Complexity Control Method of Chaos Dynamics in Recurrent Neural Networks

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Abstract

This paper demonstrates that the largest Lyapunov exponent λ of recurrent neural networks can be controlled by a gradient method. The method minimizes a square error $e_\lambda = (\lambda - \lambda^{obj})^2$ where λ^{obj} is desired exponent. The λ can be given as a function of the network parameters P such as connection weights and thresholds of neurons' activation. Then changes of parameters to minimize the error are given by calculating their gradients $\partial\lambda/\partial P$.

In a previous paper, we derived a control method of λ via a direct calculation of $\partial\lambda/\partial P$ with a gradient collection through time. This method however is computationally expensive for large-scale recurrent networks and the control is unstable for recurrent networks with chaotic dynamics.

Our new method proposed in this paper is based on a stochastic relation between the complexity λ and parameters P of the network configuration under a restriction. Then the new method allows us to approximate the gradient collection in a fashion without time evolution. This approximation requires only $O(N^2)$ run time while our previous method needs $O(N^5T)$ run time for networks with N neurons and T evolution. Simulation results show that the new method can realize a "stable" control for large-scale networks with chaotic dynamics.

1. Introduction

Recurrent neural networks, consisting of units connected with each other, have higher degree of parameter freedom compared with that of feedforward neural networks composed of the same number of units. Harnessing the dynamics of complicated interactions among the units, the recurrent network is expected to become a useful model for identifying and controlling the nonlinear complex dynamical systems[7].

Most of learning algorithms for the recurrent networks are based on the algorithms for the feedforward networks. For example, Jordan has proposed new type of recurrent networks which can be learned by the well-known backpropagation algorithm using the supervisory signals as the feedback signals[6]. In this case, the Jordan's recurrent networks can approximate the input-output function of the target systems even if the functions are nonlinear[5].

However there is no guarantee that dynamical complexity of the recurrent networks converges to the target complexity[2][8]. This means results with respect to the behavior of the actual target systems could be different from the estimated knowledge based on the recurrent networks learned by conventional methods. Especially the chaotic systems aren't captured by the conventional learning algorithm since when the neural model begins to learn the system in an accurate form, then it is itself chaotic and update of the weights are therefore unstable.

Principle and Kuo have proposed a complexity control method which updates the weights with a forgetting function given by the largest Lyapunov exponent for feedforward networks[9]. For recurrent networks Deco and Schürman have reported that the dynamical complexity can be learned by a stochastic "sample-by-sample" update of the weights with the forgetting function[1]. There is however no direct formulation for controlling the largest Lyapunov exponent of recurrent networks.

In a previous paper, we proposed a gradient-based control method of the largest Lyapunov exponent [2]. This control method gave a control formulation explicitly, but it has several problems: the method is computationally expensive for large-scale recurrent networks and the control is unstable for recurrent networks with chaotic dynamics since a gradient collection through time diverges due to the chaotic instability.

In this paper, we propose another method in order to reduce the computational cost and realize a "stable" control for recurrent networks with chaotic dynamics. Firstly we investigate relations between the complexity and parameters of the network configuration under a restriction. The new method is based on the relation which allows us to approximate the gradient collection in a fashion without time evolution. Simulation results show that new method can control the exponent for recurrent networks with chaotic dynamics.

2. Complexity of recurrent networks

A subset of the Lyapunov exponents is used as a measure of the dynamical complexity. The complexity is defined strictly by using the complete set of the exponents. Calculation of the complete set is, however, computationally expensive. In the following, only the largest Lyapunov exponent will be concerned since it can be decided whether the systems are chaotic or not by using the largest exponent: if a system is chaotic then the largest exponent λ is greater than 0, otherwise the exponent is less than 0.

Fully connected recurrent networks composed of N units are considered. Activate functions of neurons are sigmoid. Letting t be discrete time, $t = 1, 2, \cdots$, the outputs of neu-