Single-bit Error-correcting Code 에 대한 BER

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BER for Single-bit Error -correcting Code

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요 약

bit error 확률의 특징과 bit error 의 통계를 어떻게 분석할 것인가를 다루었다. Block 에서 one single-error 를 보정한 후 bit error 확률 이 얼마나 개선될 수 것인가? 본 논문에서 이에 대한 해답을 만족시킬 것이다.

Abstract

The features of bit error rate (BER) are discussed, how to analyze statistics of the bit error rate. How many will the BER be improved after correcting one single-error in the block? The answer is satisfactory in this paper.

1. Mechanisms of error control codes

Communication systems usually rely on three types of error control mechanisms [1]. They are

1). Automatic Repeat Request (ARQ)

In ARQ systems, the receiver simply detects whether the transmitted data block includes errors or not. When errors are detected, a retransmission request in transmitted to the sender. If we can accept long transmission delay time and have Bi-directional channel, the error-free condition is achieved.

2) Forward Error Correction (FEC)

The receiver can not only find the errors, but also correct them. In FEC system, it doesn't need the Bi-directional channel and has not the transmission delay time. But it more complicated than ARQ.

3) Feedback Verifying (FV)

The receiver returns the received data to sender, and compared with the sent data in sender. If errors are found, them will be sent again. This method is very simple, but need the Bi-directional channel and the transmission efficiency is lower as every bit is sent by two times at least.

2. Outline of error control codes

An error control code is an algorithm for expressing a sequence of numbers such that any error which is introduced can be detected and corrected (within certain limitations) based on the remaining numbers. The study of error-correcting codes and the associated mathematics is known as Coding Theory.

To reduce the probability of bit error and to obtain the higher-quality data transmission, the error control codes are very effective in wireless communication systems.

To detect or correct errors, we add some redundant bits to the source information by using an encoding rule

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that maximizes the error detection or error correction or error correction abilities. Such encoding is called the Channel Coding [2].

The single-bit error-correcting code is a linear code and one of the simplest error-correcting codes. Now we define that n is the total number of bits in the block, m is the number of information bits in the block, and k the number of check bits in the block, where

$$n = m + k \quad (1)$$

To calculate the fewest number of check bits necessary to correct single-bit errors in a use the following equation:

$$(m + k + 1) <= 2^k \quad (2)$$

We define the Code Rate Efficiency

$$\eta = m/n = 1\text{-}k/n. \quad (3)$$

3. Avail of error control codes

The data interfered by noise may take place the error in transmission. Generally, there are two kinds of errors, random and burst errors. When the data is transmitted through a memory-less channel, the error bit is random one [3]. Here the former is considered.

Assume the probability, P_e, of bit errors for sending "0" to be the probability of bit errors for sending "1", the probability that the block, n bits, does not take place any error is $P(n, 0)$

$$P(n, 0)=(1- P_e)^n \quad (4)$$

The probability of the r bit errors in the block is given by

$$P(n, r)= C_n^r P_e^r (1- P_e)^{n-r} \quad (5)$$

The probability that block exists any errors is $P(n, \geq 1)$

$$P(n, \geq 1)=1- P(n, 0) \quad (6)$$

According to equation (4), (5) and (6), we have

$$\begin{aligned} P(n, \geq 2) &= 1- P(n, 0)- P(n, 1) \\ &= 1-(1- P_e)^n - n P_e(1- P_e)^{n-1} \\ &= (1- P_e)^{n-1}[1+(n-1) P_e] \end{aligned} \quad (7)$$

If one error bit is corrected in the every block, the probability that any error exists in a block is

$$\begin{aligned} P_1(n, \geq 1) &= 1- P_1(n, 0) \\ &= 1- P(n, 0)- P(n, 1) \end{aligned} \quad (8)$$

Then

$$P_1(n, 0)=1- P(n, 0)- P(n, 1) \quad (9)$$

By the Equation (7)

$$(1- P_{e1})^n=(1- P_e)^{n-1} [1+(n-1) P_e] \quad (10)$$

For calculating the BER after correcting one single-error in the block, we have the very important equation as follow

$$\boxed{P_{e1} = 1-\{(1- P_e)^{n-1} [1+(n-1) P_e]\}^{1/n}} \quad (11)$$

where P_e is the BER of un-coded condition, n the code length.

The Equation (11) expresses the relation between the BER before and behind correcting one single-error bit. Table 1 shows the relation between the code rate efficiency and BER probability as different checking bits and code length.

