

# Input Constrained Receding Horizon Control with Nonzero Set Points and Model Uncertainties

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## Abstract

An input constrained receding horizon predictive control algorithm for uncertain systems with nonzero set points is proposed. For constant nonzero set points, models with uncertainty can be converted into an augmented incremental system through the use of integrators and the problem is transformed into a zero-state regulation problem for the incremental system. But the original constraints on inputs are converted into constraints on the sum of control inputs at each time instants, which have not been dealt in earlier constrained robust receding horizon control problems. Recursive state bounding technique and worst case minimizing strategy developed in earlier works are applied to the augmented incremental system to yield an offset error free controller. The resulting algorithm is formulated so that it can be solved using LP.

## 1. Introduction

The dual-mode paradigm[4-6] provides an efficient way to solve the stability problem of receding horizon control in the presence of physical constraints. In the dual-mode strategy,  $N$  free control moves are deployed to steer the state into a feasible and invariant target set, so that all future predicted states  $x$  remain within this set and so that the feedback law itself is feasible.

In the presence of model uncertainties, application of the  $N$  free control moves is likely to result in impracticable computational complexity. The computational complexity was removed by removing the use of  $N$  free control moves but instead allowing the state feedback gain to vary[1]. The free control moves were re-introduced by [2] and [4-5] using ellipsoidal invariant sets and polyhedral invariant sets, respectively. Computational complexity was avoided by employing autonomous augmented system representation [2] and by recursive state bounding [4]. In [5], recursive state bounding technique was improved so that large target invariant sets corresponding to 'detuned' state feedback gain and good predicted performances corresponding to 'tightly tuned' state feedback gain can be combined.

In the above mentioned works on robust receding horizon control, the control objective is to bring the system state to zero state ( $x=0$ ), which is a common steady state for all models in the uncertainty set with the same input  $u=0$ . For nonzero setpoints, however, there is no single steady state to serve as a reference in a zero state regulation problem, which makes it difficult to define target invariant sets around a steady state.

In this paper, a constrained receding horizon control with nonzero set point is developed by deploying the dual-mode paradigm. Through the use of integrators, uncertain systems can be augmented so that the augmented incremental dynamics have a common steady state e.g. zero steady state error, zero increments on states and inputs, for all models in the uncertainty set. For the incremental systems, original constraints on inputs are converted into constraints on sum of control inputs at each time instants. Application of the recursive bounding technique to the augmented incremental system can be made by taking the constraints on the sum of control inputs into account with care.

## 2. System Description

Consider the system with uncertainty:

$$x(k+1) = \tilde{A} x(k) + \tilde{B} u(k), \tag{1}$$

$$\begin{aligned} -u_{\max} &\leq u(k) \leq u_{\max} \\ y(k) &= Cx(k) \end{aligned} \tag{2}$$

where  $x$ ,  $u$  and  $y$  are state, input and output vectors of dimension  $n$ ,  $m$  and  $l$ , respectively.

$\tilde{A}$ ,  $\tilde{B}$  and  $C$  are matrices of conformal dimensions and vector inequalities apply on an element-by-element basis.

$\tilde{A}$  and  $\tilde{B}$  belong to the uncertainty class:

$$\Omega = \{(\tilde{A}, \tilde{B}) : \sum_{i=1}^{n_a} \eta_i(A_i, B_i), \eta_i \geq 0, \sum_{i=1}^{n_b} \eta_i = 1\}, \tag{3}$$

where  $n_p = 2^{n_a} \times 2^{n_b}$ , and  $n_a, n_b$  are the numbers of uncertain parameters in  $\tilde{A}$  and  $\tilde{B}$ .