확률론적 해석방법을 이용한 암반사면 안정성 해석
Probabilistic Analysis for Rock Slope Stability

박혁진, Hyuck-Jin Park

한국건설기술연구원 토목연구부 선임연구원, Senior Researcher, Civil Eng. Dept., KICT

개 요 : 현장상황에 대한 불충분한 자료와 파괴 메커니즘에 대한 불완전한 이해로 인해 발생하는 가변성(variability)과 불확실성(uncertainty)은 암반사면공학뿐만 아니라 지반공학에서 흔히 접하게 되는 문제점이다. 특히 암반사면공학에서는 이러한 가변성과 불확실성이 불연속면의 방향 및 기하학적 특성, 그리고 실내실험 결과의 분산으로 나타난다. 그러나 안전율(factor of safety)의 개념을 기초로 하는 전통적인 결정론적 해석방법(deterministic analysis)은 이러한 분산을 고려하지 않은 채 단일 대표 값만을 이용하여 구조물의 안정성을 판단하여 왔다.

확률론적 해석방법(probabilistic analysis)은 이러한 가변성과 불확실성을 효과적으로 정량화하여 해석에 이용할 수 있는 방법 중의 하나로 제안되었다. 이러한 해석방법은 불연속면의 기하학적 특성과 강도 특성을 확률변수(random variable)로 취급하여 신뢰성이론(reliability theory)과 확률이론(probability theory)을 근거로 분석하였으며 이를 기초로 하여 Monte Carlo Simulation과 같은 해석법을 이용, 구조물의 붕괴가능성을 확률로 표현하였다. 확률론적 해석 방법은 기존의 안전율을 대체하여 구조물의 안정성을 붕괴확률(probability of failure)로 제안하였으며 이 붕괴확률은 안전율의 확률분포함수(probability density function)에서 안전율이 1보다 작을 가능성을 확률로 나타낸 수치이다.

본 논문에서는 확률론적 해석방법을 이용하여 불연속면 특성들의 확률특성을 고찰하였으며 이를 기초로 하여 암반사면의 안정성 해석에 응용했다.

주요어 ; 암반사면, 확률론적 해석방법, 붕괴확률, 결정론적 해석방법, 안전율

1. Introduction

Uncertainty about geologic conditions and geotechnical parameters is the most distinctive characteristic of the engineering geology-geotechnical engineering field. In rock slope stability analysis, the uncertainty and variability may be in the form of a large scatter in attitude data and the geometry of jointing and also test results. One of the most difficult jobs in rock slope engineering is selecting a single representative value from widely varied data. Therefore, many engineers and researchers have attempted to limit and quantify the variation and uncertainty in their data and have adopted various methods to indicate the uncertainty and variation in the results of analysis. Casagrande(1965) noted the nature and importance of 'the calculated risk' in geotechnical engineering. In several examples he showed how the unknown risks affected the stability of projects. Peck(1969) suggested the observational approach to maintain control over uncertainties by revising estimates of site conditions and parameters when additional information becomes available. However, traditionally most engineers have taken the variation of their data into account by selecting an appropriate factor of safety. Probability theory and statistical techniques, have been applied to engineering geology field to deal properly with variability and uncertainty. Application of probabilistic analysis has provided an objective tool for quantifying and modeling variability and uncertainty. In this paper, an application of the probabilistic method to rock slope stability analysis will be introduced.

2. Deterministic Method

Traditionally assessments of the risk of failure are made on the basis of allowable factors of safety, learned from previous experience for the system considered, in its existing environment. However, most input parameters(e.g. material strength, joint geometry and pore water pressures) in the safety factor calculation are precisely unknown because of uncertainty and variations in testing, modeling, and spatial variation. Thus each of these is a random variable and the analysis with different values for each of these parameters can result in a different value of factor of safety. Thus safety factor itself is also a random variable depending on many input variables.

