Semi-Singularity in Stiffness Generation of an Anthropomorphic Robot

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Abstract

This paper analyzes the singularity of an anthropomorphic robot associated with joint and operational stiffness generation from muscle stiffness. The singularity analysis is made simply based on the signs of the actual and the desired coupling joint stiffness. First, the relationships of the muscle stiffness and the actual joint stiffness, and the operational stiffness and the desired joint stiffness are examined. Second, according to the sign restriction on the actual coupling joint stiffness, the operational space is divided into the semi-singular(SS), the regular(R), and the semi-regular(SR) regions. Third, from the sign comparison of the actual and the desired coupling joint stiffness, the sufficient condition for the semi-singularity in operational stiffness generation is derived. The limitation on the allowable operational stiffness when a task point belongs to SS, R, and SR regions is also discussed. Simulation results are given.

I. Introduction

Anthropomorphic robots are modeled after the musculoskeletal structure of the human upper limb [1]. Anthropomorphic robots consist of several mono-articular muscles, each spanning a single revolute joint, and a few bi-articular muscles spanning two revolute joints. Compared to industrial robots, anthropomorphic robots actuated by artificial muscles are believed to be close to human in functionality as well as shape [2]. Anthropomorphic robots can act as an effective active variable stiffness generator, since they can modulate the joint and the operational stiffness in a natural way.

The stiffness modulation capability of an anthropomorphic robot plays a significant role in task execution when contact with environment is involved. The dextrous manipulation of a held object and the precise peg-into-hole insertion are typical examples. There have been some works on the stiffness generation of an anthropomorphic robots. Subject to the positiveness of the muscle stiffness, the muscle stiffness solution for given operational stiffness was sought [3]. The sign dependency of the coupling joint stiffness produced by bi-articular muscles on the joint configuration was analyzed [4].

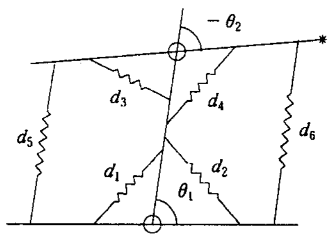

Fig. 1 An anthropomorphic robot.

An anthropomorphic robot under consideration has two pairs of mono-articular muscles and one pair of bi-articular muscles, as shown in Fig. 1. While there are six independently controllable muscles, both joint and operational stiffness contain three independent elements only. This may mislead one to guess that the joint and the operational stiffness generation from the muscle stiffness is almost free from singularity. However, such a wrong guess overlooks the sign restriction of the muscle stiffness and the configurational dependency of the stiffness generation.

In this paper, we investigate the singularity of an anthropomorphic robot, associated with the joint and the operational stiffness generation from the muscle stiffness. It is shown that the semi-singularity occurs over a considerable portion of the joint and the operational spaces. It is assumed that two links have unit length, and a pair of bi-articular muscles have the equal spacings of the half of the link length. It is also assumed that individual muscles are ideal springs with no limit on their stiffness.

II. Preliminary: Stiffness Generation

For muscle i, $i=1,\cdots,6$, let d_i and $k_{d_i}(>0)$ be the muscle length and the muscle stiffness, respectively. And, for joint j, $j=1,2$, let θ_j and $k_{\theta_{jj}}$ be the joint angle and the self joint stiffness, respectively, and let $k_{\theta_{12}}(=k_{\theta_{21}})$ be the coupling joint stiffness. The muscle stiffness matrix, $\mathbf{K}_d$, is related to the joint stiffness matrix, $\mathbf{K}_\theta$, by

$$\mathbf{K}_\theta = \mathbf{G}^t\ \mathbf{K}_d\ \mathbf{G} \tag{1}$$

where

$$\mathbf{G}^t = \begin{bmatrix} \frac{\partial d_1}{\partial \theta_1} & \frac{\partial d_2}{\partial \theta_1} & 0 & 0 & \frac{\partial d_5}{\partial \theta_1} & \frac{\partial d_6}{\partial \theta_1} \\ 0 & 0 & \frac{\partial d_3}{\partial \theta_2} & \frac{\partial d_4}{\partial \theta_2} & \frac{\partial d_5}{\partial \theta_2} & \frac{\partial d_6}{\partial \theta_2} \end{bmatrix} \tag{2}$$

$$\mathbf{K}_d = \mathrm{diag}\{d_1,\cdots,d_6\},\quad \mathbf{K}_\theta = \begin{bmatrix} k_{\theta_{11}} & k_{\theta_{12}} \\ k_{\theta_{12}} & k_{\theta_{22}} \end{bmatrix} \tag{3}$$

Since a pair of mono-articular muscles affect the spanned joint only, $\frac{\partial d_1}{\partial \theta_2} = \frac{\partial d_2}{\partial \theta_2} = \frac{\partial d_3}{\partial \theta_1} = \frac{\partial d_4}{\partial \theta_1} = 0$. Note that $\mathbf{G}$ is a function of the joint configuration, and $\mathbf{K}_d$ and $\mathbf{K}_\theta$ are positive definite.

