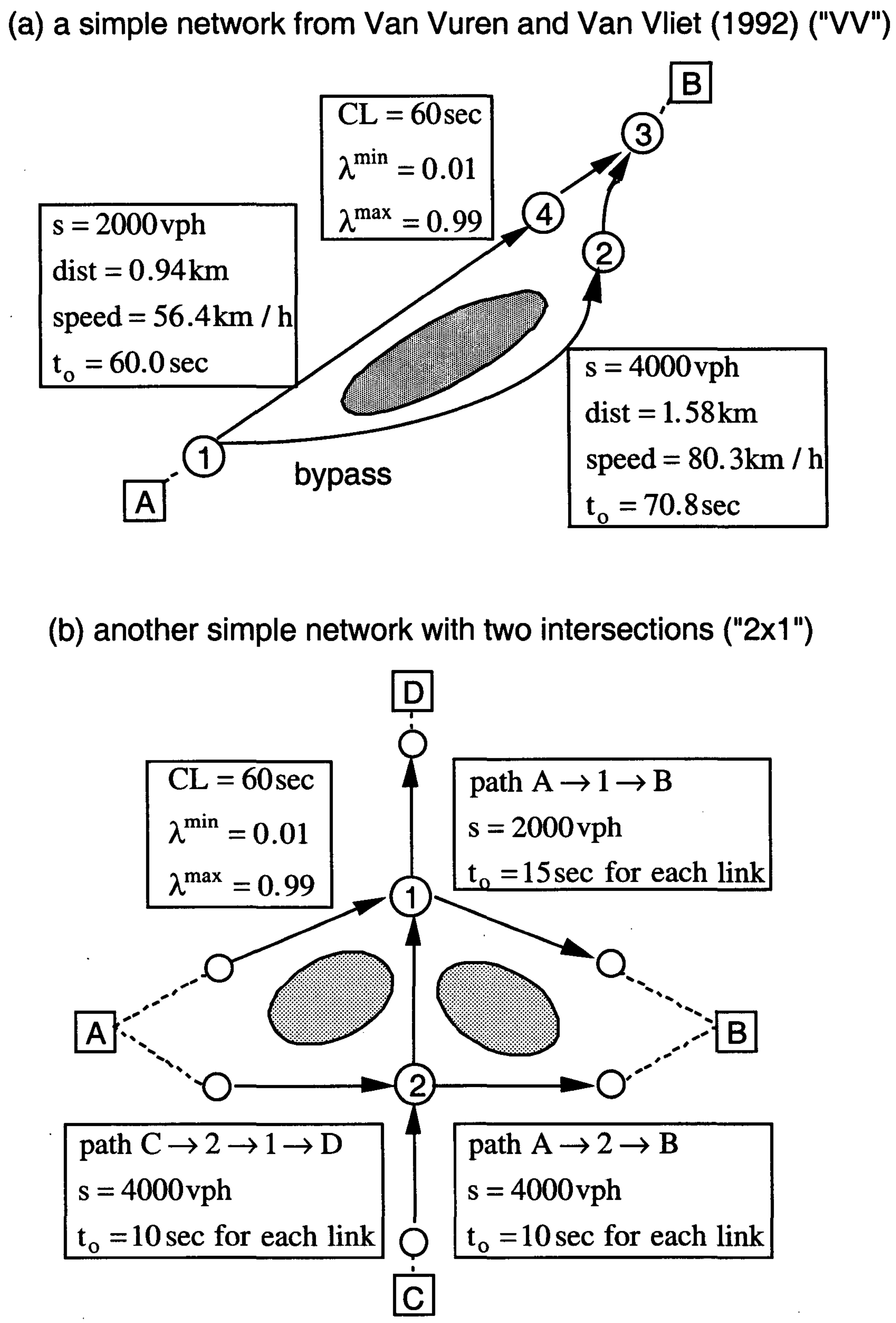

FIGURE 5 Two simple example networks denoted by VV and 2x1.

