Study on the Strain-Rate Effect using Elastoplastic-Viscoplastic Constitutive Model.

점탄소성 구성모델을 이용한 변형율 속도의 영향에 관한 연구

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개 요 : 응력-변형률 관계의 모델링에 있어서 creep, stress relaxation, strain rate effect 등의 묘사는 중요한 지반거동중의 하나인 시간 의존적 거동의 simulation은 있어서 대단히 중요한 요소라 할 수 있다. 특히 지반은 변형률 속도에 대하여 때로는 매우 다른 거동 특성을 보이기 때문에 지반의 모델링에 있어서 변형율 속도를 고려한 구성방정식의 제시는 큰 비중을 차지한다 하겠다.
본 연구에서는 변형율에 따라 변화하는 지반의 거동특성을 보다 현실에 가깝게 묘사하기 위한 구성모델을 제시하였다. 이를 위하여 Bounding Surface Model의 점탄소성 부분을 Perzyna(1966)와 Adachi and Oka(1982)의 구성방정식 이론을 이용하여 발전시켰다. 제안된 구성모델은 기존의 모델에 비하여 다양한 변형율 속도에 적용할 수 있는 모델 정수를 비교적 간단히 결정할 수 있다는 장점이 있으며, 변형율 속도의 영향뿐 아니라 creep, stress relaxation등의 현상도 잘 simulation 할 수 있다. 본 모델은 후에 엄격히 실시되는 실내시험을 통하여 검증될 예정이다.

Key Words : elastoplasticity, viscoplasticity, constitutive model, strain-rate effect, bounding surface model

1. Introduction

Soils generally exhibit time-dependent behavior. Understanding the stress-strain-time behavior of soils is essential for geotechnical engineering associated with the design and construction of foundations, slopes, tunnels, and other all geotechnical structures. Especially, in cohesive soils, time-dependent behavior, mainly related to the response of excess pore water pressure, becomes very important issue of the rapid or long term stability and deformation prediction. Cohesive soils show such time-dependent behaviors as creep, stress relaxation, and strain rate effect with respect to strength, stiffness, and deformation characteristics in addition to the dissipation of excess pore water pressure. This study focuses on the strain rate effect in cohesive soils.

The engineering properties of cohesive soils depend to a great extent on the rate of application of stress, which is called strain rate effect. For example, it is a well known fact that the undrained strength of saturated clays significantly increases with the increase in strain rate. In theoretical analysis, however, elastoplastic theory can not alone account for the time-dependent behavior of cohesive soils; therefore, many approaches, such as rheological model by applying Eyring's structural viscosity(Murayama and Shibata,1964), have been made to properly explain the time-dependent bahavior. Kaliakin and dafalias(1990a) proposed a generalized elastoplastic-viscoplastic model for cohesive soils in the frame of the bounding surface concept. In the model, creep behavior was relatively well simulated. The simulation of strain rate effect, however, still maybe in doubt; furthermore, the determination of many model parameters, that is very important in the practical use of the model, needs a extremely laborious and time-consuming work(Dafalias, Y. F. and Herrmann, L. R.,1986).

In this study, the viscoplastic part of the bounding surface model proposed by Kaliakin and dafalias(1990a) was modified and reorganized for the better simulation of strain rate effect in cohesive soils. The method regarding the simpler determination and proper use of the model parameters were studied as well. The mathematical derivation of the model and the study of the model parameters were based on Adachi and Oka(1982) and Adachi and Okano(1974). The concept, basic theory, and derivations of the model are presented in the following sections, and the model will be later verified and investigated through the result of well-calibrated laboratory tests.

2. Time Dependent Bounding Surface Model

2.1 Concept of Bounding Surface Model

The general features of the bounding surface concept are that it allows the plastic deformation to occur for stress states within the bounding surface. The material state is defined in terms of the stress tensor σ_{ij} and proper inelastic internal variables q_n which include proper measures of inelastic deformation. The bounding surface in stress space is then defined analytically by

$$F(\bar{\sigma}_{ij},\ q_n)=0 \tag{1}$$

The actual stress point σ_{ij} always lies within or on the surface. To each stress point σ_{ij} , a unique image stress point $\bar{\sigma}_{ij}$ is assigned by a properly defined mapping rule. Analytically, the radial mapping rule is expressed by