Table 1 Comparison of BER probability before and behind correcting one single-error bit

Items *n, k, η*	Before correcting errors *Pe*	After correcting single- error bit *Pe*1	Items *n, k, η*	Before correcting errors *Pe*	After correcting single- error bit *Pe*1
n=3 *k*=2 *η*=0.33	10^{-1}	9.42×10^{-3}	*n*=10 *k*=4 *η*=0.60	10^{-1}	3.02×10^{-2}
	10^{-2}	9.93×10^{-5}		10^{-2}	4.27×10^{-4}
	10^{-3}	9.99×10^{-7}		10^{-3}	4.48×10^{-6}
	10^{-4}	10.0×10^{-9}		10^{-4}	4.50×10^{-8}
	10^{-5}	10.0×10^{-11}		10^{-5}	4.50×10^{-10}
n=4 *k*=3 *η*=0.25	10^{-1}	1.33×10^{-2}	*n*=11 *k*=4 *η*=0.64	10^{-1}	3.22×10^{-2}
	10^{-2}	1.48×10^{-4}		10^{-2}	4.72×10^{-4}
	10^{-3}	1.50×10^{-6}		10^{-3}	4.97×10^{-6}
	10^{-4}	1.50×10^{-8}		10^{-4}	5.00×10^{-8}
	10^{-5}	1.50×10^{-10}		10^{-5}	5.00×10^{-10}
n=5 *k*=3 *η*=0.40	10^{-1}	1.69×10^{-2}	*n*=12 *k*=4 *η*=0..67	10^{-1}	3.42×10^{-2}
	10^{-2}	1.96×10^{-4}		10^{-2}	5.16×10^{-4}
	10^{-3}	2.00×10^{-6}		10^{-3}	5.46×10^{-6}
	10^{-4}	2.00×10^{-8}		10^{-4}	5.50×10^{-8}
	10^{-5}	2.00×10^{-10}		10^{-5}	4.00×10^{-10}
n=6 *k*=3 *η*=0.50	10^{-1}	2.00×10^{-2}	*n*=13 *k*=4 *η*=0.70	10^{-1}	3.59×10^{-2}
	10^{-2}	2.44×10^{-4}		10^{-2}	5.59×10^{-4}
	10^{-3}	2.49×10^{-6}		10^{-3}	5.96×10^{-6}
	10^{-4}	2.50×10^{-8}		10^{-4}	6.00×10^{-8}
	10^{-5}	2.50×10^{-10}		10^{-5}	6.00×10^{-10}
n=7 *k*=3 *η*=0.57	10^{-1}	2.29×10^{-2}	*n*=14 *k*=4 *η*=0.71	10^{-1}	3.76×10^{-2}
	10^{-2}	2.90×10^{-4}		10^{-2}	6.02×10^{-4}
	10^{-3}	2.99×10^{-6}		10^{-3}	6.45×10^{-6}
	10^{-4}	3.00×10^{-8}		10^{-4}	6.49×10^{-8}
	10^{-5}	3.00×10^{-10}		10^{-5}	6.50×10^{-10}
n=8 *k*=4 *η*=0.50	10^{-1}	2.55×10^{-2}	*n*=15 *k*=4 *η*=0.73	10^{-1}	3.92×10^{-2}
	10^{-2}	3.37×10^{-4}		10^{-2}	6.45×10^{-4}
	10^{-3}	3.49×10^{-6}		10^{-3}	6.94×10^{-6}
	10^{-4}	3.50×10^{-8}		10^{-4}	6.99×10^{-8}
	10^{-5}	3.50×10^{-10}		10^{-5}	7.00×10^{-10}
n=9 *k*=4 *η*=0.56	10^{-1}	2.79×10^{-2}	*n*=16 *k*=5 *η*=0.69	10^{-1}	4.07×10^{-2}
	10^{-2}	3.82×10^{-4}		10^{-2}	6.87×10^{-4}
	10^{-3}	3.98×10^{-6}		10^{-3}	7.43×10^{-6}
	10^{-4}	4.00×10^{-8}		10^{-4}	7.49×10^{-8}
	10^{-5}	4.00×10^{-10}		10^{-5}	7.50×10^{-10}

