Study of the Robust Stability of the Systems with Structured Uncertainties using Piecewise Quadratic Lyapunov Function

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Abstract

The robust stability problems for nominally linear system with nonlinear, structured perturbations are considered with Lyapunov direct method. The Lyapunov direct method has been utilized to determine the bounds for nonlinear, time-dependent functions which can be tolerated by a stable nominal system. In most cases quadratic forms are used either as components of vector Lyapunov function or as a function itself. The resulting estimates are usually conservative. As it is known, often the conservatism of the bounds we propose to use a piecewise quadratic Lyapunov function. An example demonstrates application of the proposed method.

1. Introduction

The design of robust controllers for multivariable linear systems remains one of the important issues of modern technology, see Davison[1], Desoer et al. in [2], and the Siljak review in [3]. Recently a significant research has taken place in applications of Lyapunov's direct method in robust design; it is prompted by the fact that this approach can easily accommodate nonlinear and time-varying perturbations. Using Lyapunov's direct method Siljak in [4] and Patel et al. in [5] established the procedures to calculate quantitative measures of robustness, i. e., the bounds on perturbations such that for the perturbations so bounded the system remains stable. Yeadavalli and Liang in [6] using transformations obtained improvements in bounds estimation. Results of [5] and [6] are based on the selection of some quadratic form as the Lyapunov function. As it is known this approach has its limitations; Becker and Grimm in [7] proved that applying the small gain theorem provides the stability bond which cannot be improved by state transformations. Radziszewski in [8] examining two-dimensional structurally perturbed system discussed quadratic forms as the class of Lyapunov function candidates and determined the best Lyapunov function in this class. It was found that robustness bounds obtained using this function were worse than those obtained by other methods, applicable for two-dimensional system. Siljak in [3] shows that the bound estimation strongly depends on selection of the system state space (250% improvement of the estimation). Further, he suggests use of vector Lyapunov functions to reduce conservatism of robustness estimation and provides an example of application. In this paper piecewise quadratic Lyapunov functions are proposed to improve the bound estimation.

2. Problem Statement and Definitions

Consider a nominally linear system with structured p e r t u r b a t i o n

$$\dot{x} = A_N + \sum_{j=0}^{q} p_j A_j x \tag{1}$$

with $p_j = p_j(x, t)$ and A_N having negative real parts of its eigenvalues. Let $p \triangleq [p_1, ---, p_q]^T$. We define the general robust stability problem for (1) as the assignment to determine the set R belonging to the parameter space R^q such that if $p(x, t) \in R$ for all x, t then the trivial solution $x=0$ of the system (1) is stable in the sense of Lyapunov. Most often a reduced problem is discussed when instead of the set R a parallelepiped or a ball embedded in R is to be determined, see Siljak's review in [3] for details. We limit our interest to Lyapunov direct method approach. Denote $\sum_{j=1}^{q} p_j A_j = A_u(p)$ and Π a parallelepiped in R^q $\Pi = [p \in R^q : p^- \le p \le p^+]$ with $p^1, p^2,$ -- being the 2^q vertices of Π. Usually some quadratic form

$$V(x) = x^T S x \tag{2}$$

where S is a symmetric, positive-definite matrix, is selected as a Lyapunov function candidate. The derivative $\frac{dV(x)}{dt}$ of this function along the solution of (1) is a quadratic form of x and linearly depends on parameters $p_j(t, x)$. Thus when searching for the solution of robust stability problem (1) in a form of a parallelepiped $\Pi \in R^q$ the 2^q quadratic forms

$$QF_j = x^T (A_N^T S + S A_N + A_u^T(p^j) S + S A_u(p^j)) x \tag{3}$$

generated by 2^q vertices $p^1 \cdot p^2$-- of the parallelepiped Π are considered(see for instance the paper [9] by Hersberger and Belanger). If the forms QF_j are non-positive for all x then Π is a solution to robust