Optimal Preventive Maintenance Policy Based on Aperiodic Model

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Abstract

Preventive maintenance(PM) is an action taken on a repairable system while it is still operating, which needs to be carried out in order to keep the system at the desired level of successful operation. The PM improves the reliability of the system by predicting the possible failures and thereby preventing such failures from its occurrence. In this paper, we develop the optimal preventive maintenance policies based on the aperiodic PM model. We investigate an aperiodic preventive maintenance policy and propose several optimal PM policies which minimize the expected cost over an infinite time span. Park, Jung and Yum(2000) determine the optimal period and the optimal number of PMs based on Canfield's(1986) periodic model. Our techniques to derive the optimal preventive maintenance policies based on our aperiodic PM model is similar to those in Park, Jung and Yum(2000), which can be considered as the special case of our results.

1. Introduction

Preventive maintenance is carried out to slow the degradation process of the repairable system and to keep the system operating without failure during its mission period. Since most of the repairable systems are subject to deteriorate with time in practice, more efficient maintenance of the system is critical to reduce the failure of the system and to improve the productivity of the system. Thus, it is desirable to develop a PM model, under which the system keeps its operation at the prespecified level of reliability and the operating cost is minimized over a finite or an infinite time span.

Kim, Yum and Park(2000) extend the periodic PM model proposed by Canfield(1986) to the case when the system undergoes the PMs at different intervals. Canfield(1986) discusses a periodic PM model of a system for which the PM slows the rate of degradation, while the hazard rate keeps monotone increase. Park, Jung and Yum(2000) determine the optimal period and the optimal number of PMs for Canfield's(1986) periodic PM model so that the expected cost rate per unit time for an infinite time span is minimized. In this paper, we consider an aperiodic PM model (Nakagawa(1986) called it sequential PM model), for which the system is maintained preventively at constant intervals x_k for $k = 1, 2, \cdots, N$ and it is replaced by a new system at the Nth PM. Each PM is assumed to relieve stress temporarily and to slow the degradation process of the system. If the system fails between PMs, it undergoes only minimal repair and hence, the hazard rate remains unchanged after any of these minimal repairs is completed. Since the effect of PM may depend not only on the age

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of the system, but also on the number of PMs performed previously, it is more reasonable to perform the PMs at different intervals to make the PM policy more effective.

In Section 2, we derive the expression for the expected cost rate per unit time based on the aperiodic PM model. Section 3 discusses the optimal aperiodic PM policy when the number of PMs needed prior to replacing the system by a new one is assumed to be known. Section 4 provides numerical examples for illustrative purposes.

2. Expected Cost Rate Based on The Aperiodic Model

In this section, we utilize the aperiodic PM model, which is proposed in Kim, Yum and Park(2000), to derive the explicit expression for the expected cost rate per unit time. Firstly, we express the number of failures as a function of hazard rate and then, we derive the expression for the expected cost by multiplying by a given cost structure. For the aperiodic model, the hazard rate keeps monotonically increasing, although the rate of degradation is reduced after each PM. If the system fails, it undergoes only minimal repair and hence, the hazard rate remains undisturbed by any of these minimal repairs. For such a model, the system which responds to PM has an increasing hazard rate indicating that the system degrades with time.

If the system wears out with time, the rationale for the proposed aperiodic PM model is that it may need more frequent preventive maintenances to the extent that budget and man power permit. Let x_k and y_k be the time interval between the $(k-1)$st and the kth PM and the kth PM time, respectively. That is, $y_k = \sum_{i=1}^{k} x_i$. In addition, we let τ_k and ρ_k be the restoration interval and the improvement factor at the kth PM. Thus, we assume that $\tau_k = \rho_k x_k$,where $0 \le \rho_k \le 1$. We use $h(t)$ and $h_{pm}^{k}(t)$ to denote the hazard rate without PM and the hazard rate between the kth and the $(k+1)$st PM, respectively. When the kth PM is performed effectively at time y_k, the hazard rate function changes from $h_{pm}^{k-1}(t)$ to $h_{pm}^{k}(t)$, where $h_{pm}^{k-1}(t) \ge h_{pm}^{k}(t)$ for all $t > y_k$ for $k = 1, 2, \cdots, N$. It implies that the hazard rate of the system after the PM is smaller than the hazard rate would become without the PM. We assume that the system is maintained preventively at different interval x_k for $k = 1, 2, \cdots, N$ and each PM reduces the operational stress to that existing τ_k time units previous to the PM intervention x_k, where τ_k is a restoration interval and is less than or equal to the length of PM period x_k. For more details on assumptions and notations, we refer to Kim, Yum and Park(2000). For the aperiodic PM model, the hazard rate $h_{pm}^{k}(t)$ can be expressed as

