Controller Design for Input-Saturated Linear Systems

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Abstract

In this paper, we provide an approach of controller synthesis for input-saturated linear systems by a linear parameter varying (LPV) framework. Using directly the saturation nonlinearity as scheduling parameters, we propose an LPV-stabilizer with parameter -dependent dynamic state-feedback controller concept. Especially, the synthesis conditions are formulated in terms of linear matrix inequalities (LMIs) that can be solved very efficiency.
Keywords : Saturating Actuator, LPV, LMI, Circle Criterion

1 Introduction

For the recent ten years, limited capacity of actuator has long been accepted as a major challenge in controller design. A typical approach to formulate such an actuator saturation problem is, assuming a linear time-invariant plant, to model the saturation block as a sector-bounded nonlinearity and apply small-gain type conditions [1, 2]. And many of the existing non-linear techniques for dealing with input saturation utilize such a controller that works within a saturation level [3].

Our suggestion for dealing actuator saturation problem is different from previous works in the following aspects. First, the proposed design directly accounts for the saturation nonlinearity by representing each status of the actuator with gain-scheduled parameters. Second, we design a parameter-dependent controller with circle criterion, hence, the resulting controller become a gain-scheduling controller with respect to the saturation scheduling parameters.

2 Problem Formulation

Consider the linear continuous-time system denoted by

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state and $u(t) \in \mathcal{R}^1$ is the system input (we treat single input case). A and B are known real constant matrices of appropriate dimensions. Furthermore, pair (A, B) is assumed to be controllable.

The system input $u(t)$ is assumed to take values in the compact set $\Omega \subset \mathcal{R}^1$:

$$\Omega = \{u(t) \in \mathcal{R}^1 : -u_{max} \le u(t) \le u_{max}\}, \tag{2}$$

with u_{max} positive value of $\mathcal{R}^1$. From (2), the saturation function $sat(\bar{u}(t))$ is defined as

$$sat(\bar{u}(t)) = \begin{cases} \bar{u}(t) & |\bar{u}(t)| \le u_{max} \\ sign(\bar{u}(t))u_{max} & |\bar{u}(t)| > u_{max} \end{cases}. \tag{3}$$

The saturation function $sat(\bar{u}(t))$ represents the limited actuator effects on the controller output $\bar{u}(t)$.

Next, we define the following saturation scheduling parameter

$$\rho(\bar{u}(t)) = \frac{\bar{u}(t) - sat(\bar{u}(t))}{\bar{u}(t)}, \tag{4}$$

and $\rho(\bar{u}(t)) = 0$, if $\bar{u}(t) = 0$, that is $\rho(\bar{u}(t)) \in [0\ 1)$. Then, we can find that the saturation scheduling parameter $\rho(\bar{u}(t))$ satisfies the following sector condition (see Fig 1) :

$$\rho(\bar{u}(t))(\rho(\bar{u}(t)) - \delta(a)) \le 0,\ \forall \bar{u}(t) \in [-a,\ a], \tag{5}$$

where

$$\delta(a) = \begin{cases} \frac{a-u_{max}}{a} & a \ge u_{max} \\ 0 & a \le u_{max} \end{cases}. \tag{6}$$

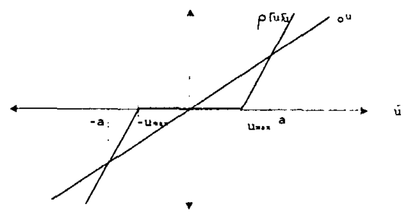

Figure 1: Saturation scheduling parameter

And then with the scheduling parameter, the system (1) can be written in a LPV form :

$$\dot{x}(t) = Ax(t) + B(1 - \rho)\bar{u}(t). \tag{7}$$

Consequently, the goal of this paper is to design the following parameter-dependent dynamic state-feedback controller

$$\begin{aligned} \dot{x}_k(t) &= A_k(\rho)x_k(t) + B_k(\rho)x(t) \\ \bar{u}(t) &= C_k x_k(t) + D_k x(t) \end{aligned}, \tag{8}$$