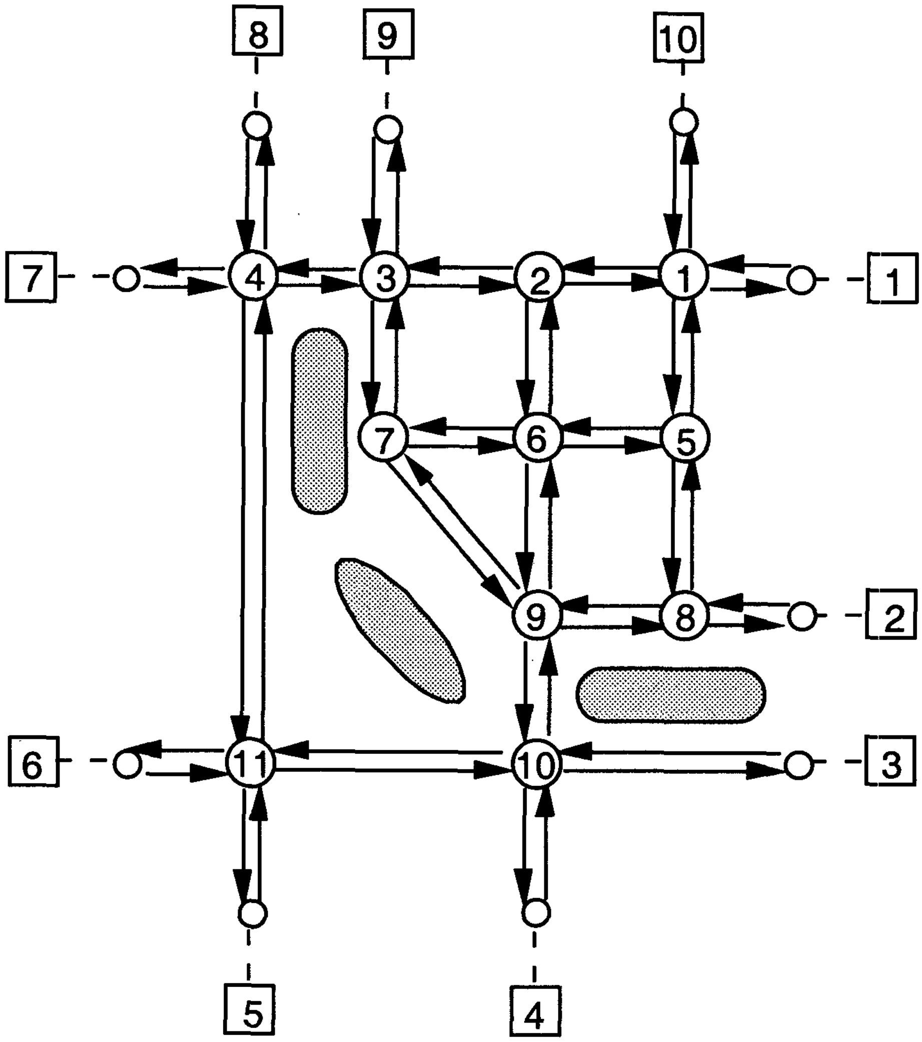

FIGURE 6 A medium size network denoted by MED.