$$h_{pm}^{k}(t) = h_{pm}^{k-1}\Big(\sum_{i=1}^{k} x_i\Big) + h\Big(t - \sum_{i=1}^{k} \tau_i\Big) - h\Big(\sum_{i=1}^{k} (x_i - \tau_i)\Big), \quad \text{for } \sum_{i=1}^{k} x_i < t \le \sum_{i=1}^{k+1} x_i. \tag{1}$$

By substituting recursively, (1) can be rewritten as

$$h_{pm}^{k}(t) = \begin{cases} h(t), & \text{for } 0 \le t \le x_1 \\ \sum_{i=1}^{k}\Big\{h\Big(\sum_{j=1}^{i-1}(x_j - \tau_j) + x_i\Big) - h\Big(\sum_{j=1}^{i}(x_j - \tau_j)\Big)\Big\} & \\ \qquad + h\Big(t - \sum_{i=1}^{k}\tau_i\Big), & \text{for } \sum_{i=1}^{k} x_i < t \le \sum_{i=1}^{k+1} x_i \ . \end{cases} \tag{2}$$

The expected cost rate per unit time for using the aperiodic PM policy with different restoration intervals during $[0, \sum_{k=1}^{N} x_k)$ can be obtained in the following manner:

Expected cost rate per unit time

$$= \Big[\Big(\text{expected cost rate of minimal repair in } [0, \textstyle\sum_{k=1}^{N} x_k)\Big) + \Big(\text{expected cost of PM in } [0, \textstyle\sum_{k=1}^{N} x_k)\Big) + \Big(\text{expected cost of replacement}\Big)\Big] \Big/ \textstyle\sum_{k=1}^{N} x_k,$$

where N is the total number of PMs carried out before the replacement of the system takes place. Each expected cost described in the above formula is obtained as follows:

i) Expected cost of minimal repair in $[0, \sum_{k=1}^{N} x_k) = C_{mr}\Big[\int_0^{x_1} h(t)dt + \sum_{k=1}^{N-1} \int_{y_k}^{y_{k+1}} h_{pm}^k(t)dt\Big]$.

This formula is obtained by combining the results given in Boland(1982) and the hazard rate structure under our aperiodic PM model, given in (1). The expression within the bracket, $\int_0^{x_1} h(t)dt + \sum_{k=1}^{N-1} \int_{y_k}^{y_{k+1}} h_{pm}^k(t)dt$, can be interpreted as the number of minimal repairs performed during $[0, \sum_{k=1}^{N} x_k)$ until the Nth PM.

ii) Expected cost of PM in $[0, \sum_{k=1}^{N} x_k) = (N-1)C_{pm}$.

iii) Expected cost of replacement $= C_{re}$.

Using the formulas given in (i), (ii) and (iii), the expected cost rate per unit time for running our aperiodic PM policy can be obtained as

$$\begin{aligned} C_1(x_1, x_2, \cdots, x_N, N) &= \frac{1}{y_N}\Big[C_{mr}\Big\{\int_0^{x_1} h(t)dt + \sum_{k=1}^{N-1} \int_{y_k}^{y_{k+1}} h_{pm}^k(t)dt\Big\} + (N-1)C_{pm} + C_{re}\Big] \\ &= \frac{1}{y_N}\Big[C_{mr}\Big\{\int_0^{x_1} h(t)dt + \sum_{k=1}^{N-1} \int_{y_k}^{y_{k+1}} \Big\{\sum_{i=1}^{k}\Big\{h\Big(\sum_{j=1}^{i-1}(x_j - \tau_j) + x_i\Big) - h\Big(\sum_{j=1}^{i}(x_j - \tau_j)\Big)\Big\} \\ &\quad + h\Big(t - \sum_{i=1}^{k} \tau_i\Big)\Big\}dt\Big\} + (N-1)C_{pm} + C_{re}\Big], \end{aligned} \tag{3}$$

where $h(t)$ is the hazard rate function until the first PM is performed. The formula (3) can be rewritten in the following expression, which is more useful to derive the optimal aperiodic PM policy in Section 3.

