Approach to BMI Problems Using Evolution Strategy

Tae-Jin Chung   Chan-Soo Chung

Dept. of Electrical Engineering, Soongsil University
1-1, Sangdo-5-Dong, Dongjak-Gu, Seoul, 156-743, Korea
TEL : +82-2-817-0004 FAX : +82-2-817-0961
E-mail : tarzan@ee.ssu.ac.kr

Abstract

Biaffine Matrix Inequalities(BMIs) are known to give more general and flexible frameworks in control designs than Linear Matrix Inequalities(LMIs). However, BMIs are nonconvex constraints and very difficult to solve. In this paper, BMI problems are solved using Evolution Strategy(ES). Numerous BMI problems are solved to verify performances of ES solver for BMI problems and compared with those of Genetic Algorithms and Branch-and-Cut algorithm.

Keywords

Biaffine Matrix Inequalities, Evolution Strategy

1 Introduction

Linear Matrix Inequalities(LMIs) have gained significant interest in the control community in recent years. Their history probably dates back to Lyapunov's work in the end of the last century. Since then, hundreds of papers have been published on this subject. Nowadays, it is known that a wide variety of problems arising in the system and control theory can be reduced to handful problems involving LMIs [1]. $\mathcal{H}_\infty$ control, $\mathcal{H}_2$ control, control of time-delayed system, and robust control problems are successfully solved in terms of LMIs.

More recently, Biaffine Matrix Inequalities(BMIs) gained much interest in the control community. The BMIs are the biaffine extension of the LMIs. Problems involving BMIs naturally come out when controllers are designed in robust control synthesis problems, more specifically, when controllers for dynamical systems, e.g., satellites, aircrafts or disk drivers, are designed in order to guarantee worst-case stability and performance in face of modeling uncertainties and disturbance inputs. It has been shown that a wide range of difficult control problems are reducible to problems involving BMIs, and they have been intensively studied by researchers in the past 5 years since they have more flexible and more general frameworks to describe dynamical systems than LMIs.

The first paper that formally introduced the term BMI in control theory is probably due to Safonov, Goh, and Ly in 1994 [2]. Later Goh *et al.* presented the first implemented algorithm for the Biaffine Matrix Inequality Eigenvalue Prolbem (BMIEP). They proposed a convex relaxation of the problem, which becomes an SDP, and a Branch-and-Bound (B&B) algorithm to obtain the global optimum. The same authors also proposed two algorithms to approximate local optima. Other B&B algorithms were proposed by VanAntwerp, and Fujioka and Hoshijima who essentially improved the lower bound through a better convex relaxation of the BMIEP [3]. The D. C. optimization techniques were used by Tuan *et al.* on the BMI feasible problem. Theoretical aspects of the BMI were studied by Meshahi and Papavassilopoulos [4]. They established equivalent formulations of the BMI feasible problem as the generalizations of Linear Programming (LP) and Linear Complementarity Problem (LCP) over a specific conic space of matrices. Althouth the LCP is very actively studied in mathematical programming, it is difficult to solve. Moreover, the formulation in [4] need a rank 1 constraints for the solution that make the problem much more difficult. Therefore, the implementation of this approach has not been published yet. More recently, the generalized Benders decomposition for bilinear and biconvex programmings has been extended to