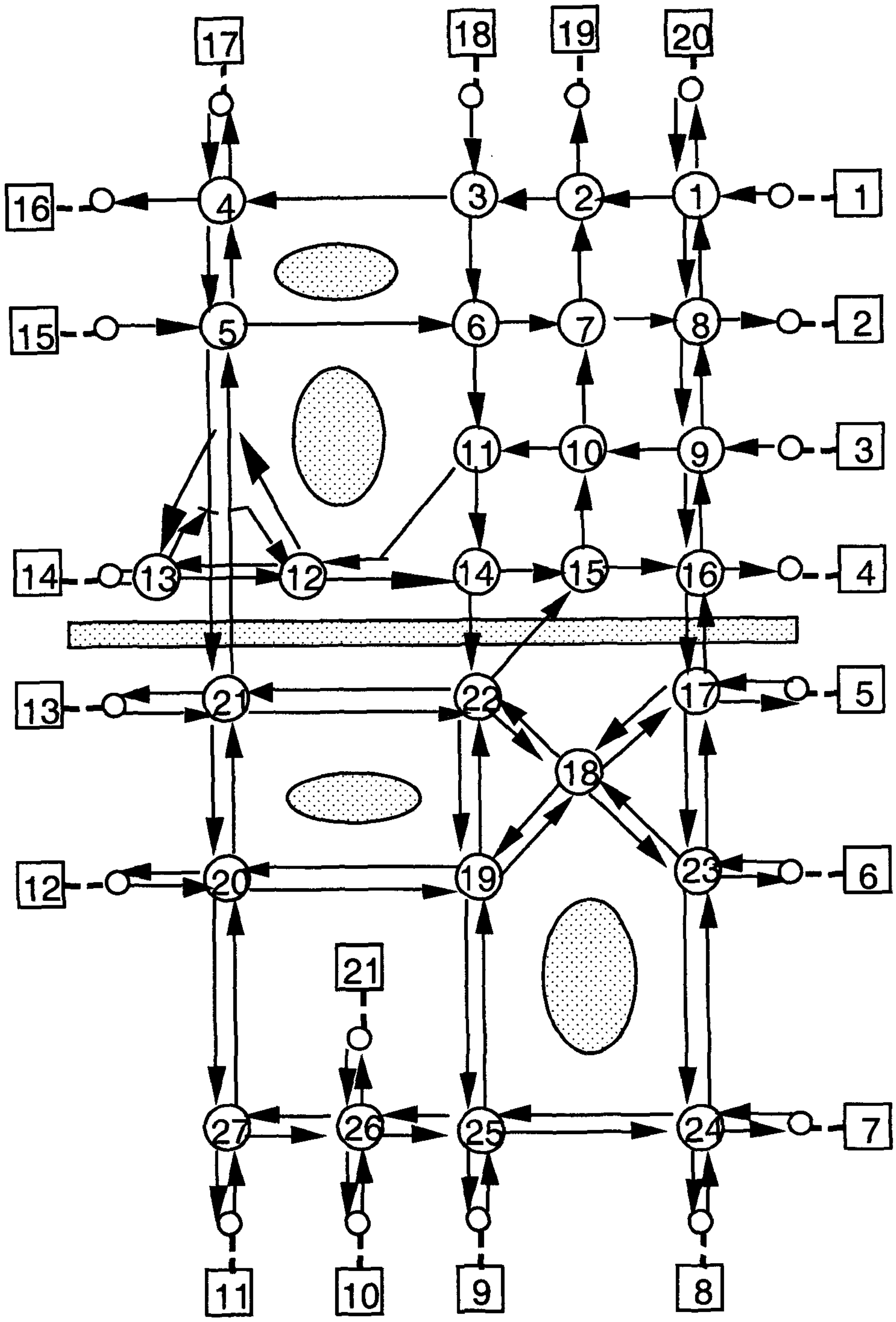

FIGURE 7 A subnetwork from the city of Austin, Texas, USA, denoted by AST.