However, the conventional factor of safety does not reflect the degree of uncertainty of these parameters. In deterministic analysis, the factor of safety requires fixed values for parameters that actually exhibit a degree of uncertainty. In most cases, the mean value of these parameters is assigned as a fixed value but some engineers tend to select values higher 어 lower than the mean, due to uncertainty and variation in input parameters. This can yield very different factor of safety values for the same project. Consequently, inconsistency is likely to exist among engineers and between applications by the same engineer. In addition, the same factor of safety can be associated with a large range of reliability level and thus FS is not a consistent measure of safety. For example, when FS = 1.5, the probability of failure can increase from 10^{-5} to 10^{-2} when the standard deviation increases from 0.15 to 0.25. Therefore, Tabba(1984) pointed out the following shortcomings of deterministic analysis ; 1) Inability to account for variations in properties and conditions. 2) Difficulty in portraying the relative importance of various sets of data in the overall stability condition. 3) Inability to predict failure in cases where failure has actually occurred.

3. Probabilistic Method

As mentioned previously, in the deterministic approach, the factor of safety and all ingredient parameters take on fixed values in spite of the fact that all parameters and even the factor of safety shows a degree of uncertainty. Therefore, despite the simplicity of this approach, it does not properly deal with uncertainty and variability in parameters and analytical or empirical models.

As an alternative to the deterministic approach, the probabilistic analysis has been introduced to consider and quantify the uncertainty and variability in parameters and in the analytical model. In this analysis, the factor of safety is considered as a random variable and can be replaced by the probability of failure to measure the level of slope stability. The probability of failure is simply defined as the probability of having FS<1 under the probability density function(PDF) of factor of safety. Probability of failure is interpreted as a measure of relative likelihood of occurrence of failure.

In general, probabilistic analysis is performed in two steps : The first step consists of analysis of available geotechnical data to determine the basic statistical parameters(that is, mean and variance) and probability density function which enables us to represent and predict the random property of geotechnical parameters. The mean value of the PDF represents the best estimate of the random variable and the standard deviation or coefficient of variance(c.o.v.) of the PDF represents an assessment of the uncertainty.

In the second step, risk analysis of slope stability is accomplished using the basic statistical parameters and probability density distribution developed from the previous step. Two methods of risk analysis are commonly used, the Monte Carlo simulation and First Order Second Moment method(FOSM). The Monte Carlo method is used when the PDF of each component variable are completely prescribed. In this procedure, values of each component are generated randomly by its respective PDFs and then these values are used to evaluate the factors of safety. By repeating this calculation, the probability of failure can be estimated by the proportion of calculations where the safety factor is less than one. This calculation is reasonably accurate only if the number of simulations is very large. The advantage of this method is that the complete probability distributions for the factor of safety are obtained. The disadvantage is that large numbers of simulation are required when the failure probability is relatively small. When the PDFs of the component variables are not available, but their mean and c.o.v. are available, the FOSM can be used to calculate approximately the probability of failure. It yields a good approximation if the uncertainties of the variables are small. Advantages of this method are : the calculation is simple and only information of moments(that is, mean and variance as first moment and second moment) are needed rather than a complete distribution function. The disadvantage is that mathematical calculations are difficult when the number of component variables is large.

4. Site Introduction

The study area for probabilistic analysis is the rock cut along Interstate highway 40 in North Carolina, which has experienced several failure occurrences after highway construction was completed. This area along Interstate 40 shows excellent exposures of a series of metasediments of Late Pre-Cambrian age. Major rock type in this area is gray, thin bedded to laminated feldspathic metasandstone and green slate with thin interbeds of fine metasandstone. Bedding is instinct in this formation and the rock is highly jointed. This area has experienced several large slides during construction and after construction. The investigation for relocation of highway concluded that wedge failure was the most common failure. On July, 1997, a large rockslide occurred in this area after heavy rain and two discontinuities formed an unstable wedge and failed. The large number of discontinuity orientation and geometries were measured from the field in this area and their stochastic properties were analyzed by author(Park, 1999).

