Constrained H_∞ Optimal Control

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Abstract: In this paper, we show based on Lyapunov theorem that the closed loop system with the constrained H^∞ optimal controller is exponentially stable. Then the on-line feedback implementation of the constrained H^∞ optimal control based on quadratic programs is proposed.

Keywords: H_∞ Optimal Control, Constrained Control, Dynamic Game, Linear Discrete-Time Systems

1. Introduction

In 1980, Zames [6] proposed the H_∞ disturbance attenuation problem. Since then, H_∞ optimal control problem has been the hottest research topic in control community. The solution of H_∞ optimal control problem has been initially sought within operator theoretic framework. However the resulting solution procedure was complicated. Later, using state space approach, a very simple and efficient solution technique based on Riccati equations has been obtained by Doyle, Glover, Khargonekar and Francis [4]. However, constraints are always present in any practical control problems. For instance, the physical restriction of the actuator limits the value the input can assume. Moreover due to safety, environmental regulation and so on, the states of the plant are desired to lie within a designated area in the state space. Under the presence of these constraints, the closed loop system becomes nonlinear and the current analysis is no longer valid. However due to the difficulty caused by the nonlinearity, the constrained H_∞ optimal control problem remained unsolved.

In this paper, we first establish that the constrained H_∞ optimal control is exponentially stable. Especially, if the plant is stable, the closed loop system is shown to be globally exponentially stable. The constrained H_∞ optimal control problem is a constrained infinite dimensional dynamic game problem where infinite number of variables and constraints exist. Hence the problem cannot be solved as it is. However we show that, for given the initial state, the constrained H_∞ optimal control problem is equivalent to another a constrained infinite dimensional dynamic game problem where only finite number of constraints exist. Then we establish that the latter problem can be reduced to a constrained finite dimensional dynamic game problem. Employing open loop information pattern solution, further reduction of this problem into a finite dimensional quadratic programming problem is also shown to be possible for some measured state. The feedback implementation of proposed technique based on the quadratic programming problems is also discussed. Finally the proposed technique is elucidated with an example.

2. Preliminaries

2.1 H_∞ *Optimal Control*

In this section, we briefly summarize some standard results in H_∞ optimal control as exposed in [1], [2].

Consider the system

$$x_{k+1} = Ax_k + Bu_k + Dd_k, \tag{1}$$
$$y_k = Cx_k + n_k,$$

where $x_k \in \mathbf{R}^n$ is the state vector; $u_k \in \mathbf{R}^m$ is the manipulated input; $d_k \in \mathbf{R}^p$ is the unknown disturbance; $y_k \in \mathbf{R}^l$ is the output; $n_k \in \mathbf{R}^q$ is the measurement noise. Throughout the paper, the system is assumed to be stabilizable and detectable.

Associated with the linear system, consider the H_∞ optimal control problem [2]:

$$J_u(x_0) = \min_{\mathbf{u}} \max_{\mathbf{d}} \left\{ \sum_{k=0}^{\infty} x_k^T Q x_k + \sum_{k=0}^{\infty} u_k^T u_k \right. \tag{2}$$
$$\left. -\gamma^2 \sum_{k=0}^{\infty} d_k^T d_k \right\}$$

subject to (1) where $Q > 0$. $Q > 0$ is assumed throughout the paper. Using $\frac{1}{\gamma}D$ instead of D, we can assume $\gamma = 1$. We adopt this assumption throughout the paper.

The H_∞ optimal control problem admits the feedback but not necessarily saddle point solution as follows.