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Items n, k, η	Before correcting errors Pe	After correcting single- error bit $Pe1$	Items n, k, η	Before correcting errors Pe	After correcting single- error bit $Pe1$
n=17 k=4 η=0.71	10^{-1}	4.20×10^{-2}	n=24 k=5 η=0.79	10^{-1}	4.99×10^{-2}
	10^{-2}	7.28×10^{-4}		10^{-2}	1.01×10^{-3}
	10^{-3}	7.92×10^{-6}		10^{-3}	1.13×10^{-5}
	10^{-4}	7.99×10^{-8}		10^{-4}	1.15×10^{-7}
	10^{-5}	8.00×10^{-10}		10^{-5}	1.15×10^{-9}
n=18 k=5 η=0.72	10^{-1}	4.34×10^{-2}	n=25 k=5 η=0.80	10^{-1}	5.09×10^{-2}
	10^{-2}	7.69×10^{-4}		10^{-2}	1.04×10^{-3}
	10^{-3}	8.41×10^{-6}		10^{-3}	1.18×10^{-5}
	10^{-4}	8.49×10^{-8}		10^{-4}	1.20×10^{-7}
	10^{-5}	8.50×10^{-10}		10^{-5}	1.20×10^{-9}
n=19 k=5 η=0.74	10^{-1}	4.46×10^{-2}	n=26 k=5 η=0.81	10^{-1}	5.17×10^{-2}
	10^{-2}	8.10×10^{-4}		10^{-2}	1.08×10^{-3}
	10^{-3}	8.90×10^{-6}		10^{-3}	1.23×10^{-5}
	10^{-4}	8.99×10^{-8}		10^{-4}	1.25×10^{-7}
	10^{-5}	9.00×10^{-10}		10^{-5}	1.25×10^{-9}
n=20 k=5 η=0.75	10^{-1}	4.58×10^{-2}	n=27 k=5 η=0.81	10^{-1}	5.26×10^{-2}
	10^{-2}	8.50×10^{-4}		10^{-2}	1.12×10^{-3}
	10^{-3}	9.39×10^{-6}		10^{-3}	1.28×10^{-5}
	10^{-4}	9.49×10^{-8}		10^{-4}	1.30×10^{-7}
	10^{-5}	9.50×10^{-10}		10^{-5}	1.30×10^{-9}
n=21 k=5 η=0.76	10^{-1}	4.69×10^{-2}	n=28 k=5 η=0.82	10^{-1}	5.34×10^{-2}
	10^{-2}	8.89×10^{-4}		10^{-2}	1.15×10^{-3}
	10^{-3}	9.88×10^{-6}		10^{-3}	1.33×10^{-5}
	10^{-4}	9.99×10^{-8}		10^{-4}	1.35×10^{-7}
	10^{-5}	1.00×10^{-9}		10^{-5}	1.35×10^{-9}
n=22 k=5 η=0.77	10^{-1}	4.80×10^{-2}	n=29 k=5 η=0.83	10^{-1}	5.52×10^{-2}
	10^{-2}	9.29×10^{-4}		10^{-2}	1.19×10^{-3}
	10^{-3}	1.04×10^{-5}		10^{-3}	1.38×10^{-5}
	10^{-4}	1.05×10^{-7}		10^{-4}	1.40×10^{-7}
	10^{-5}	1.05×10^{-9}		10^{-5}	1.40×10^{-9}
n=23 k=5 η=0.78	10^{-1}	4.90×10^{-2}	n=30 k=5 η=0.83	10^{-1}	5.49×10^{-2}
	10^{-2}	9.67×10^{-4}		10^{-2}	1.23×10^{-3}
	10^{-3}	1.08×10^{-5}		10^{-3}	1.42×10^{-5}
	10^{-4}	1.10×10^{-7}		10^{-4}	1.45×10^{-7}
	10^{-5}	1.10×10^{-9}		10^{-5}	1.45×10^{-9}