5. Analysis of Stochastic Properties of Discontinuity Parameters

5.1 Discontinuity Orientation

One of the major reasons to carry out a statistical analysis for discontinuity properties is to find a proper probability distribution representing discontinuity parameters and their random properties. There have been a number of studies to determine the appropriate probability density distribution for a discontinuity orientation distribution. Fisher(1953) proposed a distribution on the basis of the assumption that a population of orientation values was distributed about a true value. This assumption is similar to the idea of discontinuity normals being distributed about some true value within a set. He assumed that the probability, $P(\theta)$ that an orientation value selected randomly from the population makes an angle of between θ and $d\theta$ with the true orientation is given by

$$P(\theta) = \eta\, e^{k\cos\theta} d\theta \tag{1}$$

where k is a constant controlling the shape of the distribution and is commonly referred to as Fisher's constant, which is a measure of the degree of clustering within the population and η is a variable expressed as follow;

$$\eta = \frac{k \sin\theta}{e^{k} - e^{-k}} \tag{2}$$

In view of its simplicity and flexibility, the Fisher distribution provides a valuable model for discontinuity orientation data(Priest, 1993). Therefore, Fisher distribution is commonly adopted in many probabilistic analysis and also used in this study. After the measurement in the field, discontinuity orientations were clustered and then Fisher constants were evaluated for each set of orientation.

5.2 Discontinuity Length

The length of a discontinuity is defined as the distance over which the tensile and cohesive strength of the rock substance has been reduced or lost. Knowledge of the length of discontinuities in a rock mass is important in the prediction of rock behavior analysis of rock slopes because the discontinuity lengths influence the size of blocks that may be formed.

A lognormal and an exponential distribution have been proposed as representative distribution models by many different researchers(Barton, 1978; Einstein et al., 1980). However, according to Priest and Hudson(1981), the lognormal distribution is a biased distribution caused by scanline sampling. Therefore, the negative exponential distribution is an appropriate distribution and it has a advantage that sampling bias caused by scanline method can be canceled out by adopting this distribution. Consequently, the exponential distribution is commonly used in probabilistic analysis to represent a stochastic property of the discontinuity length.

5.3 Discontinuity Spacing

The purpose of discontinuity spacing measurement is to obtain the size of the blocks which compose a rock mass. Stability analysis and design are strongly dependent on the block size because weight forces and forces due to water pressure, and failure mechanism depend on the block size. Although mean discontinuity spacing provides a direct measure of spacing data, several previous studies have tried to represent the distribution of measured data by statistical analysis and description because the spacing data is considered as a random variable. On the basis of field measurements and theoretical considerations, Priest and Hudson (1976) concluded that the distribution of discontinuity spacing for various sedimentary rock types could be modeled by the negative exponential probability density function. This conclusion has been supported by others, such as Wallis and King (1980) and Baecher (1983).

According to author's measurements and statistical analysis (Park, 1999), the lognormal distribution is better than the exponential distribution. Approximately 300 spacing values were collected in field by author and then in order to determine the appropriate distribution of spacing in the study, Chi-square goodness of fit tests were performed for lognormal and negative exponential distributions, which are two possible distribution models for spacing. This is because those theoretical distributions are bounded at zero and are skewed to the right and those characteristics are similar to the properties of the spacing distribution. In addition, some publications such as Rouleau and Gale (1985) and Sen and Kazi (1984), proposed the lognormal probability distribution for discontinuity spacing. A total of 52 data points in the same set were used to evaluate the goodness of fit. The calculated $\Sigma(n_i - e_i)^2/e_i$ values of both distributions are smaller than $C_{0.95,\ 20}$, 31.4, obtained from the Chi-Square distribution table at 5% significant level with 20 degree of freedom. Therefore, both the lognormal distribution and the exponential distribution appear to be valid for spacing at the significant level of 5%. However, because the calculated $\Sigma(n_i - e_i)^2/e_i$ value for lognormal distribution, 27.4 is smaller than that for exponential distribution, 29.4, the lognormal distribution is better than the exponential distribution.