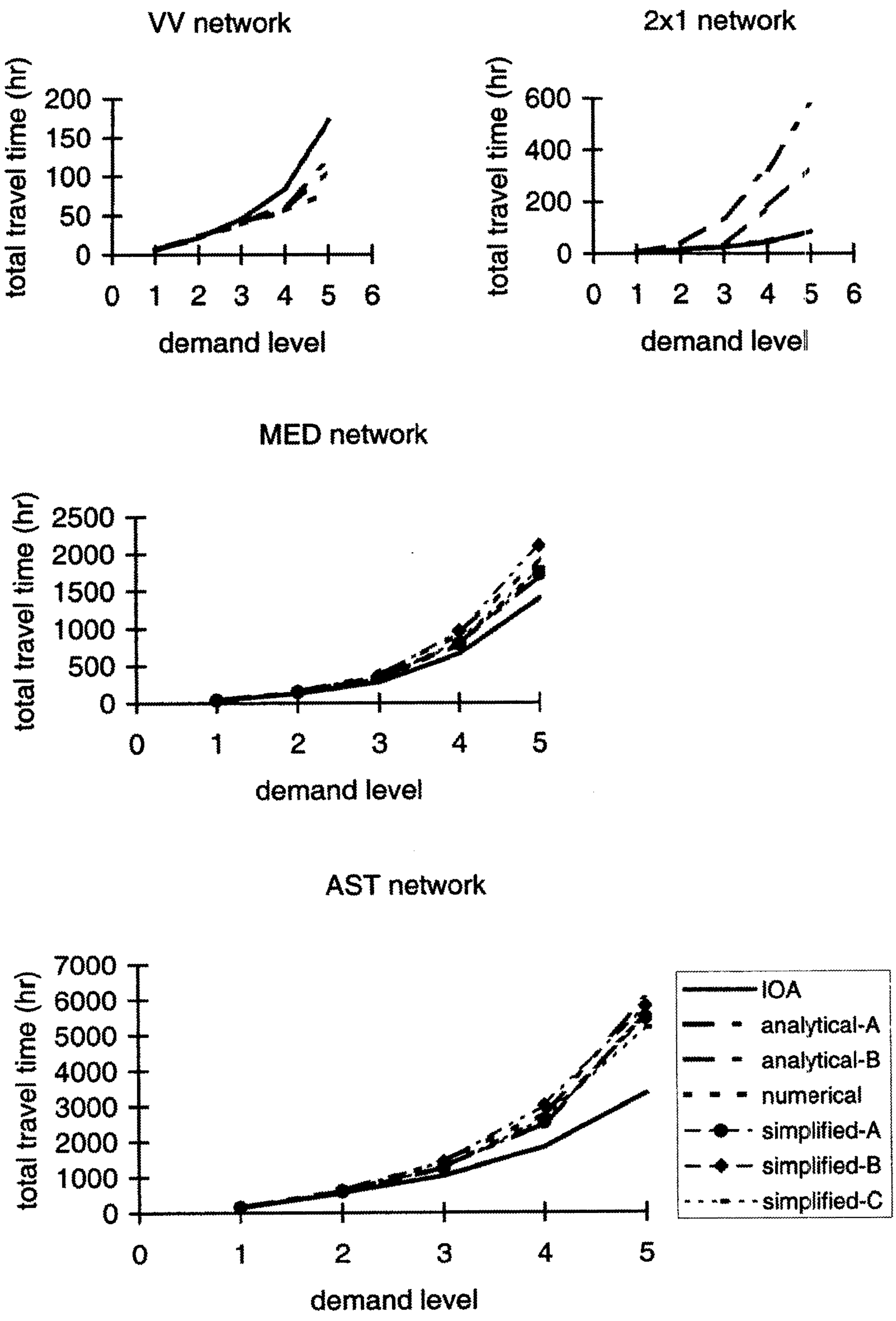

FIGURE 8 Means of total travel times by the codes, four networks and five demand levels.

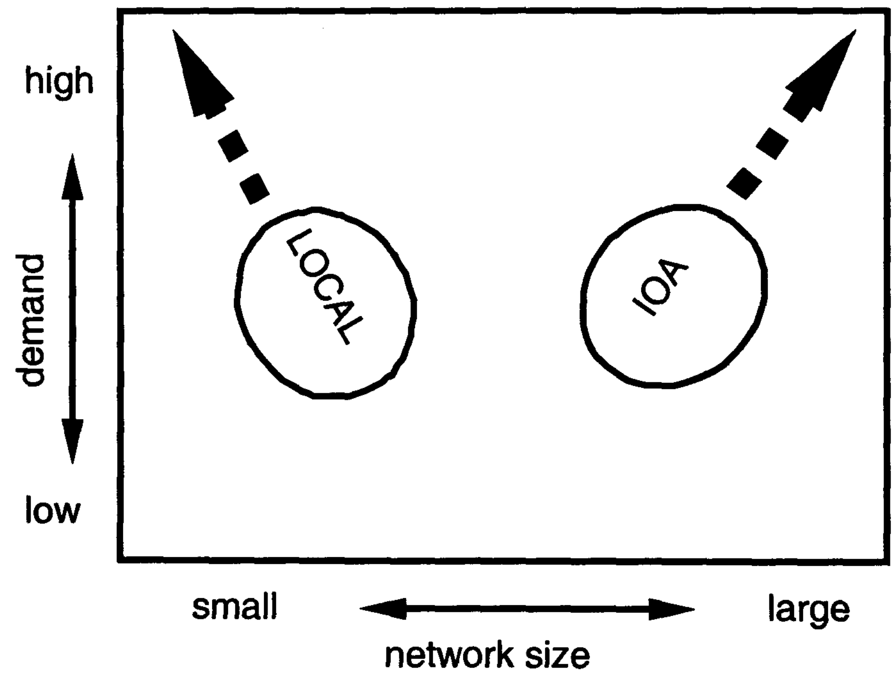

FIGURE 9 Relative superior region of two searches with respect to demand level and network size.

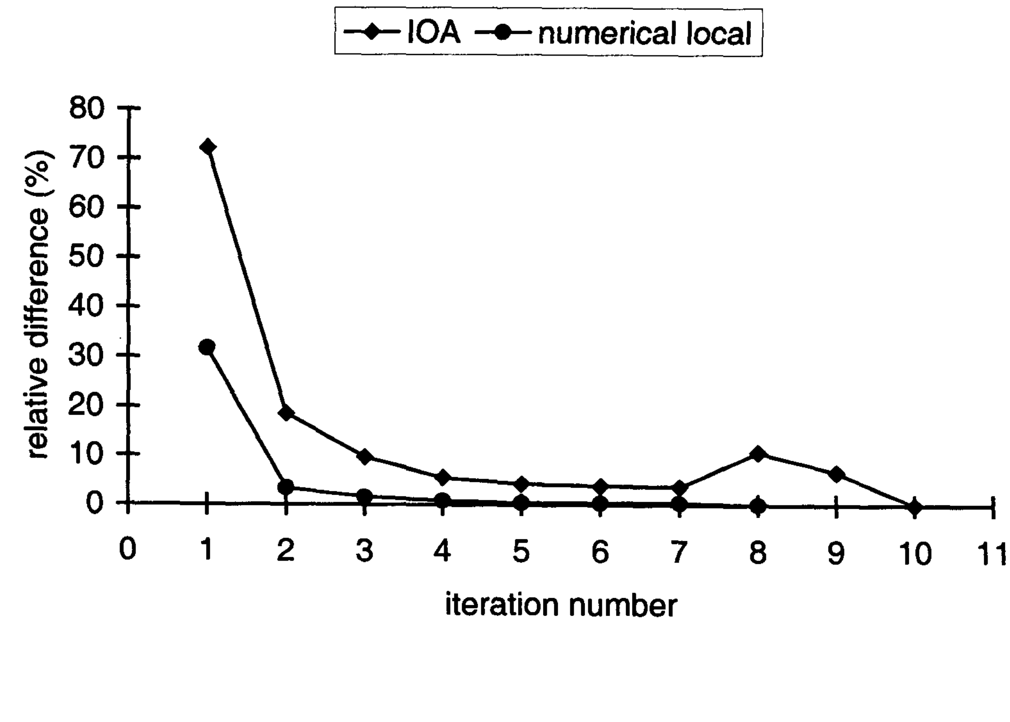

FIGURE 10 Convergence pattern of the IOA and numerical local search by relative objective value reduction between iterations for the 2x1 network at OD level 3.