$$\begin{aligned} C_2(x_1, x_2, \cdots, x_N, N) &= \frac{1}{y_N}\Big[C_{mr}\Big\{\sum_{k=1}^{N-1}\Big\{h\Big(\sum_{i=1}^{k-1}(x_i - \tau_i) + x_k\Big) - h\Big(\sum_{i=1}^{k}(x_i - \tau_i)\Big)\Big\}\Big(\sum_{j=k+1}^{N} x_j\Big) \\ &\quad + \int_0^{x_1} h(t)dt + \sum_{k=1}^{N-1} \int_{y_k}^{y_{k+1}} h\Big(t - \sum_{i=1}^{k} \tau_i\Big)dt\Big\} + (N-1)C_{pm} + C_{re}\Big]. \end{aligned} \tag{4}$$

The equivalence of (3) and (4) is straightforward to verify. If we include the improvement factor in the expression, then the expected cost rate per unit for running our aperiodic PM policy can be written as

$$C_3(x_1,x_2,\cdots,x_N,N)=\frac{1}{y_N}\Big[C_{mr}\Big\{\sum_{k=1}^{N-1}\Big\{h\Big(\sum_{i=1}^{k-1}(1-\rho_i)x_i+x_k\Big)-h\Big(\sum_{i=1}^{k}(1-\rho_i)x_i\Big)\Big\}\Big(\sum_{j=k+1}^{N}x_j\Big)$$

$$+\int_0^{x_1}h(t)dt+\sum_{k=1}^{N-1}\int_{y_k}^{y_{k+1}}h\Big(t-\sum_{i=1}^{k}\rho_i x_i\Big)dt\Big\}+(N-1)C_{pm}+C_{re}\Big]. \qquad (5)$$

In Sections 3, we determine a sequence of optimal periods for the aperiodic PM policy for a fixed N in order to minimize the expected cost rate over an infinite time span.

3. Optimal Period for Aperiodic PM Policy when Number of PM is Fixed

In this section, we find the optimal periods $\{x_k : k = 1,2,\cdots,N\}$ for our aperiodic PM policy when N is fixed. The optimal PM schedule can be obtained by taking the first derivatives of $C_3(x_1,x_2,\cdots,x_N,N)$ of (5) with respect to $x_1,x_2,\cdots,x_N$, equating each of them to zero, and then solving the resulting equations simultaneously. Suppose that the set of solutions, $\{x_k^* : k=1,2,\cdots,N\}$, exist, then $C_3(x_1,x_2,\cdots,x_N,N)$ is minimized at $\{x_k^* : k = 1,2,\cdots,N\}$ for fixed N. A necessary condition that a sequence $\{x_k\}$ be a minimum cost schedule is that $\partial C_3(x_1,\cdots,x_k,\cdots,x_N,N)/\partial x_k = 0$ for each k. For $k = 1,2,\cdots,N$, we set

$$\frac{\partial}{\partial x_k}C_3(x_1,\cdots,x_k,\cdots,x_N,N)=\frac{1}{y_N}\Big[C_{mr}\Big\{\sum_{i=1}^{k-1}\Big\{h\Big(\sum_{j=1}^{i-1}(1-\rho_j)x_j+x_i\Big)-h\Big(\sum_{j=1}^{i}(1-\rho_j)x_j\Big)\Big\}$$

$$+\Big\{h'\Big(\sum_{i=1}^{k-1}(1-\rho_i)x_i+x_k\Big)-(1-\rho_k)h'\Big(\sum_{i=1}^{k}(1-\rho_i)x_i\Big)\Big\}\Big(\sum_{i=k+1}^{N}x_i\Big)$$

$$+(1-\rho_k)\sum_{i=k+1}^{N-1}\Big\{h'\Big(\sum_{j=1}^{i-1}(1-\rho_j)x_j+x_i\Big)-h'\Big(\sum_{j=1}^{i}(1-\rho_j)x_j\Big)\Big\}\Big(\sum_{j=i+1}^{N}x_j\Big)$$