$$\bar{\sigma}_{ij}=b(\sigma_{ij}-a_{ij})+a_{ij} \tag{2}$$

Within the radial mapping model, a second "image" stress $\hat{\sigma}_{ij}$ on the boundary of the elastic nucleus is defined in addition to $\bar{\sigma}_{ij}$. The Euclidean distance between σ_{ij} and $\hat{\sigma}_{ij}$ is presented as $\hat{\delta}$ and is used to define the concept of a normalized over stress $\Delta \hat{\sigma}_{ij}$

$$\Delta\hat{\sigma}=\frac{\hat{\delta}}{r-\frac{r}{s}}=\frac{s}{b(s-1)}-1 \tag{3}$$

2.2 General Elastoplastic-Viscoplastic Equation

Assuming small deformation and rotations, the strain rate can be decomposed into an elastic and an inelastic part, the later consisting a delayed(viscoplastic) and an instantaneous(plastic) part. Analytically these are expressed by

$$\dot{\varepsilon}_{ij}=\dot{\varepsilon}_{ij}^{e}+\dot{\varepsilon}_{ij}^{i}=\dot{\varepsilon}_{ij}^{e}+\dot{\varepsilon}_{ij}^{p}+\dot{\varepsilon}_{ij}^{v} \tag{4}$$

Concerning the each elastic, plastic and viscoplasic response and Using the general function of the state, defined at previous work(Perzyna,1966), above expression can be refomed as

$$\dot{\varepsilon}_{ij}=C_{ijkl}\ \dot{\sigma}_{kl}^{e}+\langle\Phi\rangle R_{ij}^{v}+\langle L\rangle R_{ij}^{p} \tag{5}$$

where C_{ijkl} represents the fourth order tensor of elastic compliance, Φ is the proper continuous scalar function of the overstress and L is the scalar loading index.

According to these equations, the viscoplastic contribution enters the constitutive relations through the continuous scalar overstress function Φ. Although more elaborate form of Φ have been preposed(Perzyna,1966), the following expressions, which have been found to be quite suitable for predicting the time dependent response of several geologic materials(Katona,1984), are used in the current development.

$$\Phi=\frac{1}{V}\exp(J/NI)(\Delta\hat{\sigma})^{n} \tag{6}$$

where I and J are invariants of stress tensors $\Delta\hat{\sigma}_{ij}$ as previous defined and $N=M/(3\sqrt{3})$. M is a slope of critical state line.

2.3 Specific Forms of Bounding Surface

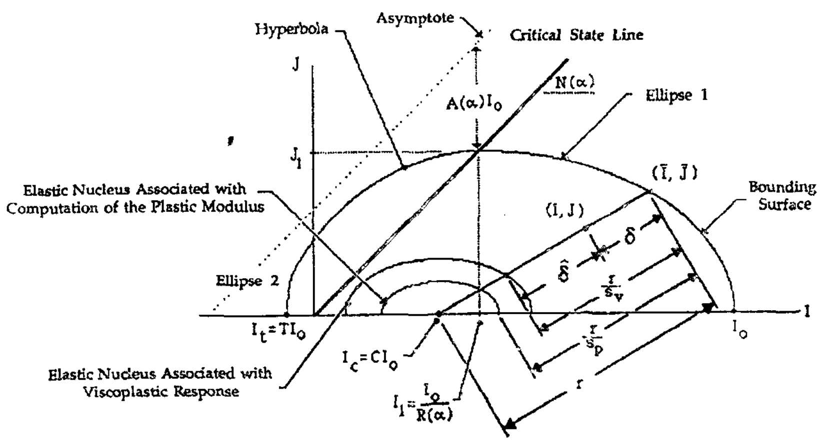

Fig. 1. Schematic Illustration of the composite form of the bounding surface in stress invariants space(Kaliakin and Dafalias, 1990a).

The definition of the bounding surface may assume many particular forms provided it satisfied certain requirements. Concerning the ideas of critical state soil mechanics, a specific form of the surface consisting of two ellipses and a hyperbola with continuous tangents at their connecting points was developed(Dafalias, Y. F. and Herrmann, L. R.,1986). These composite surface in stress invariant surface, along with associated parameters, is shown in Fig. 1.

