불확실 시간지연 시스템에 대한 지연량을 고려한 성능보장 제어

Delay-dependent Guaranteed Cost Control for Uncertain Time-delay Systems

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Abstract: This paper considers delay-dependent guaranteed cost control for uncertain time-delay systems with norm-bounded parametric uncertainties. A new delay-dependent condition for the existence of the guaranteed cost control law is presented in terms of linear matrix inequalities (LMI). An algorithm involving convex optimization is proposed to design a controller which guarantees the suboptimal minimum of the guaranteed cost of the closed-loop system for all admissible uncertainties.

Key Words: Guranteed cost control (GCC); Delay; Delay-dependent; LMI

1 Introduction

Since time-delay is often a source of instability in many engineering systems, considerable attention has been paid to the problem of stability analysis and controller synthesis for time-delay systems [1]-[3]. Especially, in accordance with the advance of robust control theory, a number of robust stabilization methods have been proposed for uncertain time-delay systems.

The existing robust stabilization results for time-delay systems can be classified into two types:delay-independent stabilization and delay-dependent stabilization [4]-[7]. The delay-independent stabilization provides a controller which can stabilize a system irrespective of the size of the delay. On the other hand, the delay-dependent stabilization is concerned with the size of the delay and usually provides an upper bound of the delay such that the closed-loop system is stable for any delay less that the upper bound.

In addition to the simple stabilization, additional criteria such as quadratic cost minimization, H_∞ norm minimization, and pole placement, can be considered when designing a controller. In this paper, a new design method for delay-dependent guaranteed cost control via state-feedback is presented for uncertain time-delay systems. Through guaranteed cost control, one can stabilize the systems systems while maintaining an adequate level of performance represented by the quadratic cost. Guaranteed cost control for uncertain systems with delay has been considered for continuous-time systems in [8]-[10] and for discrete-time systems in [11]. However, all these results are for the systems which are stabilizable independently of delay. In this paper, the results in [4] are extended such that a guaranteed cost controller for delay-dependent time delay systems can be obtained.

This paper is structured as follows: In Section 2, problem formulation and some preliminaries are given. In Section 3, guaranteed cost control for nominal time-delay systems are considered first. In Section 4, guaranteed cost control for uncertain time-delay systems is presented. In Section 5, a numerical example is given and conclusions follow in Section 6.

Notation: $\mathbf{R}^+$ is the set of nonnegative real numbers. $\mathcal{C}_h = \mathcal{C}([-h,0],\mathbf{R}^n)$ denotes the Banach space of continuous vector functions which maps the interval $[-h,0]$ into $\mathbf{R}^n$ with the topology of uniform convergence. $\|\cdot\|$ refers to either the Eulclidean vector norm or the induced matrix two-norm. $\|\phi\|_c = \sup_{-h\le t\le 0}\|\phi(t)\|$ is a norm of a function $\phi \in \mathcal{C}_h$.

2 Problem formulation and Preliminaries

Consider the uncertain time-delay systems described by the following state equation:

$$\begin{aligned} \dot{x}(t) &= (A + D\Delta(t)E)x(t) + (A_1 + D\Delta(t)E_1)x(t-h) \\ &\quad +(B + D\Delta(t)E_u)u(t), \\ x(t) &= \phi(t),\ t \in [-h,0], \\ \Delta(t) &= \mathrm{diag}\{\Delta_1(t),\cdots,\Delta_r(t)\}, \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state, $u(t) \in \mathbf{R}^m$ is the control input, A_1, A_2, B, D, E_1, E_2 and E_u are all real constant matrices and h is a scalar representing the delay in the system. $\Delta_i(t) \in \mathbf{R}^{n_i\times n_i}$ is uncertain time-varying matrix satisfying the bound $\Delta_i(t)^T\Delta_i(t) \le I$, $i = 1,\cdots,r$. Given positive-definite symmetric matrices Q and R, the following quadratic cost is considered:

$$J = \int_0^\infty \{x(t)^TQx(t) + u(t)^TRu(t)\}dt. \tag{2}$$

We are interested in designing a memoryless state-feedback controller

$$u(t) = Kx(t), \tag{3}$$

which stabilizes the uncertain systems (1) and minimizes the upper bound of the quadratic cost in (2).