$$+h\Big(\sum_{i=1}^{k-1}(1-\rho_i)x_i+x_k\Big)+(1-\rho_k)\sum_{i=k+1}^{N}\Big\{h\Big(\sum_{j=1}^{i-1}(1-\rho_j)x_j+x_i\Big)-h\Big(\sum_{j=1}^{i-1}(1-\rho_j)x_j\Big)\Big\}\Big\}$$

$$-C_3(x_1,x_2,\cdots,x_N,N)\Big]=0.$$

The solutions, in general, are extremely difficult to obtain analogously. However, for certain situations, it is possible to show the existence and uniqueness of the solutions. For example, if $\rho_k=\rho=1$ is assumed, we can show that there exist finite and unique periods $\{x_k^* : k = 1,2,\cdots,N\}$ which satisfy the following equation (6). Thus, the periods $\{x_k^* : k = 1,2,\cdots,N\}$ are the optimal periods which minimize the expected cost rate per unit time, given in (5).

$$\frac{\partial}{\partial x_i}h(x_i)=\sum_{k=i+1}^{N}h(x_k)\Big/\sum_{k=i+1}^{N}x_k,\ \text{for } i=1,\cdots,N-1,$$

and

$$\frac{\partial}{\partial x_N} C_3(x_1, \cdots, x_k, \cdots, x_N, N) = 0. \tag{6}$$

Because of the difficulties of obtaining the explicit solutions of (5) in general, we consider a special case to describe the proposed method to determine the optimal sequential periods in the next section.

4. Numerical Examples

In this section we determine the optimal periods for the aperiodic PM policy and compare its corresponding values of $C_3(x_1, \cdots, x_N, N)$ for various choice of ρ, C_{mr}, C_{pm} and C_{re} for fixed N. We also investigate the patterns of $\{x_1^*, \cdots, x_N^*\}$, which are determined so that the expected cost rate is minimized for the given values of ρ, C_{mr}, C_{pm} and C_{re} for fixed N.

For the purpose of comparison of our aperiodic PM policy with Park, Jung and Yum's (2000) periodic PM policy, which optimizes Canfield's (1986) PM policy, we use the same life distribution as that in Park, Jung and Yum (2000) as an example. Suppose that the failure time of the system is a Weibull distribution with the following form of hazard rate function.

$$h(t) = \gamma\theta^{-\gamma}t^{\gamma-1}$$

for $t > 0$, where $\theta > 0$ and $\gamma > 0$ are scale and shape parameters, respectively. By varying the values of ρ, we study the effect of the restoration interval on the optimal PM period and the expected cost rate. As the value of ρ increases to 1, the PM becomes more effective and thus it is expected that the optimal number of PMs preceding the next replacement of the system gets larger and the expected cost rate decreases. It is also noted that as the value of ρ gets closer to 1, the length of sequential period gets longer and it follows that the expected cost is reduced. Under the aperiodic PM model, the hazard rate function is piecewise continuous and it is expressed as

$$h_{pm}^k(t) = \begin{cases} \gamma\theta^{-\gamma}t^{\gamma-1}, & \text{for } 0 \le t \le x_1 \\ \gamma\theta^{-\gamma}\sum_{i=1}^{k}\Big\{\Big(\sum_{j=1}^{i-1}(1-\rho_j)x_j + x_i\Big)^{\gamma-1} - \Big(\sum_{j=1}^{i}(1-\rho_j)x_j\Big)^{\gamma-1}\Big\} & \\ \qquad + \Big(t - \sum_{i=1}^{k}\rho_i x_i\Big)^{\gamma-1}, & \text{for } \sum_{i=1}^{k} x_i < t \le \sum_{i=1}^{k+1} x_i \ . \end{cases} \tag{7}$$

Therefore, using (7), the expected cost rate per unit time for running our aperiodic PM policy is obtained as

$$\begin{aligned} &C_4(x_1, x_2, \cdots, x_N, N) \\ &= \frac{1}{y_N}\Big[\frac{\gamma}{\theta^\gamma}C_{mr}\Big\{\sum_{k=1}^{N-1}\Big\{\Big(\sum_{i=1}^{k-1}(1-\rho_i)x_i + x_k\Big)^{\gamma-1} - \Big(\sum_{i=1}^{k}(1-\rho_i)x_i\Big)^{\gamma-1}\Big\}\Big(\sum_{j=k+1}^{N} x_j\Big) \\ &\quad + \int_0^{x_1} t^{\gamma-1}dt + \sum_{k=1}^{N-1}\int_{y_k}^{y_{k+1}}\Big(t - \sum_{i=1}^{k}\rho_i x_i\Big)^{\gamma-1}dt\Big\} + (N-1)C_{pm} + C_{re}\Big]. \end{aligned} \tag{8}$$

We find the optimal periods $\{x_k : k = 1, 2, \cdots, N\}$ for our aperiodic PM policy for any fixed integer $N \geq 1$. Differentiating $C_4(x_1, x_2, \cdots, x_N, N)$ of (8) with respect to x_k for each k and setting it equal to 0, we have

$$\frac{\partial}{\partial x_k} C_4(x_1, \cdots, x_k, \cdots, x_N, N) = \frac{1}{y_N}\Big[C_{mr}\frac{\gamma}{\theta^\gamma}\Big\{\sum_{i=1}^{k-1}\Big\{\Big(\sum_{j=1}^{i-1}(1-\rho_j)x_j+x_i\Big)^{\gamma-1} - \Big(\sum_{j=1}^{i}(1-\rho_j)x_j\Big)^{\gamma-1}\Big\}$$
$$+ (\gamma-1)\Big\{\Big(\sum_{i=1}^{k-1}(1-\rho_i)x_i + x_k\Big)^{\gamma-2} - (1-\rho_k)\Big(\sum_{i=1}^{k}(1-\rho_i)x_i\Big)^{\gamma-2}\Big\}\Big(\sum_{i=k+1}^{N} x_i\Big)$$
$$+ (1-\rho_k)(\gamma-1)\sum_{i=k+1}^{N-1}\Big\{\Big(\sum_{j=1}^{i-1}(1-\rho_j)x_j + x_i\Big)^{\gamma-2} - \Big(\sum_{j=1}^{i}(1-\rho_j)x_j\Big)^{\gamma-2}\Big\}\Big(\sum_{j=i+1}^{N} x_j\Big)$$
$$+\Big(\sum_{i=1}^{k-1}(1-\rho_i)x_i+x_k\Big)^{\gamma-1}+(1-\rho_k)\sum_{i=k+1}^{N}\Big\{\Big(\sum_{j=1}^{i-1}(1-\rho_j)x_j+x_i\Big)^{\gamma-1} - \Big(\sum_{j=1}^{i-1}(1-\rho_j)x_j\Big)^{\gamma-1}\Big\}\Big\}$$
$$- C_4(x_1, x_2, \cdots, x_N, N)\Big] = 0. \tag{9}$$

Although the solutions for $x_1, \cdots, x_N$ is extremely difficult to obtain from the equations (9), in general, it may be possible for certain situations. Especially, if we take $\rho_k = \rho = 1$, then we may obtain the following relationship of optimal periods $\{x_k^* : k = 2, \cdots, N\}$ by solving the equations (9). That is, differentiating $C_4(x_1, x_2, \cdots, x_N, N)$ with respect to x_k and setting it equal to 0 imply

$$x_i = a_i x_{i-1}, \quad \text{for } i = 2, 3, \cdots, N,$$

where

$$a_i = \left[(\gamma-1)\Big\{1 + \sum_{k=i+1}^{N}\prod_{j=i+1}^{k} a_j\Big\}\Big/\Big\{1 + \sum_{k=i+1}^{N}\prod_{j=i+1}^{k} a_j^{\gamma-1}\Big\}\right]^{1/(\gamma-2)}$$

for $i = 2, \cdots, N$ and thus, the values of x_i's are obtained recursively.

For $i = 1$, we obtain

$$x_1 = \Big[((N-1)C_{pm} + C_{re})\theta^\gamma\Big/\Big\{C_{mr}H_1\Big(1 + \sum_{l=2}^{N}\prod_{j=2}^{l} a_j\Big)\Big\}\Big]^{1/\gamma},$$

where

$$H_1 = \gamma\Big\{(\gamma-1)\Big(\sum_{l=2}^{N}\prod_{j=2}^{l} a_j\Big)+1\Big\} - \Big[\gamma\Big\{\sum_{l=2}^{N}\prod_{j=2}^{l} a_j + \sum_{k=2}^{N-1}\Big\{\Big(\prod_{j=2}^{k} a_j\Big)^{\gamma-1}\Big(\sum_{l=k+1}^{N}\prod_{j=2}^{l} a_j\Big)\Big\}\Big\}$$
$$+ \sum_{k=1}^{N-1}\Big(\prod_{j=2}^{k+1} a_j\Big)^{\gamma} + 1\Big]\Big/\Big\{1 + \sum_{l=2}^{N}\prod_{j=2}^{l} a_j\Big\}.$$

For $i = N$, we obtain $x_N = a_N x_{N-1} = \prod_{j=2}^{N} a_j x_1$ and $a_N = (\gamma-1)^{1/(\gamma-2)}$.

Tables 1-3 present the values of the optimal periods $\{x_k^* : k = 1, 2, \cdots, N\}$ and its corresponding expected cost rate $C_4(x_1^*, \cdots, x_N^*)$ of the aperiodic PM policy for various choice of ρ and γ when $N = 3,5,7$ and $C_{mr} = 1, C_{pm} = 1.5$ and $C_{re} = 5$. These tables show that the expected cost rate of the aperiodic PM policy is less than that of the periodic PM policy and as expected, the expected cost rate decreases as the measure of improvement effect, ρ, increases. Note also that the expected cost rate increases as the scale parameter, γ, increases for the same level of ρ and fixed C_{re}. These tables also show that as the measure of improvement effect, ρ, decreases, the first PM period gets shorter for the same level of γ and that as the fixed PM number, N, increases, the values of the first PM period gets smaller and its corresponding cost rate consistently increases. It is also apparent from these tables that as the amount of C_{re} increases, the values of the optimal periods increase and its corresponding cost rate is increasing as well for a fixed N.

Table 1. Periodic and aperiodic PM policies with $N = 3, C_{mr} = 1, C_{pm} = 1.5, C_{re} = 5$

		Aperiodic PM policy				Periodic PM policy	
γ	ρ	x_1^*	x_2^*	x_3^*	$C_4(x_1^*, x_2^*, x_3^*)$	x^*	$C(x^*)$
3	0.1	0.35243	0.42292	0.84584	7.40193	0.53931	7.41686
	0.3	0.36925	0.44310	0.88620	7.06486	0.56229	7.11379
	0.5	0.38982	0.46778	0.93556	6.69214	0.58976	6.78242
	0.7	0.41589	0.49907	0.99815	6.27250	0.62353	6.41507
	0.9	0.45073	0.54088	1.08176	5.78766	0.66667	6.00000
	1.0	0.47333	0.56799	1.13599	5.51138	0.69336	5.76900
5	0.1	0.43757	0.31960	0.44466	8.32067	0.40034	8.32630
	0.3	0.45950	0.36161	0.50599	7.53522	0.44135	7.55258
	0.5	0.48228	0.41425	0.58786	6.73678	0.49218	6.77262
	0.7	0.50426	0.47867	0.69885	5.94607	0.55301	6.02764
	0.9	0.52168	0.55252	0.84790	5.20263	0.61676	5.40457
	1.0	0.52712	0.59083	0.93789	4.86417	0.64439	5.17282
7	0.1	0.50411	0.27769	0.32259	8.45116	0.36739	8.46807
	0.3	0.52886	0.33524	0.39000	7.44226	0.41598	7.47905
	0.5	0.55087	0.40790	0.48534	6.46309	0.47866	6.49958
	0.7	0.56771	0.49292	0.61873	5.55767	0.55564	5.59913
	0.9	0.57682	0.58355	0.79592	4.77096	0.63248	4.91893
	1.0	0.57856	0.62923	0.90041	4.42717	0.66172	4.70154

Table 2. Periodic and aperiodic PM policies with $N = 5, C_{mr} = 1, C_{pm} = 1.5, C_{re} = 5$

		Aperiodic PM policy						Periodic PM policy	
γ	ρ	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	$C_4(x_1^*, \cdots, x_5^*)$	x^*	$C(x^*)$
3	0.1	0.23518	0.25596	0.28717	0.34460	0.68921	9.10534	0.36194	9.11757
	0.3	0.24964	0.27169	0.30482	0.36579	0.73157	8.57805	0.38286	8.61923
	0.5	0.26847	0.29218	0.32781	0.39337	0.78675	7.97648	0.40966	8.05545
	0.7	0.29467	0.32070	0.35981	0.43177	0.86354	7.26719	0.44599	7.39929
	0.9	0.33547	0.36511	0.40963	0.49156	0.98311	6.38328	0.50000	6.60000
	1.0	0.36726	0.39969	0.44844	0.53813	1.07625	5.83087	0.53963	6.11527
5	0.1	0.34365	0.23136	0.19931	0.19736	0.31903	10.6530	0.25794	10.6616
	0.3	0.36292	0.26417	0.23104	0.22954	0.36854	9.4424	0.29049	9.4669
	0.5	0.38521	0.30732	0.27627	0.27691	0.44102	8.1518	0.33570	8.1918
	0.7	0.41138	0.36444	0.34376	0.35203	0.55611	6.7810	0.40172	6.8455
	0.9	0.43966	0.43666	0.44538	0.47913	0.75548	5.37885	0.49675	5.5360
	1.0	0.45075	0.47483	0.50987	0.57149	0.90719	4.71839	0.54928	5.0066
7	0.1	0.42258	0.21658	0.16272	0.14694	0.21601	11.0174	0.23251	11.0389
	0.3	0.44445	0.26369	0.20168	0.18186	0.26305	9.4730	0.26931	9.5304
	0.5	0.46711	0.32583	0.26089	0.23757	0.33730	7.8795	0.32261	7.9559
	0.7	0.49086	0.40399	0.35218	0.33280	0.46537	6.2748	0.40459	6.3439
	0.9	0.51090	0.49246	0.48459	0.49753	0.69998	4.77882	0.52621	4.8776
	1.0	0.51623	0.53612	0.56470	0.61415	0.87884	4.12642	0.58849	4.3614

Table 3. Periodic and aperiodic PM policies with $N = 7, C_{mr} = 1, C_{pm} = 1.5, C_{re} = 5$

γ	ρ	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*
3	0.1	0.18280	0.19326	0.20667	0.22492	0.25235
	0.3	0.19525	0.20642	0.22074	0.24024	0.26954
	0.5	0.21198	0.22410	0.23965	0.26082	0.29263
	0.7	0.23647	0.24999	0.26734	0.29095	0.32644
	0.9	0.27854	0.29448	0.31491	0.34273	0.38453
	1.0	0.31633	0.33442	0.35763	0.38922	0.43668
5	0.1	0.29786	0.19541	0.16159	0.14723	0.14359
	0.3	0.31488	0.22365	0.18791	0.17198	0.16791
	0.5	0.33531	0.26148	0.22613	0.20920	0.20509
	0.7	0.36141	0.31357	0.28523	0.27064	0.26871
	0.9	0.39540	0.38537	0.38051	0.38229	0.39446
	1.0	0.41133	0.42555	0.44340	0.46709	0.50156

γ	ρ	x_6^*	x_7^*	$C_4(x_1^*, \cdots, x_7^*)$	x^*	$C(x^*)$
3	0.1	0.30282	0.60565	10.6681	0.28094	10.6786
	0.3	0.32345	0.64689	9.98796	0.29929	10.0236
	0.5	0.35115	0.70231	9.19987	0.32364	9.2696
	0.7	0.39172	0.78345	8.24706	0.35852	8.3677
	0.9	0.46143	0.92286	7.00121	0.41591	7.2132
	1.0	0.52402	1.04804	6.16495	0.46416	6.4633
5	0.1	0.15239	0.26065	12.8799	0.19396	12.8891
	0.3	0.17807	0.30238	11.3138	0.22045	11.3405
	0.5	0.21760	0.36562	9.61313	0.25891	9.6558
	0.7	0.28642	0.47343	7.74539	0.32029	7.8053
	0.9	0.42964	0.69211	5.71937	0.42757	5.8470
	1.0	0.56218	0.89240	4.72526	0.50000	5.0000

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