5.4 Discontinuity Strength Parameters

Compared to other discontinuity parameters, limited research has been accomplished previously regarding statistical evaluation of joint strength parameters. However, although limited work has been accomplished, two different distributions are suggested for shear strength parameters. Mostyn and Li(1993) considered c and ϕ as normally distributed. However, in the paper by Muralha and Trunk (1993), a lognormal distribution is adopted for c and ϕ. They insisted that the lognormal distribution had an advantage of assuming that the shear strength will not yield negative values. Therefore, in this current study, both normal and lognormal distributions are considered as possible distribution models to represent random properties of strength parameters and both distributions are tested for validity. According to author's test(Park, 1999), both distributions appear to be valid models for internal friction angle, but the normal distribution model is superior to the lognormal model according to the test. Therefore, in the probabilistic analysis of the stochastic procedure for shear strength, the normal distribution is used as a probability density function to simulate the random characteristics of shear strength.

6. Probabilistic Analysis for Rock Slope Stability

6.1 Monte Carlo Simulation

The Monte Carlo simulation is frequently used to evaluate the failure probability of a mechanical system, in particular, when direct integration is not practical or when the equation to integrate is difficult to obtain. In this research, the Monte Carlo method is employed because the deterministic model for rock slope failure is not easy to solve by analytical means. This simulation is the most widely used among the probabilistic analysis methods and many others applied it to evaluate slope stability(Kulatilake et al., 1985; Muralha and Trunk. 1993). The Monte Carlo simulation approach is to assume that for a given stability analysis, each variable takes a single value selected randomly from its measured distribution, independent of the other variables. The group of randomly selected parameters is combined with the fixed input data to generate a single random value for factor of safety. This process is repeated many times to generate a large number of different factor of safety, which can be plotted in histogram form. The simulation procedure used in this study is expressed in a flowchart in Figure 1.

6.2 Probabilistic Assessment

The Monte Carlo simulation performed in the previous procedure yields a list of factor of safety for every possible, kinematically unstable rock block. Absolute values of the factor of safety of less than one indicate blocks that will fail. The probability of failure is expressed as

$$P_f = \frac{N_F}{N_T} \tag{3}$$

where N_F is the number of iteration that the blocks are failed, that is, factor of safety is less than 1, and N_T is the total number of iterations that the blocks analyzed. However, N_T can be interpreted in two different way; N_T is either the total number of iteration performed or only those iterations that form kinematically unstable blocks. Defending on this definition, the probability of failure will be different. However, in this paper, for clear definition and results of probability of failure, the probability of failure is defined as the multiplication of the probability of kinematic instability and the probability of kinetic instability. That is,

$$P_f = \frac{N_m}{N_T} \times \frac{N_f}{N_m} \tag{4}$$

where N_m is the number of iterations that a block is kinematically unstable, N_T is the total number of iterations and N_f is the number of iterations that a block has factor of safety less than one. This multiplication is based on the concept of composite models. Therefore based on this probabilistic theory, the probability of failure is defined as the ratio of the number of iterations that factor of safety is less than one, which is based on premise that the wedge is kinematically unstable, and the number of total iterations. This method provides a clear definition based on probability theory and simplified the evaluation of factor of safety and without confusion.