3. Development of the Constitutive Relations

3.1 Derivation of Constitutive Equations

We define the static equilibrium state as a state at which deviatoric strain rate components $\dot{e}_{ij}$ as well as volumetric strain rate $\dot{v}$ becomes zero. Therefore, any deformation processes with definite strain rate are regarded as in non-eqilibrium state, namely, in dynamic state.

Perzyna(1963) pointed out that the difference of the dynamic and static behaviors of materials occurred due to the strain rate sensitivity of the materials and defined this rate sensitive behaviors as viscoplastic. Then, he assumed the existence of the static yield function as follows,

$$F(\sigma_{ij},\ \varepsilon_{ij}^{p}) = f(\sigma_{ij},\ \varepsilon_{ij}^{p})/k_s = 1 \tag{7}$$

Where k_s is the work hardening parameter.

By using Drucker's postulate(1959), Perzyna(1966) proposed the following flow rule for viscoplastic deformation in a simple case of infinitesimal strain field

$$\dot{\varepsilon}_{ij} = \langle \Phi(F) \rangle \frac{\partial f}{\partial \sigma_{ij}} \tag{8}$$

In the equation, the symbol $\langle \Phi(F) \rangle$ is defined as

$$\langle \Phi(F) \rangle = \begin{cases} 0 & \text{for } F \leq 0 \\ \Phi(F) & \text{for } F > 0 \end{cases} \tag{9}$$

In order to construct constitute equations, we have to assign the yield function. According to the original critical state energy theory(Roscoe et al., 1963), the following static yield function is assumed to be valid.

$$f_s = \sqrt{2J_2}^{(s)}/M^* \ \sigma_m'^{(s)} + \ln \sigma_m'^{s} = k_s \tag{10}$$

where $\sqrt{2J_2} = \sqrt{S_{ij}S_{ij}}$ is the second invaliant of deviatoric stress S_{ij}, $M^*$is defined as the value of stress ratio $\sqrt{2J_2}/\sigma_m'$ at the critical state and the superscript (s) denotes the values at the static equilibrium states.

The strain-hardening parameter k_s is assumed to be given by $\ln \sigma_{my}'$, namely, σ_{my}' represents strain-hardening effect in the change of stress state from σ_m' =0 to $\sigma_m' = \sigma_{my}'$. Thus we define

$$k_s = \ln \sigma_{my}'^{(s)} \tag{11}$$

The dynamic yield function f_d should be the same functional form of f_s because $F=0$ expresses the statical yield condition. f_d is express as

$$f_d = \sqrt{2J_2}/M^* \sigma_m' + \ln \sigma_m' = k_d \tag{12}$$

where k_d , in the same way as the static strain-hardening parameter k_s, is

$$k_d = \ln \sigma_{my}'^{(d)} \tag{13}$$

Fig. 2 is a schematic diagram of the static and dynamic yield surface. In the figure, $P_i^{(d)}$ is a dynamic state at the end of one day isotropic consolidation under prescribed pressure $\sigma_{myi}'^{(d)}$. On the other hand, $P_i^{(s)}$ is the corresponding static state to $P_i^{(d)}$ with same strain-hardening, namely in the same inelastic volumetric strain state, and lies on the static isotropic consolidation line to which attained by infinite time duration of isotropic consolidation. The state path $P_i^{(d)} \rightarrow P^{(d)}$ represents a shear deformation process with an inelastic strain rate. The state path $P_i^{(d)} \rightarrow P_u^{(d)}$ shows the increases of pore water pressure when taking back to undrained condition after the end of one day consolidation and the path $P_i^{(d)} \rightarrow P_\infty^{(d)}$. represents the secondary consolidation (delayed compression)

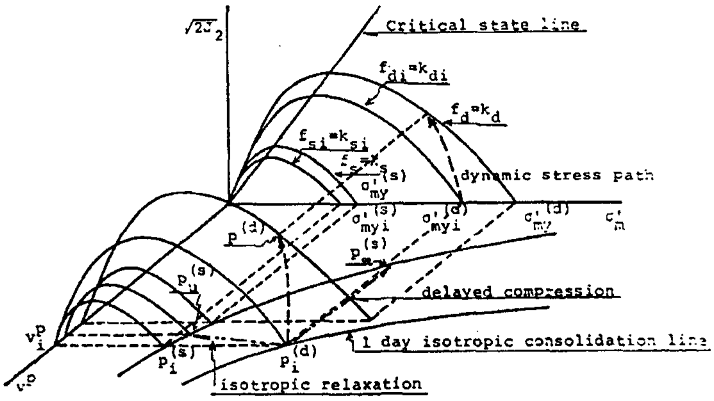

Fig. 2. Schematic diagram of both static and dynamic surface (Adachi and Oka, 1982).