Let x and y be the task coordinates. Let k_{xx}, k_{yy} and $k_{xy}(=k_{yx})$ be the self and the coupling operational stiffness, respectively. The operational stiffness matrix, $\mathbf{K}_o$, is related to the joint stiffness matrix, $\mathbf{K}_\theta$, by

$$\mathbf{K}_\theta = \mathbf{J}^t\ \mathbf{K}_o\ \mathbf{J} \tag{4}$$

where

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \end{bmatrix},\quad \mathbf{K}_o = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{xy} & k_{\theta_{yy}} \end{bmatrix} \tag{5}$$

Note that $\mathbf{J}$ is dependent on the joint configuration, and $\mathbf{K}_o$ is positive semidefinite.

While the muscle stiffness matrix has six independent elements, both joint and operational stiffness matrices have three independent elements. This may mislead one to guess that the joint and the operational stiffness generation from the muscle stiffness is almost free from singularity. However, such a guess overlooks the sign restriction of the muscle stiffness and the configurational dependency of the stiffness generation.

III. Semi-Singularity in Joint Stiffness Generation

From (1), the actual joint stiffness generated from the muscle stiffness can be written as

$$^a k_{\theta_{11}} = g_{11}^2 k_{d_1} + g_{21}^2 k_{d_2} + g_{51}^2 k_{d_5} + g_{61}^2 k_{d_6} \tag{6}$$

$$^a k_{\theta_{22}} = g_{31}^2 k_{d_3} + g_{42}^2 k_{d_4} + g_{52}^2 k_{d_5} + g_{62}^2 k_{d_6} \tag{7}$$

$$^a k_{\theta_{12}} = g_{51} g_{52} k_{d_5} + g_{61} g_{62} k_{d_6} = G_5 k_{d_5} + G_6 k_{d_6} \tag{8}$$

where $g_{ij} = \frac{\partial d_i}{\partial \theta_j}$, $i=1,\cdots,6$, $j=1,2$, and $G_i = g_{i1} g_{i2}$, $i=5,6$. The magnitude of g_{ij}, $i=1,\cdots,6$, $j=1,2$ is the same as the normal distance from joint j to muscle i. Note that the actual coupling joint stiffness, $^a k_{\theta_{12}}$, is produced by bi-articular muscles only, while the actual self joint stiffness, $^a k_{\theta_{11}}$ and $^a k_{\theta_{22}}$, are produced by both mono- and bi-articular muscles.

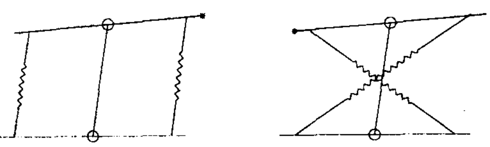

Fig. 2 Two alignments of bi-articular muscles.

Since $k_{d_i}>0$, $i=1,\cdots,6$, $^a k_{\theta_{11}}$ and $^a k_{\theta_{22}}$ are always positive. However, $^a k_{\theta_{12}}$ can be either positive or negative depending on the joint configuration, G_5 and G_6, and the muscle stiffness, k_{d_5} and k_{d_6}. Fig. 2 shows two alignments of a pair of bi-articular muscles with respect to the link. If bi-articular muscle i, $i=5,6$, is aligned to cross the extended link, g_{i1} and g_{i2} are of opposite signs so that $G_i<0$; otherwise, g_{i1} and g_{i2} are of the same sign so that $G_i>0$.

Fig. 3 shows the plots of G_5 and G_6 and their positive and negative zones. With reference to the sign combination of G_5 and G_6, the joint space can be divided into four regions with $(G_5, G_6)=(+,+)$, $(-,-)$, $(+,-)$, and $(-,+)$, as shown in Fig. 4. In the regions of $(+,+)$ and $(-,-)$, $^a k_{\theta_{12}}$ is always restricted to be positive and negative, respectively. When the sign restriction is imposed on $^a k_{\theta_{12}}$, the semi-singularity in joint stiffness generation is said to occur. In the regions of $(+,-)$ and $(-,+)$, where $G_5G_6<0$, the sign of ${}^ak_{\theta_{12}}$ can be taken arbitrarily by controling the relative magnitudes of k_{d_5} and k_{d_6}, which is good for flexible joint stiffness generation.

