Recursive State Space Model Identification Algorithms Using Subspace Extraction via Schur Complement

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abstract

In this paper, we present recursive algorithms for state space model identification using subspace extraction via Schur complement. It is shown that an estimate of the extended observability matrix can be obtained by subspace extraction via Schur complement. A relationship between the least squares residual and the Schur complement matrix obtained from input-output data is shown, and the recursive algorithms for the subspace-based state-space model identification (4SID) methods are developed. We also proposed the above algorithm for an instrumental variable (IV) based 4SID method. Finally, a numerical example of the application of the algorithms is illustrated.

1 Introduction

The 4SID methods have attracted much attention because of being essentially suitable for the identification of multivariable systems. This is one reason why the methods are essentially suitable for multivariable system identification. The main characteristic of the methods is to calculate the shift-invariant subspace, defined by the span of the column of the matrices determined by the input-output data. The MOESP algorithm [1], [2] is known as the ordinary 4SID method. The QR factorization and the singular value decomposition (SVD) are the principal computational tools. We have considered the QR factorization in the MOESP algorithm, and show alternative derivation of the estimate of the extended observability matrix by subspace extraction via Schur complement [4]. Then we also propose the above derivation for IV-based 4SID method. A relationship between the least squares residual and the Schur complement matrix obtained from input-output data is shown, and we propose a recursive formula for the error covariance matrix in the 4SID method.

2 Problem formulation

Consider the following nth order discrete time linear time-invariant state-space model with m inputs and l outputs collected in u_k and y_k respectively:

$$x_{k+1} = Ax_k + Bu_k \quad (1)$$

$$y_k = Cx_k + Du_k + v_k, \quad (2)$$

where v_k is an unmesurable stochastic disturbance. The noise term is assumed to be a zero-mean stationary random process uncorrelated with u_k. The unknown system matrices A, B, C, and D have appropriate dimensions. It is assumed that the model is minimal, that is, the system is completely reachable and observable. We introduce the stacked input vector

$$u_i(k+N) := \left[u_{k+N}^T, u_{k+N+1}^T, \cdots, u_{k+N+i-1}^T\right]^T, \quad (3)$$

where $u_i(k+N) \in \mathbb{R}^{mi\times 1}$. The stacked vectors of the output and noise vectors are defined similarly. We define the Hankel matrix

$$U_{k,i,N} := [u_i(k), u_i(k+1), \cdots, u_i(k+N-1)]. \quad (4)$$

The corresponding definitions for the input and noise matrices are similar. In relation to the order of the system n the pair i, N satisfy $i > n$ and $N \gg n$. We define the state vector sequence as

$$X_{k,N} := [x_k \; x_{k+1} \; \cdots \; x_{k+N-1}] \quad (5)$$

Then, we obtain the following relation:

$$Y_{k,i,N} := \Gamma_i X_{k,N} + H_i U_{k,i,N} + V_{k,i,N}. \quad (6)$$

where

$$\Gamma_i := \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{i-1} \end{bmatrix}, H_i := \begin{bmatrix} D & & & O \\ CB & D & & \\ \vdots & \ddots & \ddots & \\ CA^{i-2}B & \cdots & CB & D \end{bmatrix} \quad (7)$$

The matrix Γ_i is The extended observability matrix. We will omit the subscripts for U and Y unless otherwise mentioned.

An estimated realization of the system matrices are denoted by

$$[A_T, B_T, C_T, D_T] = [TAT^{-1}, TB, CT^{-1}, D]$$