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Items n, k, η	Before correcting errors Pe	After correcting single- error bit $Pe1$	Items n, k, η	Before correcting errors Pe	After correcting single- error bit $Pe1$
n=31 k=5 η=0.84	10^{-1}	5.56×10^{-2}	n=38 k=6 η=0.84	10^{-1}	6.00×10^{-2}
	10^{-2}	1.26×10^{-3}		10^{-2}	1.50×10^{-3}
	10^{-3}	1.47×10^{-5}		10^{-3}	1.81×10^{-5}
	10^{-4}	1.50×10^{-7}		10^{-4}	1.85×10^{-7}
	10^{-5}	1.50×10^{-9}		10^{-5}	1.85×10^{-9}
n=32 k=6 η=0.81	10^{-1}	5.63×10^{-2}	n=39 k=6 η=0.85	10^{-1}	6.05×10^{-2}
	10^{-2}	1.30×10^{-3}		10^{-2}	1.53×10^{-3}
	10^{-3}	1.52×10^{-5}		10^{-3}	1.85×10^{-5}
	10^{-4}	1.55×10^{-7}		10^{-4}	1.90×10^{-7}
	10^{-5}	1.55×10^{-9}		10^{-5}	1.90×10^{-9}
n=33 k=6 η=0.82	10^{-1}	5.70×10^{-2}	n=40 k=6 η=0.85	10^{-1}	6.11×10^{-2}
	10^{-2}	1.33×10^{-3}		10^{-2}	1.56×10^{-3}
	10^{-3}	1.57×10^{-5}		10^{-3}	1.91×10^{-5}
	10^{-4}	1.60×10^{-7}		10^{-4}	1.95×10^{-7}
	10^{-5}	1.60×10^{-9}		10^{-5}	1.95×10^{-9}
n=34 k=6 η=0.82	10^{-1}	5.76×10^{-2}	n=41 k=6 η=0.85	10^{-1}	6.16×10^{-2}
	10^{-2}	1.37×10^{-3}		10^{-2}	1.60×10^{-3}
	10^{-3}	1.62×10^{-5}		10^{-3}	1.95×10^{-5}
	10^{-4}	1.65×10^{-7}		10^{-4}	1.99×10^{-7}
	10^{-5}	1.65×10^{-9}		10^{-5}	2.00×10^{-9}
n=35 k=6 η=0.83	10^{-1}	5.83×10^{-2}	n=42 k=6 η=0.86	10^{-1}	6.21×10^{-2}
	10^{-2}	1.40×10^{-3}		10^{-2}	1.63×10^{-3}
	10^{-3}	1.66×10^{-5}		10^{-3}	2.00×10^{-5}
	10^{-4}	1.70×10^{-7}		10^{-4}	2.04×10^{-7}
	10^{-5}	1.70×10^{-9}		10^{-5}	2.04×10^{-9}
n=36 k=6 η=0.83	10^{-1}	5.89×10^{-2}	n=43 k=6 η=0.86	10^{-1}	6.25×10^{-2}
	10^{-2}	1.43×10^{-3}		10^{-2}	1.66×10^{-3}
	10^{-3}	1.71×10^{-5}		10^{-3}	2.04×10^{-5}
	10^{-4}	1.75×10^{-7}		10^{-4}	2.09×10^{-7}
	10^{-5}	1.75×10^{-9}		10^{-5}	2.10×10^{-9}
n=37 k=6 η=0.84	10^{-1}	5.94×10^{-2}	n=44 k=6 η=0.86	10^{-1}	6.30×10^{-2}
	10^{-2}	1.47×10^{-3}		10^{-2}	1.69×10^{-3}
	10^{-3}	1.76×10^{-5}		10^{-3}	2.09×10^{-5}
	10^{-4}	1.80×10^{-7}		10^{-4}	2.14×10^{-7}
	10^{-5}	1.80×10^{-9}		10^{-5}	2.15×10^{-9}