Before moving on, we introduce some lemmas which are essential for the development of our results.

Lemma 1. *For any matrices D, E, Δ, and Λ with appropriate dimensions such that $\Delta = \mathrm{diag}\{\Delta_1,\cdots,\Delta_r\}$, $\Delta_i^T\Delta_i \le I$ and $\Lambda = \mathrm{diag}\{\lambda_1 I,\cdots,\lambda_r I\}$, $\lambda_i > 0$ for $i = 1,\cdots,r$, it follows that*

$$D\Delta E + E^T\Delta^TD^T \le D\Lambda D^T + E^T\Lambda^{-1}E. \tag{4}$$

Lemma 2. [4] *Assume that $a(\cdot) \in \mathbf{R}^{n_a}$, $b(\cdot) \in \mathbf{R}^{n_b}$ and $\mathcal{N}(\cdot) \in \mathbf{R}^{n_a\times n_b}$ are defined on the interval Ω. Then, for any matrices $X \in \mathbf{R}^{n_a\times n_a}$ and $Z \in \mathbf{R}^{n_b\times n_b}$, the following holds:*

$$-2\int_\Omega a^T(\alpha)\mathcal{N}b(\alpha)d\alpha \le \int_\Omega \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y-\mathcal{N} \\ Y^T-\mathcal{N}^T & Z \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} \tag{5}$$

where

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0.$$

Lemma 3. *Suppose that $V_1, V_2 : \mathbf{R}^+ \to \mathbf{R}^+$ are continuous nondecreasing functions such that $V_1(s)$ and $V_2(s)$ are positive for $s > 0$ and $V_1(0) = V_2(0) = 0$ and that $V : R \times \mathcal{C}_h \to R$ is a continuous cost functional satisfying*

$$V_1(\|x(t)\|) \le V(t,x_t) \le V_2(\|x_t\|_c), \tag{6}$$

where x_t denotes $x_t(\theta) = x(t+\theta)$ for $-h \le \theta \le 0$. Given $Q = Q^T > 0$ and $R = R^T > 0$, if there exists a control law $u(t)$ such that

$$\dot{V}(t,x_t) < -\{x(t)^TQx(t) + u(t)^TRu(t)\}, \tag{7}$$

then the systems (1) controlled by $u(t)$ is quadratically stable and $V(0,x_0)$ upper bounds the quadratic cost incurred by $u(t)$, that is,

$$\int_0^\infty \{x(t)^TQx(t) + u(t)^TRu(t)\}dt < V(0,x_0). \tag{8}$$

3 Guaranteed cost control for nominal systems

Let us consider a nominal state-delayed system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_1x(t-h) + Bu(t), \\ x(t) &= \phi(t),\ t \in [-h,0]. \end{aligned} \tag{9}$$

We start with the guaranteed cost stability analysis of the unforced system (9) with $u(t) = 0$. The following theorem presents a delay-dependent guaranteed cost stability condition.

Theorem 1. *If there esist $P > 0$, $S > 0$, X, Y, and Z such that*

$$\begin{bmatrix} (1,1) & PA_1 - Y & hA^TZ \\ A_1^TP - Y^T & -S & hA_1^TZ \\ hZA & hZA_1 & -hZ \end{bmatrix} < 0, \tag{10}$$

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0, \tag{11}$$

where

$$(1,1) \triangleq PA + A^TP + hX + Y + Y^T + S + Q,$$

then the unforced system (9) with $u(t) = 0$ is asymptotically stable and the cost function in (2) satisfies the bound:

$$J \le x(0)^TPx(0) + \int_{-h}^0\int_\beta^0 \dot{x}(\alpha)^TZ\dot{x}(\alpha)d\alpha d\beta + \int_{-h}^0 x^T(\alpha)Sx(\alpha)d\alpha \tag{12}$$

Proof. Define the cost functional $V(t,x_t)$ as follows:

$$V(t,x_t) \triangleq V_1 + V_2 + V_3 \tag{13}$$