7. Results of Probabilistic Analysis

The input parameters used in this analysis are listed in Table 1. Three major joint sets were measured in the field and joint set 1 and 2 were identified as joint. Joint set 3 was identified as a bedding plane and in most cases, the length of bedding planes is assumed to be infinite. In this study, in order to show the length of bedding planes is much greater than other joint sets, a value of 60 m is assigned. The height of slope is approximately 60 m and orientation of the slope are 140°/45°. Based on input values and the simulation procedure, probabilistic analyses were performed and the results of analysis for wedge failure are listed in Table 2. According to the deterministic analysis results in Table 2, two discontinuity combinations(J1&J2 and J1&J3) are analyzed as stable and one(J2&J3) is as unstable. In case of combination J2&J3, both the deterministic and the probabilistic analysis are interpreted as unstable. However, other two combinations(J1&J2 and J1&J3) are analyzed

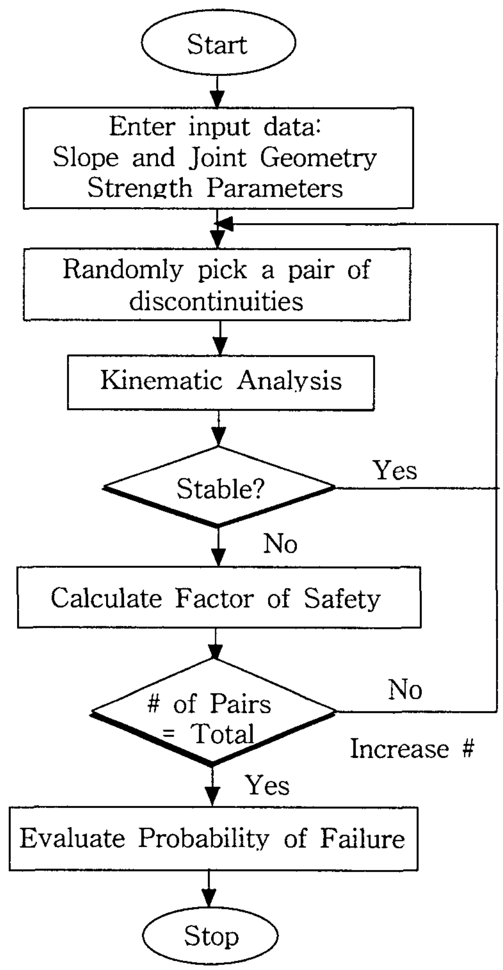

Fig.1 The flow chart for probabilistic analysis

Table 1 Input values for discontinuity properties

Set I.D.	Mean Orientation	Fisher Const.	Mean Firction Angle (degree)	STD of Fiction Angle	Mean Length (m)	Spacing (m)
J1	163/63	29	30	3	0.75	0.39
J2	196/56	119	30	3	0.48	3.9
J3	227/37	36	30	3	60	0.9

as stable in the deterministic analysis but as unstable in the probabilistic analysis. The probabilities of kinematic instability for J1&J2 and J1&J3 are 19.0 % and 63.8 %. This means that, in case for J1&J3 combination, the intersection line of the combination evaluated by a fixed representative orientation is located in stable area but if we consider the scattering of orientations, 64 % of intersection lines generated randomly are unstable kinematically. The probabilities of kinetic instability are 2.5 % and 75.4 % respectively. Consequently, the total probabilities of wedge failure for J1&J2 and J1&J3, evaluated by multiplication of two probabilities, are 0.47% and 48.1 %, respectively. Therefore, there is a big difference between the deterministic and probabilistic analysis in J1&J3 especially. Based on 10% of the acceptable failure probability for rock slope suggested by Hoek(1991), J1&J2 could be determined as stable and this result is coincident with the deterministic analysis. However, J1&J3 combination is interpreted as quite unstable on the basis of probabilistic analysis, but is analyzed as stable on the basis of deterministic analysis. Consequently, the deterministic analysis based on a fixed representative value of discontinuity parameters fails to indicate the possibility of failure. This is because the deterministic analysis cannot consider the scatter of discontinuity parameters. That is, even though the representative value of discontinuity parameters does not indicate unstable condition, many other scattered data could show the instability.