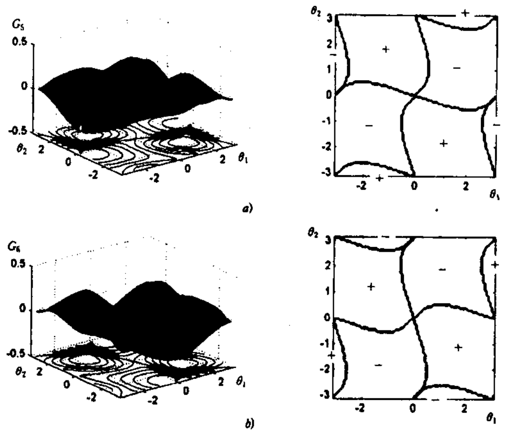

Fig. 3 Plots of a) G_5 and b) G_6 and their positive and negative zones.

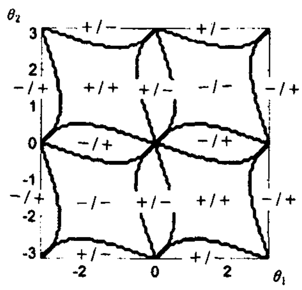

Fig. 4 The division of the joint space into four regions with $(G_5, G_6)=(+,+)$, $(-,-)$, $(+,-)$, and $(-,+)$.

For a given task point, there are two possible joint configurations, including elbow-up(EU) and elbow-down(ED). The choice of the joint configuration may help to avoid the semi-singularity in joint stiffness generation. With the change in joint configuration allowed, the operational space can be divided into three regions: the semi-singular(SS), the regular(R), and the semi-regular(SR) regions, as shown in Fig. 5. In SS region, both joint configurations result in the coupling joint stiffness of the same sign, that is, $(G_5^{EU}, G_6^{EU})=(+,+)$ and $(G_5^{ED}, G_6^{ED})=(+,+)$, or $(G_5^{EU}, G_6^{EU})=(-,-)$ and $(G_5^{ED}, G_6^{ED})=(-,-)$. In R region, two joint configurations result in the coupling joint stiffness of opposite signs, that is, $G_5^{EU}G_6^{EU}<0$ and $G_5^{ED}G_6^{ED}<0$. The operational space excluding SS and R regions is SR region in which either of two joint configurations is free from the semi-singularity in joint stiffness generation. Notice that SS and R regions respectively occupy larger and smaller portions of the operational space than one may expect.

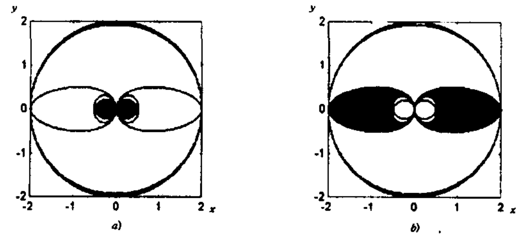

Fig. 5 The division of the operational space: a) the regular and b) the semi-singular regions.

IV. Sufficient Condition for Semi-Singularity in Operational Stiffness Generation

From (4), the desired joint stiffness required to generate the operational stiffness can be written as

$$ {}^dk_{\theta_{11}} = a_1^2k_{xx}+b_1^2k_{yy}+2a_1b_1k_{xy} \quad (9)$$

$$ {}^dk_{\theta_{22}} = a_2^2k_{xx}+b_2^2k_{yy}+2a_2b_2k_{xy} \quad (10)$$

$$\begin{aligned} {}^dk_{\theta_{12}} &= a_1a_2k_{xx}+b_1b_2k_{yy}+(a_1b_2-a_2b_1)k_{xy} \\ &= c_1k_{xx}+c_2k_{yy}+c_3k_{xy} \end{aligned} \quad (11)$$

where $a_j=\frac{\partial x}{\partial\theta_j}$ and $b_j=\frac{\partial y}{\partial\theta_j}$, $j=1,2$, and $c_1=a_1a_2$, $c_2=b_1b_2$, and $c_3=a_1b_2-a_2b_1$. Since $k_{xx}>0$, $k_{yy}>0$ and $|k_{xy}|<\sqrt{k_{xx}k_{yy}}$, from (9) and (10),

$$ {}^dk_{\theta_{jj}}>(|a_j|\sqrt{k_{xx}}-|b_j|\sqrt{k_{yy}})^2\geq 0,\ j=1,2 \quad (12)$$

