# Sliding Mode Robust Control of Uncertain Delay Systems: Generalize Transformation Approach

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**Abstract**

In this paper, the theoretical development to stabilize a class of uncertain time-delay systems via sliding mode control is presented. The system under consideration is described in state space model containing state delay, uncertain parameters and disturbance. The main idea is to reduce the state of delayed system, by employing the generalize linear transformation, into an equivalent one with no delay inside, which is easier to analyze its behavior and stability. Then, the sliding control approach is employed to find the stabilizing control law. Finally, a numerical simulation is illustrated to show the algorithm for applying the proposed theorems and the effectiveness of the designed control law in stabilizing the controlled systems.

## 1. Introduction

Time delays can be found in various engineering systems such as chemical processes, pneumatic/hydraulic systems, biological systems and economic systems. Compared to systems without delay, the presence of delay makes it more difficult to achieve the stability of the systems. On the other hand, it is reasonable to include uncertain parameters and disturbance in practical control systems containing modeling errors, linearization approximations, etc. Therefore, the problem of robust stabilization of delayed systems with uncertain parameters has received considerable attention of many researchers, and many solution approaches have been proposed. Therein Cheres (1989) stabilize time-delay systems by assuming that the delayed state is the uncertainties of the systems. Phoojaruenchanachai (1998, 1999) employed the improved linear transformation to reduce the delay systems to delay-free systems and used it to design the control law.

In this paper, a class of linear systems containing known delay, uncertain parameters and additive disturbances are presented in the most general autonomous delay systems. Determination of controller parameters can be divided into two parts. First, base on the improved theorem of Fiagbedzi and Pearson (1990), the generalize linear transformation is utilized to reduce the original problem into an equivalent one which is easier to find the solution. Next, Sliding mode control (SMC), which its design is based on Utkin (1978), is derived. Finally, the numerical example is illustrated to show applicability of the proposed method.

## 2. Problem Formulation

In this section, we will present the main idea to design control law for delayed systems, with uncertain parameters and disturbance, via the generalize linear transformation.

Consider a class of uncertain time-delay system $(S_d)$ which is defined by the following equations

$$\dot{x}(t) = \int_{-r}^{0} d\alpha(\theta)x(t+\theta) + \int_{-r}^{0} d\beta(\theta)x(t+\theta) + [B+\Delta B(t)]u(t) + B\omega(t), \quad (1)$$

where $x(t)\in R^n$ is the current value of the system state, $u(t)\in R^m$ is the control function, $\omega(t)\in R^l$ is the additive disturbance, $\alpha(\cdot) \in L_1([-r,0];\ R^{n\times n})$ is a matrix valued functions of bounded variation on $[-r,0]$, $\beta(\theta) \in L_1([-r,0];\ R^{n\times n})$ is an integrable matrix whose elements are continuous, unknown but bounded functions on $[-r,0]$, $\Delta B(t)$ is a matrix whose elements are continuous, unknown but bounded functions, $r\in R^+$ is a max-known constant delay time and the initial function of system be specified as $x_0(\eta)\in C_d([r,0]; R^n)$. The integrals indicated above are Stieltjes integrals, the distributed integral from, and it is enough to consider one uniform delay, $r$, since the functions of bounded variation can be extended, without change of variation, to $[-r,0]$.

**Assumptions**

Before proposing our controllers, the following assumptions are made throughout here.

***Assumption 2.1:***

The nominal system of $(S_d)$, i.e., the system $(S_d)$ which $\int_{-r}^{0} d\beta(\theta)=0$, $\Delta B(t)=0$, $\omega(t)=0$ is spectrally stabilizable.

***Assumption 2.2:***

The continuous matrix functions $H(\theta)$ and $E(t)$ of appropriate dimensions, for all $t\in R^+$, exist such that

a) $\beta(\theta)=BH(\theta)$,

b) $\Delta B(t)=BE(t)$,

c) $I+\frac{1}{2}\left(E(t)+E^T(t)\right)\geq \delta I$ for some scalar $\delta > 0$,

d) scalar $\mu(x_t)$ and $\mu_E(t)$ exist such that

$$\mu(x_t)\geq \left\| \int_{-r}^{0} dH(\theta)x(t+\theta)+\omega(t)\right\|,$$

and

$$\mu_E(t)\geq \|E(t)\|$$

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