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Items n, k, η	Before correcting errors Pe	After correcting single- error bit $Pe1$	Items n, k, η	Before correcting errors Pe	After correcting single- error bit $Pe1$
n=45 k=6 η=0.87	10^{-1}	6.34×10^{-2}	n=52 k=6 η=0.88	10^{-1}	6.63×10^{-2}
	10^{-2}	1.72×10^{-3}		10^{-2}	1.93×10^{-3}
	10^{-3}	2.14×10^{-5}		10^{-3}	2.47×10^{-5}
	10^{-4}	2.19×10^{-7}		10^{-4}	2.54×10^{-7}
	10^{-5}	2.20×10^{-9}		10^{-5}	2.55×10^{-9}
n=46 k=6 η=0.87	10^{-1}	6.39×10^{-2}	n=53 k=6 η=0.89	10^{-1}	6.66×10^{-2}
	10^{-2}	1.75×10^{-3}		10^{-2}	1.96×10^{-3}
	10^{-3}	2.19×10^{-5}		10^{-3}	2.51×10^{-5}
	10^{-4}	2.24×10^{-7}		10^{-4}	2.59×10^{-7}
	10^{-5}	2.25×10^{-9}		10^{-5}	2.60×10^{-9}
n=47 k=6 η=0.87	10^{-1}	6.43×10^{-2}	n=54 k=6 η=0.89	10^{-1}	6.70×10^{-2}
	10^{-2}	1.78×10^{-3}		10^{-2}	1.99×10^{-3}
	10^{-3}	2.23×10^{-5}		10^{-3}	2.56×10^{-5}
	10^{-4}	2.29×10^{-7}		10^{-4}	2.64×10^{-7}
	10^{-5}	2.30×10^{-9}		10^{-5}	2.65×10^{-9}
n=48 k=6 η=0.88	10^{-1}	6.47×10^{-2}	n=55 k=6 η=0.89	10^{-1}	6.73×10^{-2}
	10^{-2}	1.81×10^{-3}		10^{-2}	2.01×10^{-3}
	10^{-3}	2.28×10^{-5}		10^{-3}	2.61×10^{-5}
	10^{-4}	2.34×10^{-7}		10^{-4}	2.69×10^{-7}
	10^{-5}	2.35×10^{-9}		10^{-5}	2.70×10^{-9}
n=49 k=6 η=0.88	10^{-1}	6.51×10^{-2}	n=56 k=6 η=0.89	10^{-1}	6.77×10^{-2}
	10^{-2}	1.84×10^{-3}		10^{-2}	2.04×10^{-3}
	10^{-3}	2.33×10^{-5}		10^{-3}	2.65×10^{-5}
	10^{-4}	2.39×10^{-7}		10^{-4}	2.74×10^{-7}
	10^{-5}	2.40×10^{-9}		10^{-5}	2.75×10^{-9}
n=50 k=6 η=0.88	10^{-1}	6.55×10^{-2}	n=57 k=6 η=0.89	10^{-1}	6.80×10^{-2}
	10^{-2}	1.87×10^{-3}		10^{-2}	2.07×10^{-3}
	10^{-3}	2.37×10^{-5}		10^{-3}	2.70×10^{-5}
	10^{-4}	2.44×10^{-7}		10^{-4}	2.79×10^{-7}
	10^{-5}	2.45×10^{-9}		10^{-5}	2.80×10^{-9}
n=51 k=6 η=0.88	10^{-1}	6.59×10^{-2}	n=58 k=6 η=0.90	10^{-1}	6.83×10^{-2}
	10^{-2}	1.90×10^{-3}		10^{-2}	2.10×10^{-3}
	10^{-3}	2.42×10^{-5}		10^{-3}	2.75×10^{-5}
	10^{-4}	2.49×10^{-7}		10^{-4}	2.84×10^{-7}
	10^{-5}	2.50×10^{-9}		10^{-5}	2.85×10^{-9}

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Items n, k, η	Before correcting errors Pe	After correcting single- error bit $Pe1$	Items n, k, η	Before correcting errors Pe	After correcting single- error bit $Pe1$
n=59 k=6 η=0.90	10^{-1}	6.86×10^{-2}	n=62 k=6 η=0.90	10^{-1}	6.95×10^{-2}
	10^{-2}	2.12×10^{-3}		10^{-2}	2.20×10^{-3}
	10^{-3}	2.79×10^{-5}		10^{-3}	2.93×10^{-5}
	10^{-4}	2.89×10^{-7}		10^{-4}	3.04×10^{-7}
	10^{-5}	2.90×10^{-9}		10^{-5}	3.05×10^{-9}
n=60 k=6 η=0.90	10^{-1}	6.89×10^{-2}	n=63 k=6 η=0.90	10^{-1}	6.97×10^{-2}
	10^{-2}	2.15×10^{-3}		10^{-2}	2.23×10^{-3}
	10^{-3}	2.84×10^{-5}		10^{-3}	2.98×10^{-5}
	10^{-4}	2.94×10^{-7}		10^{-4}	3.09×10^{-7}
	10^{-5}	2.95×10^{-9}		10^{-5}	3.10×10^{-9}
n=61 k=6 η=0.90	10^{-1}	6.92×10^{-2}	n=64 k=7 η=0.89	10^{-1}	7.01×10^{-2}
	10^{-2}	2.18×10^{-3}		10^{-2}	2.26×10^{-3}
	10^{-3}	2.89×10^{-5}		10^{-3}	3.03×10^{-5}
	10^{-4}	2.99×10^{-7}		10^{-4}	3.14×10^{-7}
	10^{-5}	3.00×10^{-9}		10^{-5}	3.15×10^{-9}

4. Conclusion

1). For calculating the BER after correcting one single-error in the block, the Equation (11) is very important.

2). Although correcting one single-error only, the probability of bit error is greatly decreased, generally one to four order of magnitude, shown in table 1.

3). The code rate efficiency, 1-k/n, is reduced as check bits r increasing when we keep the bit rate constant.

Conferences

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