# Robust Control of a Glass-Fiber Reinforced Composite Beam using $\mu$-Synthesis Algorithm

°Yeo-Hung Yun*, Young-Choon Lee*, Tae-Kyu Kwon**, Kee-Ho Yu*** and Seong-Cheol Lee***

*Graduate Students, **Research Assistant(NAHTEC), ***Professor (RRC)
Department of Mechanical Engineering, Chonbuk National University, 561-756, Chonju Korea
(Tel : 82-63-270-2320, Fax : 82-63-270-2315, E-mail: meconlee@moak. chonbuk. ac. kr)

## Nomenclature

$b$ : width of a PVDF
$C^E_{pq}$ : stiffness matrix
$C^s_p$ : capacitance of a sensor
$E_k$ : electric field
$D_i$ : electric displacement
$d_{kp}$, $e_{iq}$ : piezo coefficients
$E$ : effective young's modulus
$i, j, k$ :subscript of piezo elements(=1,⋯,3)
$I$ : inertia moment
$K_s$ : constant of a PVDF(= $Y_p d_{31} b y_c / C^s_p$ )
$K_a$ : constant of a PZT (= $(1/2) b d_{31} Y_p (t_a + t_b)$)
$p, q$ : subscript of piezo elements(=1,⋯,6)
$r$ : radius of curvature
$s^E_{pq}$ : compliance matrix
$S$ : sensitivity function(= $(I+G_{nom}K)^{-1}$)
$S_p$ : strain matrix
$T_q$ : stress matrix
$T$ : complementary sensitivity(= $KG_{nom}(I+KG_{nom})^{-1}$)
$y_c$ : distance from the sensor to neutral axis
$Y_p$ : young's modulus of piezoelectric materials
$y'(x)$ : modal function of a normalized beam
$v_p(t)$ : output voltage of a PVDF sensor
$\rho$ : density of a beam
$\varepsilon^T_{ik}$ : permeability coefficient
$\eta_i(t)$ : generalized modal coordinate
$\phi_i(t)$ : modal function of a composite beam
$\beta_i$ : satisfaction coefficient( $\beta_i^4 = \rho A \omega_i^2 / EI$ )
$\omega_i$ : natural frequencies of a cantilever beam

## Abstract

A study on the robust control of a composite beam with a distributed PVDF sensor and piezo-ceramic actuator is presented in this paper. 1st and 2nd natural frequencies are considered in the modeling, because robust control theory which has robustness to structured uncertainty is adopted to suppress the vibration. If the controllers designed by $H_\infty$ theory do not satisfy control performance, it is improved by $\mu$-synthesis method with $D$-$K$ iteration so that the $\mu$-controller based on the structured singular value satisfies the nominal performance and robust performance. Simulation and experiment were carried out with the designed controller and the verification of the robust control properties was presented by results.

## 1. Introduction

Glass-Fiber-Reinforced(GFR) polymeric composite materials provides the desirable properties of high stiffness and strength as well as low specific weight. Therefore, this material is now used in a variety of components for automotive, aerospace, marine, and architectural structure. Smart structure incorporating piezoelectric sensors and actuators have found a wide range of applications in the fields of active vibration control[1], shape control[2], self-sensing control[3] and noise control in the past decade. Piezoelectric elements have been used successfully in the closed loop control of a variety of active structures including beams[4], plates[5], and shell[6]. Through a number of studies have been reported on the vibration control using these materials, but there are few studies concerning the active robust vibration control of a random directional glass fiber reinforced thermoplastic composite.

In this work, using a PVDF sensor and piezo-ceramic actuator, a study on the robust control of a cantilever composite beam is presented. The state equation of a composite beam is obtained by using a modal approach and modal coordinates. A robust controller is designed to suppress the vibration of a reinforced beam using $\mu$-synthesis theory. Experiment was carried out and the results were compared with the simulation results.

## 2. Theoretical Analysis

The PVDF sensor and piezo-ceramic actuator are embedded to a random directional glass fiber reinforced thermoplastic composite. The properties of materials used in this study are shown in Table 1. Assuming that piezo-material is isotropic, the general piezo-material equations can be expressed in the following form.

$$S_p = s^E_{pq} T_q + d_{kp} E_k \tag{1}$$

$$D_i = d_{iq} T_q + \varepsilon^T_{ik} E_k \tag{2}$$

$$T_p = c^E_{pq} S_q + e_{kp} E_k \tag{3}$$

$$D_i = e_{iq} S_q + \varepsilon^s_{ik} E_k \tag{4}$$

Supposed that composite material is a Bernoulli-Euler beam, the Eq. (1) and Eq. (2) can be written by: