Stabilization Control for Limit Cycle of an Inverted Pendulum System

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Abstract

In this paper, a kind of limit cycle of an inverted pendulum system is discussed. We propose a stabilization control law for such a limit cycle of an inverted pendulum system that the pendulum rotates periodically. Besides, the stabilization control law is extended so as to ensure not only stability of the limit cycle but also an L_2-gain disturbance attenuation in the presence of modeling error and viscosity friction.

1 Introduction

Inverted pendulum systems have been widely used to verify an effectiveness of the designed stabilization control system. Many control schemes have been proposed to stabilize the pendulum at the unstable equilibrium [1], [2]. Recently, nonlinearities in the dynamics of the system are incorporated positively in controller design [3], [4]. Literatures [1] and [4] also proposed a swing-up control law that swings up the pendulum automatically from its dangling state around the stable equilibrium to neighborhood of the unstable equilibrium. However, realization of such a limit cycle of the inverted pendulum system that the pendulum rotates periodically, and stabilization control of this kind of limit cycle have not been studied enough. To the best of our knowledge, [5] and [6] are only literatures that developed a realization and a stabilization control of this kind of limit cycle for the inverted pendulum system. In [5], the limit cycle is sampled for every period, and is stabilized by linear optimal control for its linearized time-invariant discrete-time model.

Limit cycle phenomenon is unique feature of nonlinear systems. Therefore, in this paper, we use no linearization to derive a stabilization control law for the limit cycle. If non-conservative force does not act in the inverted pendulum system, once the pendulum obtain a certain energy, it eternally rotates periodically. In practice, however, motion of the pendulum declines due to the presence of viscosity friction in actual systems. This complicates stability analysis and controller synthesis of the limit cycle. First, along the research line of derivation of the swing-up control law given in [4], a realization law of above mentioned limit cycle of the inverted pendulum system is derived based on the law of mechanical energy conversion. Then, a stabilization control law for the produced limit cycle is designed based on the Lyapunov stability theory. Moreover, taking modeling error and viscosity friction into consideration as disturbance, the stabilization control law is extended so as to ensure not only stability of the limit cycle but also an L_2-gain disturbance attenuation in the presence of modeling error and viscosity friction. There is a feature in the proposed control scheme. The designer can change a period of the limit cycle by giving a desired velocity of the pendulum at an arbitrarily point in rotatory phase, which has not been clearly analyzed in [5] and [6]. Finally, the effectiveness of the proposed control scheme is demonstrated via experimental work.

2 Model of Inverted Pendulum

Consider the Inverted Pendulum system shown in Figure 1. The dynamics are given by

$$M(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + D\dot{\boldsymbol{\theta}} + G(\boldsymbol{\theta}) = \boldsymbol{\tau}, \tag{1}$$

$$\begin{aligned}
\boldsymbol{\theta} &= \begin{bmatrix} \theta_0 & \theta_1 \end{bmatrix}^T, \quad \boldsymbol{\tau} = \begin{bmatrix} \tau_0 & 0 \end{bmatrix}^T, \\
M(\boldsymbol{\theta}) &= \begin{bmatrix} j_1 + j_2 \sin^2\theta_1 & j_3\cos\theta_1 \\ j_3\cos\theta_1 & j_2 + j_4 \end{bmatrix}, \\
C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) &= \begin{bmatrix} 2j_2\sin\theta_1\cos\theta_1\dot{\theta}_1 & -j_3\sin\theta_1\dot{\theta}_1 \\ -j_2\sin\theta_1\cos\theta_1\dot{\theta}_0 & 0 \end{bmatrix}, \\
D &= \begin{bmatrix} d_0 & 0 \\ 0 & d_1 \end{bmatrix}, \quad G(\boldsymbol{\theta}) = \begin{bmatrix} 0 \\ -j_5\sin\theta_1 \end{bmatrix},
\end{aligned}$$

where $\boldsymbol{\theta}$ is angle vector of arm and pendulum, $M(\boldsymbol{\theta})$ is moment of inertia matrix, $C(\boldsymbol{\theta},\dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}}$ is nonlinear vector containing Coriolis and Centrifugal forces, D is viscosity matrix, $G(\boldsymbol{\theta})$ is gravitational vector and τ_0 is input torque for arm. Moreover, the following notations are used for simplicity.

$$\begin{aligned}
j_1 &= J_0 + m_0 l_{c0}^2 + m_1 l_0^2, \\
j_2 &= m_1 l_{c1}^2, \quad j_3 = m_1 l_0 l_{c1}, \\
j_4 &= J_1, \quad j_5 = m_1 g l_{c1}.
\end{aligned}$$