Taking into account the elastic stress-strain relations, we obtain the following constitutive equations for normally consolidated clays from Eq. (8) and (12).

$$\begin{aligned} \dot{\varepsilon}_{ij} &= \frac{1}{2G}\dot{s}_{ij} + \frac{\chi}{3(1+e)}\frac{\dot{\sigma}_m'}{\sigma_m'}\delta_{ij} + \Phi(F)\frac{\partial f_d}{\partial \sigma_{ij}'} \\ &= \frac{1}{2G}\dot{s}_{ij} + \frac{\chi}{3(1+e)}\frac{\dot{\sigma}_m'}{\sigma_m'}\delta_{ij} + \frac{1}{M^*\sigma_m}\Phi(F)\frac{S_{ij}}{\sqrt{2J_2}} \\ &\quad + \frac{1}{M^*\sigma_m}\Phi(F)\left[M^* - \frac{\sqrt{2J_2}}{\sigma_m'}\right]\delta_{ij} \end{aligned} \tag{14}$$

where G is the elastic shear modulus, χ is swelling index, e is void ratio and δ_{ij} is Kronecker's delta.

According to the outcome of the previous works (Adachi and Okano 1974; Oka, 1979), the function form of $\Phi(F)$ in Eq. (14) is assumed to be follows

$$\Phi(F)=c_0\exp[m'\ln(\sigma_{my}'^{(d)}/\sigma_{my}'^{(s)})] \tag{15}$$

by substituting both dynamic and static yield function of Eq. (10) and (12) into Eqs (14), $\Phi(F)$ is rewritten as follows

$$\Phi(F)=c_0\exp\left\{m'\left[\frac{\sqrt{2J_2}}{M^*\sigma_m'}+\ln\sigma_m'-\frac{\sqrt{2J_2}^{(s)}}{M^*\sigma_m'^{(s)}}-\ln\sigma_m'^{(s)}\right]\right\} \tag{16}$$

3.2 Relationship between Stress Ratio and Strain Rate

Since the volume change is negligible under undrained condition, i.e., $\dot{\varepsilon}_{kk}=0$, we obtain the relationship between mean effective stress σ_m' and inelastic volumetric strain υ^p from Eq.(13)

$$\dot{\varepsilon}_{ij}=\dot{\upsilon}=\frac{x}{(1+e)}\frac{\dot{\sigma}_m'}{\sigma_m'}\delta_{ij}+\frac{1}{M^*\sigma_m'}\Phi(F)\times\left[M^*-\frac{\sqrt{2J_2}}{\sigma_m'}\right]=0 \tag{17}$$

Fig.4 shows this fact schematically. Namely, the inelastic volumetric strain υ^p are same at both stress states represented as P_1 and P_2 lying on two different stress paths which correspond to strain rates $\dot{\varepsilon}_{11}^{(1)}$ and $\dot{\varepsilon}_{11}^{(2)}$, respectively, as shown in Fig. 3.

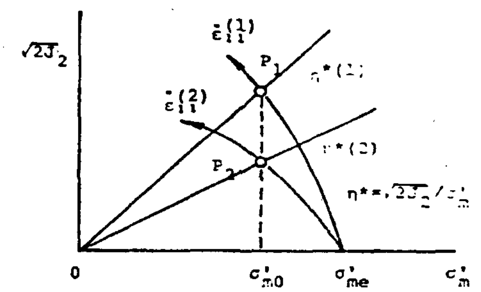

Fig. 3. The same inelastic volumetric strain (same strain-hardening) states on different stress path (Adachi and Oka, 1982).

