

# Structural Dynamic System Reconstruction for Modal Parameter Estimation

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**Abstract :** We present a modal parameter estimation technique by developing a residual based system reconstruction and using the system matrix coordinate transformation. The modal parameters can be estimated from the residues of the system transfer functions expressed in modal coordinate basis, derived from the state space system matrices. However, for modal parameter estimation of multivariable and high order structural systems over broad frequency bands, this non-iterative algorithm gives high accuracy in estimating the natural frequencies and damping ratios. From vibration tests on cross-ply and angle-ply composite laminates, the natural frequencies and damping ratios can be estimated using the modal coordinates of the structural system reconstructed from the experimental frequency response. These results are compared with those of finite element analysis and single-degree-of-freedom curve-fitting.

**Keywords :** system reconstruction, parameter estimation, frequency response function, natural frequency, damping ratio, vibration of composite laminates.

## 1 Introduction

The modal parameters of flexible structures have traditionally been extracted using FRF curve-fitting methods such as multiple reference orthogonal polynomial polyreference frequency domain techniques [1]. Although adequate in vibration analysis of single band and lower order systems, these may be inadequate for state-space based controller design and damage detection.

Subspace system identification techniques have been proposed to provide numerically reliable state space realization of high-dimensional multivariable systems. Juang et al developed eigensystem realization algorithm using data correlation (ERA/DC) [2] used singular value decomposition to reduce the effect of noise in stochastic systems. Liu and Skelton [3] proposed the covariance equivalent realization (q-COVER) method using  $q + 1$  output covariance derivatives. Liu and Miller [4] developed the methods of observability range space extrac-

tion (ORSE) method by generalizing the q-COVER and ERA identification. In the frequency domain, Juang et al also developed the eigensystem realization algorithm [5], in which the system response must disappear completely within the time-window. Liu et al developed the frequency domain ORSE identification algorithm by extending the concept of the ORSE identification to the frequency domain. Kim and Hwang recently presented a modal-coordinate based structural reconstruction method [7].

The present paper uses the subspace identification method, to provide a structural system reconstruction using vibration spectral estimates that allow reconstruction of transfer function in the term of the modal parameter for transfer function poles and zeros. Next, the bases of the identified system matrices are related to the modal bases of the structural dynamic system. The transfer function described in modal-parameter space is obtained, providing the modal parameters (natural frequencies, modal dampings, eigenshapes) of the structural dynamic system. From the modal based transfer function, the eigenvalues representing natural frequencies and damping ratios are represented for vibration analysis of composite laminated plates. The effectiveness of this method can be shown by comparing simulation results with FEM-based modal analysis.

## 2 Residual Spectrum-Based Subspace Identification

Since the measurement data and their digitally processed spectra are discrete, we can derive the state space model of a sampled data system with zero-order hold from the continuous system model with parameters  $[\bar{A}, \bar{B}, \bar{C}, \bar{D}]$ , such that

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k), \quad \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) \quad (2.1)$$

and

$$\mathbf{A} = e^{\bar{\mathbf{A}}\Delta t}, \quad \mathbf{B} = \int_0^{\Delta t} e^{\bar{\mathbf{A}}(\Delta t-\tau)} \bar{\mathbf{B}} d\tau, \quad \mathbf{C} = \bar{\mathbf{C}}, \quad \mathbf{D} = \bar{\mathbf{D}} \quad (2.2)$$