A controller design using modal decomposition of matrix pencil

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Abstract

This paper proposes LQ optimal controller design method based on the modal decomposition. Here, the design problem of linear time-invariant systems is considered by using pencil model. The mathematical model based on matrix pencil is one of the most general representation of the system. By adding some conditions the model can be reduced to traditional system models. In pencil model, the state feedback is considered as an algebraic constraint between the state variable and the control input variable. The algebraic constraint on pencil model is called purely static mode, and is included in infinite mode. Therefore, the information of the constant gain controller is included in the purely static mode of the augmented system which consists of the plant and the control conditions. We pay attention to the coordinate transformation matrix, and LQ optimal controller is derived from the algebraic constraint of the internal variable. The proposed method is applied to the numerical examples, and the results are verified.

1 Introduction

In control theory, a dynamical system is considered as a mapping to the output signal from an incoming signal, and analysis and design of the system have been carried out using a transfer function and a state space representation. In order to describe the system by the state space representation, however, the physical information of the original system may be lost by the model reduction and simplification process. Moreover, the model cannot be constructed when neither causality nor dynamical dimension is decided. In order to cope with such problems, some kinds of implicit representations have been proposed against the background of behavioral approach. When we shall confine our attention to linear time-invariant systems, pencil model is considered as one of the most general representation of the system [1]. The model is described using matrix pencil, and keeps the system information included in first principles. But pencil model is a verbose representation from the viewpoint of controller design problems. In the case when we design a controller based on pencil model, it is difficult to treat the model by a traditional method. Therefore, as a beginning, this paper proposes LQ optimal controller design method based on the modal decomposition.

The paper is structured as follows. In section 2 the pencil model is introduced, and some kind of pencils are defined in order to discuss the LQ control problem. Section 3 describes an optimal controller design method by using the modal decomposition. The proposed method is applied to numerical examples in section 4. Finally, the conclusions are outlined in section 5.

2 Preliminaries

2.1 Pencil model

This section reviews the definition of pencil model. Let's consider the system model defined as follows.

$$\begin{bmatrix} M & - & \mathcal{D}N \end{bmatrix} z = 0 \quad (1)$$

$$w = Sz \quad (2)$$

where $M \in \mathbf{R}^{q\times n}$, $N \in \mathbf{R}^{q\times n}$ and $S \in \mathbf{R}^{r\times n}$ are constant real valued matrices. The internal variable $z \in \mathbf{R}^n$ and the external variable $w \in \mathbf{R}^r$ are time continuous function. The letter $\mathcal{D}$ denotes a differential operator in the sense of distribution.