Table 2 Results of wedge failure for deterministic analysis and probabilistic analysis

Set No. 1	Set No. 2	Factor of Safety	Probability of Failure		Total Probability of Failure
			Kinematic	Kinetic	
J1	J2	Stable	0.190	0.025	0.0047
J1	J3	Stable	0.638	0.754	0.481
J2	J3	0.33	0.332	0.698	0.232

8. Conclusions

The result of comparison between deterministic analysis and probabilistic analysis in the study area indicates that the analysis result of probabilistic analysis could be quite different from that of the deterministic analysis. The deterministic analysis based on a fixed value of discontinuity parameters fails to indicate the possibility of slope failure. Consequently, the deterministic analysis is unable to represent the actual condition of rock slope because this analysis does not consider random properties of parameters and therefore, this misinterpretation can cause serious problems. By contrast, the probabilistic analysis is more representative of the actual behavior of parameters and provides analysis results. Therefore, it is recommended that the probabilistic analysis should be used especially in cases when significant scatter in the parameters is observed.

9. References

1. Baecher, G. B.(1983), "Statistical analysis of rock mass fracturing", *Journal of Mathematical Geology*, 15(2), pp.329-347.
2. Barton, C. M.(1978), "Analysis of joint traces", *Proceeding of 19th U.S. Symposium on Rock Mechanics*, University of Nevada.
3. Casagrande, A.(1965), "Role of the 'calculated risk' in earthwork and foundation engineering", *Journal of Soil Mechanics and Foundation Engineering Division*, ASCE, 91(4), pp.1-40.
4. Einstein, H. H., Baecher, G. B., Veneziano, D.(1980), *Risk Analysis for Rock Slopes in Open Pit Mines*, MIT, Massachusetts.
5. Fisher, R. A.(1953), "Dispersion on a sphere", *Proceedings of the Royal Society of London*, U.K., pp. 295-305.
6. Hoek, E.(1991), "When is a design in rock engineering acceptable?", *Proceeding of 7th ISRM*, pp.1485-1496.
7. Kulatilake, P.H.S.W., Finley, R. E., Ghosh, A.(1985), Effect of variability of joint orientation and strength on factor of safety of wedge stability", *Proceeding of International Symposium on Fundamentals of Rock Joints*, pp.25-33.
8. Mostyn, G. R., Li, K. S.(1993), "Probabilistic slope analysis - state of play", *Proceedings of Conference on Probabilistic Methiod in Geotechnical Engineering*, pp.89-109.
9. Muralha, J., Trunk, U.(1993),"Stability of rock blocks-Evaluation of failure probabilities by Monte Carlo and first order reliability methods", *International Symposium on Assessment and Prevention of Failure Phenomenon in Rock Engineering*, pp.759-765.
10. Park, H. J.(1999), *Risk Analysis of Rock Slope Stability and Stochastic Properties of Discontinuity Parameters in western North Carolina*, Ph. D. Thesis, Purdue University.
11. Peck, R. B.(1969), "Advantages of limitations of the observational method in applied soil mechanics", *Geotechnique*, 19(2), pp.171-187.
12. Priest, S. D.(1993), *Discontinuity Analysis for Rock Engineering*, Chapman and Hall.
13. Priest, S. D., Hudson, J. A.(1976), "Discontinuity spacing in rock", *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, 13, pp.135-148.
14. Priest, S. D., Hudson, J. A.(1981), "Estimation of discontinuity spacing and trace length using scanline surveys", *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, 18, pp.183-197.
15. Rouleau, A., Gale, J. E.(1985), "Statistical characterization of the fracture system in the Stripa Granite, Sweden", *nt. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, 22, pp.353-367.
16. Sen, Z., Kazi, A.(1984), "Discontinuity spacing and RQD estimates from finite length scanlines", *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, 21, pp.203-212.
17. Tabba, M. M.(1984), "Deterministic versus risk analysis of slope stability", *Proceeding of 4th International Symposium on Landslides*, pp.491-498.
18. Wallis, P. F., King, M. S.(1980), "Discontinuity spacing in a crystalline rock", *Int. J. Rock Mech. Min. Sci. & Geomech. Abstr.*, 17, pp.63-66.