The desired self joint stiffness, ${}^dk_{\theta_{11}}$ and ${}^dk_{\theta_{22}}$, are always positive as the actual self joint stiffness, ${}^ak_{\theta_{11}}$ and ${}^ak_{\theta_{22}}$, are. Thus, the desired self joint stiffness can always be generated from the muscle stiffness, as far as the sign is concerned. On the other hand, the desired coupling joint stiffness, ${}^dk_{\theta_{12}}$, can be positive or negative depending on the joint configuration, c_1, c_2 and c_3, and the operational stiffness, k_{xx}, k_{yy} and k_{xy}. When ${}^{d}k_{\theta_{12}}$ differs from the actual coupling joint stiffness, ${}^{a}k_{\theta_{12}}$, in sign, that is, ${}^{d}k_{\theta_{12}}>0$ and ${}^{a}k_{\theta_{12}}<0$, or ${}^{d}k_{\theta_{12}}<0$ and ${}^{a}k_{\theta_{12}}>0$, the operational stiffness generation fails.

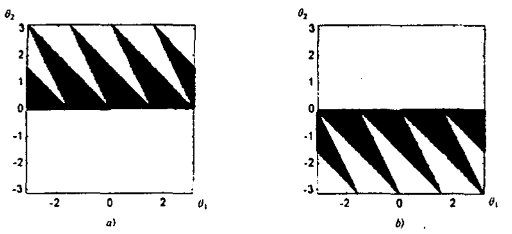

Fig. 6 The joint space regions in which a) COND_1 and b) COND_2 are true.

Assume that the change in joint configuration is not allowed for operational stiffness generation. From (11), the sufficient condition for ${}^{d}k_{\theta_{12}}>0$ can be expressed as

$$\begin{aligned} \mathrm{COND}_1 &= (c_1>0\ \&\ c_2>0\ \&\ c_3>0), \quad \text{if } k_{xy}>0 \\ \mathrm{COND}_2 &= (c_1>0\ \&\ c_2>0\ \&\ c_3<0), \quad \text{if } k_{xy}<0 \end{aligned} \tag{13}$$

Fig. 6 shows the joint space regions in which COND_1 and COND_2 are true. The sufficient condition for ${}^{d}k_{\theta_{12}}<0$ can be expressed as

$$\begin{aligned} \mathrm{COND}_3 &= (c_1<0\ \&\ c_2<0\ \&\ c_3<0), \quad \text{if } k_{xy}>0 \\ \mathrm{COND}_4 &= (c_1<0\ \&\ c_2<0\ \&\ c_3>0), \quad \text{if } k_{xy}<0 \end{aligned} \tag{14}$$

but it can be shown that there is no corresponding joint space region. On the other hand, the sufficient condition for ${}^{a}k_{\theta_{12}}<0$ is that $G_5<0$ and $G_6<0$.

From the above, the sufficient condition for the semi-singularity in operational stiffness generation is obtained by

$$\begin{aligned} \mathrm{COND}^{+} &= \mathrm{COND}_1 \& (G_5<0 \& G_6<0), \quad \text{if } k_{xy}>0 \\ \mathrm{COND}^{-} &= \mathrm{COND}_2 \& (G_5<0 \& G_6<0), \quad \text{if } k_{xy}<0 \end{aligned} \tag{15}$$

Fig. 7 shows the joint space region in which either COND^{+} and COND^{-} is true, which is the intersection of the regions shown in Fig. 6 and $(-,-)$ region shown in Fig. 5. Notice that the semi-singular joint space region satisfying the sufficient condition is considerable.

The flexibility in operational stiffness generation depends on where a task point belongs to among SS, R, and SR regions shown in Fig. 5. The operational stiffness can be geometrically represented as a stiffness ellipse in the operational space. When a task point is given in R region, regardless of the choice of the joint configuration, there is no limitation on the shape of the allowable stiffness ellipsoid. However, when a task point is given in SS or SR regions, the allowable shape of the stiffness ellipsoid becomes limited. Such a limitation is more severe in SS region than in SR region. This is because SS region does not provide alternate choice of joint configuration while SR region does.

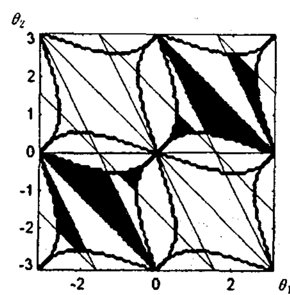

Fig. 7 The joint space region satisfying the sufficient condition for the semi-singularity in operational stiffness generation.

V. Conclusion

This paper analyzed the semi-singularity of an anthropomorphic robot, associated with the stiffness generation. The semi-singularity analysis was made simply based on the signs of the actual and the desired coupling joint stiffness. It was shown that the semi-singularity is present over a considerable portion of the workspace. Taking into account the semi-singularity in stiffness generation, the optimal muscle attachment [4, 5] and the least squares solution of muscle stiffness [3] are under study.

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