Identification Using Orthonormal Functions for Linear Dynamical Systems

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Abstract

The use of orthogonal functions with the aim of adapting the system and signal representation to the specific properties of the systems and signals has a long history. A least-squares identification method is studied that estimates a finite number of expansion coefficients in the series expansion of a transfer function, where the expansion is in terms of recently introduced generalized orthogonal functions. It is shown that there exist orthogonal functions that are generated by stable linear dynamical systems.

1 Introduction

• Physical systems are generally continuous-time in nature. However, as the data measured from these systems is generally in the form of discrete samples, and because of the speed, memory capacity and processing power of modern digital computers, most modern signal processing is performed in the discrete-time digital domain. Several problems exist with the conventional z-domain discrete-time representation.

• We will expand and generalize the orthogonal functions as basis functions for dynamical system representations. To this end, use is made of balanced realizations of inner transfer functions. The orthogonal functions can be considered as generalizations of, for example, the pulse functions, Laguerre functions, and Kautz functions, and give rise to an alternative series expansion of rational transfer functions. It is shown how we can exploit these generalized basis functions to increase the speed of convergence in a series expansion. If a model of the system G is represented by a finite-length series expansion then it is easily understandable that the accuracy of the model, in terms of the minimal possible deviation between system and model, will be essentially dependent on the choice of basis functions. In section 2 we first present the statement of problem, and we formulate the orthonormal functions in Section 3. In section 4 we introduce an identification of expansion coefficients. Finally, the proposed approximation technique is illustrated by a simple example.

2 Statement of Problem

Given the fact that every stable system has a unique series expansion in terms of a pre-chosen basis, a model representation in terms of a finite-length series expansion can serve as an approximate model, where the coefficients of the series expansion can be estimated from input-output data. A model of a linear stable time-invariant system with additive disturbance is given by:

$$y(t) = G^0(q)u(t) + v(t),$$

$$G^0(q) = \sum_{k=1}^{\infty} g_k q^{-k}, \qquad (1)$$

where $u(t)$ and $y(t)$ are the input and output signals, respectively. Time shifts are represented by the delay operator $q^{-1}u(t) = u(t-1)$. And $v(t)$ is a unit-variance, zero-mean white noise process. Let $\{\Psi_k(z)\}_{k=1,2,\cdots}$ be an orthonormal basis for the set of systems. The orthonormality of the basis is reflected by the property that