Under undrained conditions, the total strain rate component $\dot{\varepsilon}_{11}$ is equivalent $\dot{e}_{ij}$ because Eq.(16) is always satisfied.

$$\dot{\varepsilon}_{11}=\dot{e}_{ij}=\frac{\dot{S}}{2G}+\frac{1}{M^*\sigma_m'}\Phi(F)\frac{S_{ij}}{\sqrt{2J_2}} \tag{18}$$

We continue to discuss the problem in a simple case of conventional axisymmetric triaxial compression, i.e., $\sigma'_1>\sigma'_2\doteq\sigma'_3$. Under this specific condition, the following relations are reduced

$$S_{11}=2/3(\sigma_1-\sigma_3),\ \sqrt{2J_2}=\sqrt{2/3}(\sigma_1-\sigma_3),\ \varepsilon_{11}=e_{11}=2/3(\varepsilon_1-\varepsilon_3)$$

Using these relations Eq. (15) results in as follows

$$\dot{\varepsilon}_{11} = \frac{\dot{S}_{ij}}{2G} + \frac{\sqrt{2/3}}{M^* \sigma_m'} \Phi(F) \tag{19}$$

$$\Phi(F) = c_0 \exp\left\{ m' \left[\frac{q}{M\sigma_m'} + \ln \sigma_m - \frac{q^{(s)}}{M\sigma_m'^{(s)}} - \ln \sigma_m'^{(s)} \right] \right\} \tag{20}$$

where $q = (\sigma_1 - \sigma_3)$ and $M = \sqrt{3/2} M^*$.

Assuming $\dot{\varepsilon}_{11} = \dot{\varepsilon}_{11}^{p}$, namely elastic shear strain rate $\dot{e}_{11}^{e} = \dot{S}_{11}/2G$ to be negligible, the nest relation is obtained from Eqs(20) and (21) by comparing the state P_1 and P_2 shown in Fig. 4.

$$\ln\left(\dot{\varepsilon}_{11}^{(1)} / \dot{\varepsilon}_{11}^{(2)} \right) = \frac{m'}{M^*} \times \sqrt{2J_2^{(1)}} / \sigma_m' - \sqrt{2J_2^{(2)}} / \sigma_m' \tag{21}$$

where the superscripts (1) and (2) correspond to the states P_1 and P_2

If the material properties such as m and M^* can be find properly, we obtain the different deviatoric stress invariant value at the different strain rate through Eq. (21). Consequently, the values of the deviatoric stress invaliants in overstress function(Eq. (6)) can be easily obtained by above equations with the various strain rate state.

4. Determination of Model Parameter Values

The current bounding surface formation requires eighteen separate model parameters. These parameters include fourteen parameters associated with the elastoplasic response and four parameters which defined the viscoplastic response. The value of all the parameters fall within fairly narrow ranges. Furthermore, the values of several parameters can be determined from standard soil mechanics parameters(Kaliakin and Dafalias, 1991). In addition to these bounding surface model parameters, we need one material parameters for the developed viscoplastic constitute equations(Eq. 21)

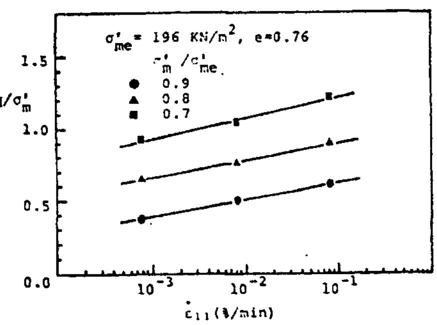

Fig. 5. Relationship between the stress ratio and logarithm of strain rate (Adachi and Oka, 1982).

Fig. 5. clearly shows that the linear relationship between the logarithm of strain

rate $\dot{\varepsilon}_{11}$ and the stress ratio q/σ_m' as an equi-inelastic volumetric strain rate. Thus, the parameter m' can be determined from the slope of equi-inelastic volumetric strain line obtained from few triaxial test perfomed at varied strain rate test.

5. Summary and Conclusions

The bounding surface model proposed by Kaliakin and Dafalias(1990a) successfully simulated the time-dependent behavior of cohesive soils, which is related to creep, stress relaxation and strain rate effect. However, determining the parameter needs an elaborate work for the prediction of the influence of strain rate effect. As the strain rate changes, all eighteen parameter should be altered.

Time dependent behavior of cohesive soils can be simulated using viscoplastic models. In the further study, using the relationship between stress invariants and strain rate which was postulated by Adachi and Oka(1982), we will evolve the viscoplastic part in the constitutive equation of the model and numerically implement this work.

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