Sliding Mode Control for Attitude Tracking of Thruster-Controlled Spacecraft

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Abstract

Nonlinear pulse width modulation(PWM) controlled system is considered to achieve control performance of thruster-controlled spacecraft. The actual PWM controlled motions occurs, very closely, around the average model trajectory. Furthermore nonlinear PWM controller design can be directly applied to thruster controlled spacecraft to determine thruster on-time. Sliding mode control for attitude tracking of three-axis thruster-controlled spacecraft is presented. Simulation results are shown which use modified Rodrigues parameters and sliding mode control law to achieve attitude tracking of a three-axis spacecraft with thrusters.

1. Introduction

In a great variety of practical cases, the control signal in variable structure systems(VSS) is a pulse-modulated signal(in particular, pulse-amplitude, pulse-frequency or pulse-width modulated control signal). Variable structure systems in which the control signal is a train of pulses constitute a sub-class of such systems, which poses specific properties. A natural name for such system is referred to as sampled-data variable structure systems.

PWM controlled systems constitute a sub-class of nonlinear sampled-data control system. The sampled output error, which is difference between the desired and the actual system output signals, is translated into a pulse control signal whose pulse width is proportional to the error signal. Typically, PWM controlled systems, as VSS in sliding mode, are known as robust with respect to parameter variations of system and external perturbation signal[1].

Sira-Ramirez has studied a different design approach by using the geometric properties of average PWM controlled system response[4-5]. By allowing a simpler analysis of nonlinear PWM controlled systems through their average response, it is also known that actual PWM controlled system response shows sliding mode trajectories around integral manifolds of the average PWM controlled system model[1]. Furthermore, thruster-controlled spacecraft system constitutes a sub-class of nonlinear PWM controlled system. Therefore nonlinear PWM controller design can be directly applied to the attitude tracking of thruster-controlled spacecraft. In real application, it is very important to determine on-time of thruster because system response directly depends on it.

In this paper, sliding mode control problem for attitude tracking of thruster-controlled spacecraft is considered. Actual thruster on-time is given by PWM controller. Spacecraft control problem is considered based on modified Rodrigues parameters to achieve non-singular attitude description and minimal parameterization. The specification of nonlinear PWM controlled system is made on the basis of the average PWM model. The main contribution of this paper is utilization of nonlinear PWM controlled system that exactly describes the behavior of thruster-controlled spacecraft.

2. Theoretical Background

In this section, a review of variable structure control and dynamic equations of motions for a three-axis stabilized spacecraft is presented.

2.1 Attitude Kinematics and Dynamics

The attitude of spacecraft is assumed to be presented by quaternion. The quaternion representation is defined as

$$\beta_0 = \cos\frac{\theta}{2} \tag{1a}$$

$$\beta_i = n_i \sin\frac{\theta}{2}, \qquad i = 1,2,3 \tag{1b}$$

where n_i is an unit vector corresponding to the axis of rotation, and θ is the angle of rotation. For minimal paramerization of the attitude, we use modified Rodrigues parameters(MRP) which is derived by applying stereographic projection of the quaternions. The transformation from quaternion to MRP vector σ is given by [2]

$$\sigma_i = \frac{\beta_i}{\beta_0 + 1}, \qquad i = 1,2,3 \tag{2}$$

Using Equation (1), the modified Rodrigues parameters are written as

$$\sigma = \tan\frac{\theta}{4}\hat{n} \tag{3}$$

Stduying Equation (3), it is evident that the MRP have a geometric singularity at $\theta = \pm 360$ degrees. But the non-uniqueness of the MRP allows one to avoid their singularities[2]. The MRP kinematic differential equation in vector form by using spacecratft's body angular velocity (ω), is

$$\dot{\sigma} = \frac{1}{4}\left[(1-\sigma^2)I_{3\times3} + 2[\sigma\times] + 2\sigma\sigma^T\right]\omega = B(\sigma)\omega \tag{4}$$

where the notation $\sigma^2 = (\sigma^T\sigma)$ is used, and $I_{3\times3}$ is the 3×3 identity matrix. The cross matrix operator $[\sigma\times]$ is defined by

$$[\sigma\times] \equiv \begin{bmatrix} 0 & -\sigma_3 & \sigma_2 \\ \sigma_3 & 0 & -\sigma_1 \\ -\sigma_2 & \sigma_1 & 0 \end{bmatrix} \tag{5}$$

The inverse transformation of $B(\sigma)$ in Equation (4) in explicit vector form is given by: