RELIABILITY PREDICTION BASED ON DEGRADATION DATA

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ABSTRACT. As monitoring, testing, and measuring techniques develop, predictive control of components and complete systems have become more practical and affordable. In this paper we develop a statistics-based approach assuming nonlinear degradation paths and time-dependent standard deviation. This approach can be extended to provide reliability estimates and limit value determination in the censoring case for predictive maintenance policy.
Key word and phrases : predictive maintenance, degradation, reliability, censoring, maximum likelihood estimation

1. INTRODUCTION

Global demands for high reliability of electronic parts, mechanical parts, manufacturing machines, operating process *etc.* are spurring companies to use condition monitoring and fault diagnostic systems for predictive maintenance. The recent developments in sensors, measuring and analyzing techniques have facilitated the continuous monitoring of the system performance. Predictive maintenance can be done when failure modes for the machine can be identified and monitored for increased intensity and when the machine can be shut down at a fixed control limit before critical fault levels are reached.

There are three main tasks to be fulfilled for predictive maintenance. The first task is to find the condition parameter which can describe the machine condition or the product performance. It is assumed that the effect of the degradation phenomenon on the machine condition or the product performance can be expressed by the condition parameter which can be obtained from a random variable called degradation criterion. Typical degradation criteria include the vibration measurements, the amount of wear of mechanical parts such as shafts and bearings, the drift of a resistor, output power drop of light emitting diodes, fatigue-crack-growth, the gradual corrosion of

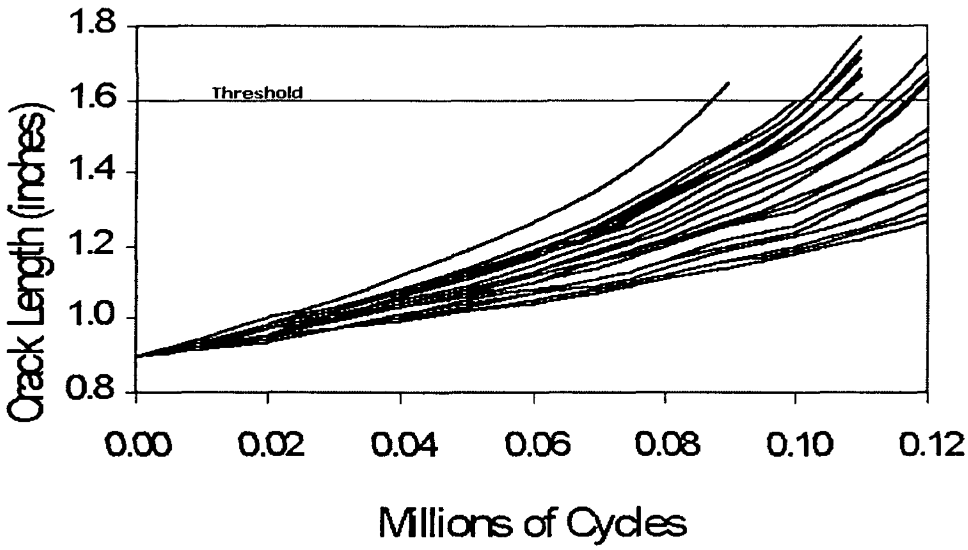

FIGURE 1. Fatigue-Crack-Growth Data from Bogdanoff and Kozin (1985).

a reinforcing bar, and the propagation delay of an electronic chip. These criteria contain useful information about product reliability. The second task is to monitor the condition parameter or the degradation criteria with suitable data aquisition system. The final task is to assess the current machine condition from the measured data and to determine the limit value, S_L, of the condition parameter and its two components: the alarm value S_a and the breakdown value S_b. If a running machine reaches the alarm value it is an indication that it is experiencing an intensive wearing. Hence the type and advancement of the fault must be identified in order to prepare the maintenance procedure. If a machine reaches the breakdown value, S_b, the shutdown of a machine for maintenance becomes necessary.

In this paper, leaving aside the determination of the condition parameter and the installation of the based data acquisition system, we will consider the reliability prediction and the limit value determination in condition monitoring.

2. Degradation Model for Condition Monitoring

The limit value for condition monitoring can be determined through reliability function. The degradation criteria, taken over time, contain useful information about reliability function. Reliability estimation models that utilize degradation data are classified as physics-based models and statistics-based models (Eghbali and Elsayed, 1997). The physics-based degradation models are those in which the degradation phenomenon is described by physics-based relationship or experimental-based results. There is no general physics-based relationship that describes the degradation phenomenon of all products. Perhaps the reason for not being able to describe it is that we are not fully aware of all the contributing factors. It is also possible that, although we know all the factors involved, the number of factors affecting the degradation phenomenon is so large that it is not feasible to take them into consideration. Statistics-based models are useful for the generalization of the degradation phenomenon.

Lu and Meeker (1993) obtained the degradation data from the figure on page 242 of Bogdanoff and Kozin (1985). Figure 1 is a plot of the crack-length measurements versus time (in million cycles) from the data. There are 21 sample paths, one for each of 21 test units. Testing stopped at .12 million cycles. They defined a critical crack length of 1.6 inches to be a failure. From this plot, we can find that the degradation criterion is a time-dependent random variable that can follow different distributions at different distinct times.

Eghbali and Elsayed (1997) develop a statistical approach based on degradation data. They assume the degradation criterion follows the same distribution family but its parameters may change with time as shown in Figure 2. The solid curve represents the mean of the degradation criterion versus time and the areas under the density functions and above the critical level line represent the failure probability at the corresponding times. Furthermore, it is assumed that the degradation paths are monotonic functions of time; they are either Monotonically Increasing Degradation Paths (MIDP) or Monotonically Decreasing Degradation Paths (MDDP).

Other important studies that have used degradation data to assess reliability can be found in Gertsbackh and Kordonskiy (1969), Nelson (1981), Carey and Koenig (1991), Chick and Mendel (1996), Feinberg and Widom (1996), Lu *et al.* (1997), Meeker *et al.* (1998), and Ettouney and Elsayed (1999).

3. Model For Degradation Data

The degradation criterion beyond the critical level can be measured or not. We will consider the case that the degradation criterion can not be measured beyond the critical level. We may put n items to observe the degradation criterion Y and observe them before a pre-assigned critical level x_c. We assume that degradation measurements are available for prespecified time t. In this paper, it is assumed that the degradation paths are Monotonically Increasing Degradation Paths(MIDP). In this case, instead

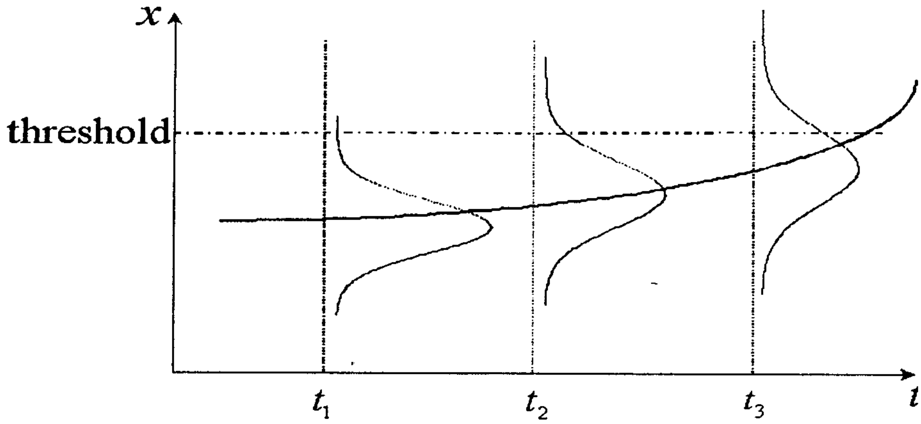

FIGURE 2. Degradation Criterion Distribution versus Time for MIDP.

of observing $Y_1, \cdots, Y_n$ we can only observe $X_1, \cdots, X_n$ where

$$X_i = \begin{cases} Y_i & \text{if } Y_i \le x_c \\ x_c & \text{if } Y_i > x_c. \end{cases}$$

We also define:

Y degradation criterion (positive random variable), $y \ge 0$
$f(y;t)$ probability density function (pdf) of the degradation criterion, Y, at a given time t
$F(y;t)$ cumulative distribution function of the degradation criterion, Y, at a given time t
$\lambda(y;t)$ failure rate function of $f(y;t)$, referred to as the *degradation failure rate function*
$\Lambda(y;t)$ cumulative failure rate function of $f(y;t)$

Here the critical degradation criterion level is fixed while m, the number of items that is smaller than x_c, is a random variable which we denote by M. Let p be the probability that the degradation criterion does not exeed the the critical degradation criterion level x_c at a given time t, then M has a binomial distribution,

$$P(M = m) = \binom{n}{m} p^m q^{n-m}, \; m = 0, 1, 2, \cdots, n,$$

where $p \equiv P(Y \le x_c) = F(x_c;t)$ and $q = 1 - p$. Suppose the items that failed are not replaced. The data consist of the m life times, $X_{(1)} < X_{(2)} < \cdots < X_{(m)}$ that is smaller than x_c and $(n-m)$ life times that may be greater than x_c. The likelihood of samples is given by

$$L = \frac{n!}{(n-m)!} \prod_{i=1}^{m} \lambda(x_{(i)};t) \exp\left(-\sum_{i=1}^{m} \Lambda(x_{(i)};t) - (n-m)\Lambda(x_c;t)\right)$$

for $m > 0$. This follows from the following consideration. For $m > 0$, let $h(x;t)$ be the conditional pdf of the degradation criterion, given that the degradation criterion does not exeed the critical level x_c at a given time t. $h(x;t)$ is given by

$$h(x;t) = \begin{cases} \frac{f(x;t)}{F(x_c;t)}, & 0 < x \le x_c \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the joint pdf of $x_{(1)}, x_{(2)}, \cdots, x_{(m)}$ is given by

$$\begin{aligned} g(x_{(1)}, \cdots, x_{(m)};t) &= m! \prod_{i=1}^{m} h(x_{(i)};t) \\ &= \frac{m! \prod_{i=1}^{m} \lambda(x_{(i)};t) \cdot \exp\left(-\sum_{i=1}^{m} \Lambda(x_{(i)};t)\right)}{\{1 - \exp(-\Lambda(x_c;t))\}^{m}.} \end{aligned}$$

The likelihood of the sample is the joint pdf of $\{x_{(1)}, x_{(2)}, \cdots, x_{(m)}\}$ and m. Hence

$$\begin{aligned} L &= g(x_{(1)}, \cdots x_{(m)};t) \binom{n}{m} p^m q^{n-m} \\ &= \frac{n!}{(n-m)!} \prod_{i=1}^{m} \lambda(x_{(i)};t) \exp\left(-\sum_{i=1}^{m} \Lambda(x_{(i)};t) - (n-m)\Lambda(x_c;t)\right). \end{aligned}$$

As Eghbali and Elsayed (1997) considered a statistical model for degradation data analysis, we will use a Weibull distribution with a time-dependent scale parameter as a degradation criterion distribution. We assume that pdf of the degradation criterion can be expressed as

$$f(x;t) = \frac{\gamma}{\theta(t)} x^{\gamma-1} \exp[\frac{-x^{\gamma}}{\theta(t)}],\ t > 0$$

where $\theta(t) = b\exp(-at)$ is the scale parameter with constant a and b ($b > 0$).

The likelihood function corresponding to this model is given by

$$\begin{aligned} L(\gamma, a, b) &= \frac{n!}{(n-m)!} \left(\frac{\gamma}{\theta(t)}\right)^m \prod_{i=1}^{m} x_{(i)}^{\gamma-1} \\ &\quad \times \exp\left(\frac{-\sum_{i=1}^{m} x_{(i)}^{\gamma} - (n-m)x_c^{\gamma}}{\theta(t)}\right). \end{aligned}$$

The maximum likelihood estimator is obtained by maximizing the log-likelihood function. We refer this estimator as the Weibull proportional degradation model.

4. Maximum Likelihood Estimation

We assume that degradation measurements are available for prespecified times - $t_1, t_2, \cdots, t_k$. Let $x_{(1)j}, x_{(2)j}, \cdots, x_{(n)j}$ be the degradation criterion at time t_j and m_j be the number of items that failed before x_c at time t_j. Then the likelihood function corresponding to our model is given by

$$\begin{aligned} L(\gamma, a, b) &= \prod_{j=1}^{k} \frac{m_{j-1}!}{(m_{j-1}-m_j)!}\left(\frac{\gamma}{\theta(t_j)}\right)^{m_j} \prod_{j=1}^{k}\prod_{i=1}^{m_j} x_{(i)j}^{\gamma-1} \\ & \prod_{j=1}^{k} \exp\left(\frac{-\sum_{i=1}^{m_j} x_{(i)j}^{\gamma} - (m_{j-1}-m_j)x_c^{\gamma}}{\theta(t_j)}\right), \end{aligned} \tag{1}$$

where $m_0 = n$. Taking the logarithm of Equation (1) we obtain the log-likelihood function corresponding to our model

$$\begin{aligned} l(\gamma, a, b) &= const. + \sum_{j=1}^{k} m_j \log\gamma - \sum_{j=1}^{k} m_j \log b + a\sum_{j=1}^{k} m_j t_j \\ & +(\gamma-1)\sum_{j=1}^{k}\sum_{i=1}^{m_j} \log x_{(i)j} - \sum_{j=1}^{k} \frac{\sum_{i=1}^{m_j} x_{(i)j}^{\gamma} - (m_{j-1}-m_j)x_c^{\gamma}}{b\exp(-at_j)}. \end{aligned}$$

Moreover, taking partial derivative of the above equation with respect γ, a and b yields the following equations.

$$\begin{aligned} \frac{\partial}{\partial\gamma} l(\gamma, a, b) &= \frac{\sum_{j=1}^{k} m_j}{\gamma} + \sum_{j=1}^{k}\sum_{i=1}^{m_j} \log x_{(i)j} \\ & -\sum_{j=1}^{k} \frac{\sum_{i=1}^{m_j} x_{(i)j}^{\gamma} \log x_{(i)j} - (m_{j-1}-m_j)x_c^{\gamma}\log x_c}{b\exp(-at_j)}, \end{aligned}$$

$$\frac{\partial}{\partial a} l(\gamma, a, b) = \sum_{j=1}^{k} m_j t_j - \sum_{j=1}^{k} \frac{t_j(\sum_{i=1}^{m_j} x_{(i)j}^{\gamma} - (m_{j-1}-m_j)x_c^{\gamma})}{b\exp(-at_j)},$$

and

$$\frac{\partial}{\partial b} l(\gamma, a, b) = -\frac{\sum_{j=1}^{k} m_j}{b} + \sum_{j=1}^{k} \frac{\sum_{i=1}^{m_j} x_{(i)j}^{\gamma} - (m_{j-1}-m_j)x_c^{\gamma}}{b^2\exp(-at_j)}.$$

We can use Newton Raphson method to compute MLE (maximum likelihood estimator) of γ, a and b. Using the result, we can estimate the reliability function and determine the limit values as follows. Let's define T_x be the time to degrade to a degradation level x, then the corresponding reliability function can be determined as

$$R_x(t) = P(T_x > t) = \exp[\frac{-x^{\gamma}}{b\exp(-at)}],$$

where $R_x(t)$ means the reliability at time t and a degradation level x. Conversely, if the desired reliability at time t, $R_x(t)$ is given, the level of the degradation criterion, x, which is equivalent to the limit value, S_L, can be determined. Hence we use the above equation to determine S_L corresponding to $R_{S_L}(t)$.

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