Real-Time Optimal Control for Nonlinear Dynamical Systems Based on Fuzzy Cell Mapping

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Abstract

The complexity of nonlinear systems makes it difficult to ascertain their behavior using classical methods of analysis. Many efforts have been focused on the advanced algorithms and techniques that hold the promise of improving real-time optimal control while at the same time providing higher accuracy.
In this paper, a fuzzy cell mapping method of real-time optimal control for nonlinear dynamical systems is proposed. This approach combines fuzzy logic with cell mapping techniques in order to find the optimal input level and optimal time interval in the finite set which change the state of a system to achieve a desired objective. In order to illustrate this method, we analyze the behavior of an inverted pendulum using fuzzy cell mapping.

1. Introduction

The minimum-time and minimum-energy optimal control concepts have been most naturally and frequently applied and extended to various control engineering problems including many practical aerospace applications. In addition, other control topics such as fuzzy logic control and various nonlinear control theories has also been developed and widely adopted. The rapid development of microprocessors has allowed the real-time implementation of optimal control concepts combined with other control concepts to many different engineering systems.
Zadeh[1] introduced fuzzy logic during the late sixties and it has also found many applications in engineering systems. Prior to Zadeh, a number of authors had investigated three and multi-valued logic systems. However, unlike these other systems which were developed for a variety of reasons, fuzzy logic has been widely accepted and has found many useful applications. The reason for making mathematical programming fuzzy is to allow the model of the object or evaluation to have ambiguity and to extract a solution that seems to be good. Fuzzy logic has been very successful when applied to nonlinear control systems. Most physical systems are nonlinear. These systems are difficult to control with conventional controllers. Fuzzy logic control systems have shown the capability of handling system nonlinearities along with modeling uncertainties and imprecision[2]. The success that fuzzy logic has shown with control systems and the nonlinear engineering world has motivated us to look at using it to analyze and better understand optimum control for nonlinear systems. More recently, many efforts have been focused on the development and implementation of advanced algorithms and techniques that hold the promise of improving real-time optimal control[3] while at the same time providing higher control accuracy.
Hsu[4] introduced the Simple Cell Mapping method(SCM) which has been introduced not only for studying nonlinear dynamical systems but for a new approach to optimum control problems. These mappings generally provide a very good approximation to the global behavior of the system although they may not be highly accurate at the local level. The loss in accuracy results from the need to associate the behavior of the cell with its center point. In order to overcome the drawbacks associated with the SCM method we introduced the Fuzzy Cell Mapping (FCM) method[5].
Both SCM and FCM can be viewed as a Markov chain. However, the FCM is different from SCM in that the SCM membership function is crisp whereas the FCM membership function is fuzzy. A Markov process is a special type of stochastic process distinguished by a certain Markov property. The probability for a future state given the present state is not changed by the information concerning past states. A Markov chain is a type of Markov process with a denumerable number of states. The time parameter is taken to be the set of nonnegative integers or the set of nonnegative real numbers. The mathematical theory of Markov chains is well developed and can be directly applied to SCM or FCM.
In this paper, we use FCM method to develop real-time optimal control for nonlinear dynamical systems. The remainder of the paper is divided into three sections. In section 2, we arrange fuzzy cell mapping and membership function. In section 3, we use α -cut of the membership function to work an example. We first analyze the system using the SCM method and then, without further numerical integrations, develop the transition possibility matrix from the membership function. We then use this matrix to analyze the system. In the last section, we summarize the results and give our conclusions. In addition, we suggest future work in this area that we believe is important.

2. Fuzzy Cell Mapping and Membership Function

Cell mapping analysis replaces integration by the simpler operation of mapping. This requires converting the continuous state space into a cell state space. There are many ways to build cells in a state space. We will follow Hsu [8] and divide the state space into finite cells of uniform size. Each cell will have a length h_i along an axis x_i. The state variable x_i can be divided into intervals such that

$$(Z_i - \frac{1}{2})h_i \le x_i < (Z_i + \frac{1}{2})h_i \tag{1}$$

where z_i is an integer. For our discussion the number of intervals the x_i axis is divided into is finite, say m_i, so that $z_i=1,2,...,m_i$.
An N-tuple z_i, $i=1,2,...,N$, is then called a cell vector and is denoted by $\underline{z}$. A point $\underline{x}(x_i, i=1,2,...,N)$ belongs to a cell $\underline{z}(z_i, i=1,2,...,N)$ if x_i belongs to z_i for all i. The state space X can be considered as a collection of cells, $\underline{z}(z_1,z_2,...,z_N)$, which constitutes the cell state space. In a cell state space, the source of the mapping is called an original cell and the target of the mapping is called an image cell. In FCM, the position variable $\underline{x}$ is the mapped location in the image cell of the original cell's center point. A membership function allocating grades of membership for X_1 to the cells, Z_1, in the image cell's state space, can be defined for two dimension. Fig. 1 shows a cell state space for two dimensions. The center point of the original cell has been mapped to the cell $\underline{z}(j,k)$ and is shown as being a distance d_1 and d_2 away from the center point of $\underline{z}(j,k)$ along the x_1 and x_2 axis, respectively. We determine the membership value of the mapped point in the image cell $\underline{z}(j,k)$, from the intersection of the mapped cell, shown as dotted lines in Fig. 8, with the image cell. The area, A_v, intersecting the mapped cell and the image cell, $\underline{z}(j,k)$, is