Adaptive Controllers for Feedback Linearizable Systems using Diffeomorphism

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Abstract— **A systematic scheme is developed for the design of new adaptive feedback linearizing controllers for nonlinear systems. The developed adaptation law estimates the uncertain time-varying parameters using the structure of diffeomorphism. Our scheme is applicable to a class of nonlinear systems which violates the restrictive parametric-pure-feedback condition [4]-[6].**

Keywords— **Feedback linearization, Adaptive control, Parameter uncertainty**

I. Introduction

Feedback linearization is one of the well known popular nonlinear control schemes to deal with nonlinear systems. For the exactly known system dynamics, the stability, regulation, and tracking problem have been successfully solved in the literature of [1]-[3]. However, in real practice, since there exist structural and/or parameter uncertainty, it is difficult to use exact feedback linearization directly. To overcome this limitation many adaptive schemes have been developed [4]-[7]. In [4], the adaptive schemes are developed for uncertain nonlinear systems via two-step transformations - diffeomorphism and backstepping-like adaptive scheme with update laws. The uncertainties are limited to the case where they enter linearly in the system. This limitation is relaxed in [5] by allowing uncertainties to be in nonlinear forms and time-varying case is included. In [6], the adaptive tracking problem is mainly focused in a more general form by including unbounded tracking dynamics. However, the restrictive parametric-pure-feedback conditions still have to be met. Recently, in [7], the singularity issue is pointed out when applying adaptive schemes and a switching control between exact and approximate feedback linearization is suggested at the expense of zero residual tracking errors.

In this paper, we develop an adaptive diffeomorphism in order to tackle a regulation problem against the unknown system parameter. The construction of the adaptation law is based on the structure of the diffeomorphism of feedback linearization methodology. Thus, the proposed adaptive scheme is an intrinsic technique of feedback linearization and retains a capacity to deal with a more general form of systems compared with other literature.

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II. Adaptive Feedback Linearization using Diffeomorphism

A. Formulation

Consider the following single-input system

$$\dot{x} \;=\; f(x,\theta) + g(x,\theta)u \tag{1}$$

where $x \in \mathbf{R}^n$ is the state vector, $u \in \mathbf{R}$ is the control input, $y \in \mathbf{R}$ is the controlled output, and $\theta \in \mathbf{R}$ is the time-varying uncertain parameter. We assume that $f(\cdot)$ and $g(\cdot)$ are real smooth functions and the nominal value of $\theta = \theta_o$ is known. Without loss of generality, θ_o is taken to be zero.

First, we assume that (1) is input-state feedback linearizable for $\theta \in \Gamma$; there exists a diffeomorphism $z = T(x,\theta)$ that transforms (1) into the Byrnes-Isidori normal form

$$\begin{aligned} \dot{z}_i &= z_{i+1} \quad \text{with} \quad 1 \le i \le n-1 \\ \dot{z}_n &= \alpha(z,\theta) + \beta(z,\theta)u \end{aligned} \tag{2}$$

Using the controller $u = \frac{1}{\beta(z,\theta)}(-\alpha(z,\theta) + v)$ where $\beta(z,\theta) \neq 0$ in domain of attraction, we obtain the linearized system

$$\dot{z} = Az + Bv \tag{3}$$

where (A, B) is in the Brunovsky canonical form and v is a new control input to stabilize the system.

Note that to stabilize the system, we simply have $v = Kz$ to make $A + BK$ Hurwitz. However, since z is obtained from $T(x,\theta)$, some states of z are not available directly. Instead, we actually use the estimated states $v = K\hat{z}$ to force $\hat{z}$ track z($\hat{\theta}$ tracks θ) faster than system dynamics.

B. Asymptotic Stabilization using Adaptive Diffeomorphism

We define $\psi(x,\theta) \in \mathbf{R}^m$ a vector made up with $1 \le m \le n$ number of the elements of $T(x,\theta)$ such that $\frac{\partial T_i(x,\theta_o)}{\partial\theta}$ is nonzero for $x \in D_x \subset \mathbf{R}^n$ where $1 \le i \le n$. In order to estimate θ, we define

$$\xi_i = \int_0^t \psi_i(x,\hat{\theta})d\tau \tag{4}$$

for $1 \le i \le m$ where $\hat{\theta}$ is the estimate of θ. Based on (4), an adaptation law of $\hat{\theta}$ is defined by

$$\hat{\theta} = \theta_o + \sum_{i=1}^{m} \phi_i \quad \text{with} \quad \phi_i = -\frac{\eta_i}{\Omega_i}\xi_i \tag{5}$$

where η_i is an adaptive gain and $\Omega_i = \frac{\partial\psi_i(x,\theta_o)}{\partial\theta}$.
To preceed further, we provide an intuition of the adaptation law (5) by considering the case when $m = 1$ for