GT 셀 형성을 위한 효율적 *p*-median 접근법
Efficient *p*-median approach to GT cell formation

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Abstract

This paper is concerned with development of an efficient p-median approach applicable to large cell formation(CF) problems. A two-phase methodology that seeks to minimize the number of exceptional elements is proposed. In phase I, two efficient p-median formulations which contain fewer binary variables than existing p-median formulations are constructed. These make it possible to implement large CF problem within reasonable computer runtime with commercially available linear integer programming codes. Given the initial cell configuration found with the new p-median formulations, in phase II bottleneck machines and parts are reassigned to reduce the number of exceptional elements. This procedure has the flexibility to provide the cell designer with alternative solutions. Test results on large CF problems show a substantial efficiency of the new p-median formulations.

1. Introduction

Group Technology(GT) is a manufacturing philosophy that identifies and exploits the similarities of product design and manufacturing process. The basic idea of GT is to decompose a manufacturing system into subsystems, facilitating better control. Cellular manufacturing (CM) is an application of GT principles to manufacturing. The most important step toward designing a CM system is cell formation(CF). CF consists of identifying part families and machine cells such that a part family is processed within a machine cell with minimum interaction with other cells. A part family consists of batches of parts requiring similar or identical processing and its corresponding machine cell consists of dissimilar machines dedicated to process the parts of that specific part family.

The objective of CF is to create mutually independent machine cells which are capable of processing a part family completely. The main input to CF problem is the $m \times n$ machine-part incidence matrix where each row corresponds to a machine and each column to a part. The machine-part incidence matrix is a binary matrix $\mathbf{A}(=[a_{ij}])$ where the element a_{ij} is 1 or 0 depending on whether or not part j requires processing on machine i.

The best block-diagonal structure from the incidence matrix means the best cells configuration with minimum intercell part moves. However, CF process often identifies exceptional elements('1' entries outside the diagonal block). which create interactions between cells. Exceptional elements are the results of bottleneck machines that are needed to process a large number of parts found in two or more part families, or bottleneck parts that require processing on machines assigned to two or more machine cells. Most of the studies on CF problem are concerned with eliminating or minimizing the exceptional elements.

Many researchers have addressed the CF problem and proposed numerous methods for grouping machines and parts. Mathematical programming approaches attempt to find the cells and families by formulating the problem into linear or nonlinear programming models. Kusiak[5] suggested a linear integer programming model, called the p-median model. Since Kusiak suggested using p-median model as a methodology for solving CF problem, many authors have reported successful applications to cell configuration with slight modifications over the original formulation([2],[9]-[11]). However, the existing p-median formulations have critical limitations in that the formulations can only be applied to small CF problems.

The purpose of this study is to develop an efficient p-median approach applicable to large CF problems. A two-phase methodology that seeks to minimize the number of exceptional elements is proposed. In phase I, two efficient p-median formulations which contain fewer binary variables than existing p-median formulations are proposed. This improvement makes it possible to implement the p-median model on large CF problems within reasonable computer runtime with commercially available linear integer programming codes. Given the initial cell configuration found using the new p-median formulation, in phase II bottleneck machines and parts are reassigned to reduce the number of exceptional elements.

2. Phase I: efficient *p*-median formulations

2.1. Quadratic 0-1 formulation

In this paper, the quadratic 0-1 formulation of Kumar *et al.*[4] which is frequently addressed by many authors([1],[6]) is adopted. To cluster m machines into the prespecified p cells, the following binary variable is used:

$$x_{hi} = \begin{cases} 1, \text{if machine } h \text{ belongs to cell } i, h, i = 1, \Lambda, m \\ 0, \text{otherwise.} \end{cases}$$

The quadratic p-median model used is as follows:

(Q0)

$$\text{Maximize} \quad \sum_{h=1}^{m} \sum_{i=1}^{m} s_{hi} x_{hk} x_{ik} \qquad (1)$$

subject to

$$\sum_{i=1}^{m} x_{hi} = 1, \; h = 1, \Lambda, m \qquad (2)$$

$$\sum_{i=1}^{m} x_{ik} \le U, \; k = 1, \Lambda, p \qquad (3)$$

$$\sum_{i=1}^{m} x_{ik} \ge L, \; k = 1, \Lambda, p \qquad (4)$$

$$x_{ik} = 0 \text{ or } 1, \; i = 1, \Lambda, m; \; k = 1, \Lambda, p. \qquad (5)$$

The objective function (1) maximizes the sum of McAuley's machine similarities[7] within cells without regard to the machine identified as a median of cell. Constraint (2) specifies that each machine needs to be assigned to one and only one cell. Constraints (3) and (4) specify the maximum and the minimum number of machines assigned to each cell, respectively. Constraint (5) ensures the binary solution.

2.2. Linearization of (Q0)

To linearize the quadratic terms in equation (1), let

$$z_{hk} = x_{hk} \sum_{i=h+1}^{m} s_{hi} x_{ik}$$

Then, based on Oral and Kettani's finding[8], the following linear formulation equivalent to (Q0) is obtained:

(L1)

$$\text{Maximize} \quad \sum_{h=1}^{m-1} \sum_{k=1}^{p} (s_{hi}^{+} x_{hk} - z_{hk}) \qquad (6)$$

subject to (2)-(5) and

$$z_{hk} \ge \sum_{i=h+1}^{m} (-s_{hi}) x_{ik} + s_{hi}^{+} x_{hk} + s_{hi}^{-} (1 - x_{hk}),$$

$$h = 1, \Lambda, m-1; \; k = 1, \Lambda, p \qquad (7)$$

$$z_{ik} \ge 0, \; h = 1, \Lambda, m-1; \; k = 1, \Lambda, p \qquad (8)$$

where $s_{hk}^{+} = \max \sum_{i=h+1}^{m} s_{hk}$ and $s_{hk}^{+} = \min \sum_{i=h+1}^{m} s_{hk}$.

The objective function value of (6) is the same as one of (1) in optimal solution. In constraint (7), s_{hk}^{+} is the sum of the largest $(p-1)$ coefficients of s_{hi} for $h = 1, \Lambda, m-1$. It then is easily seen that $s_{h1}^{+} = \Lambda \; s_{hp}^{+}$ for $h = 1, \Lambda, m-1$. On the other hand, s_{hk}^{-} can always be set to 0 for any h if $s_{hi} \ge 0$ for all h, i. Note that in constraint (8) the extra variable z_{hi} is continuous. As a result this, the linear model (L1) contains pm binary variables, $p(m-1)$ continuous variables, and only a total of $(p+1)m$ linear constraints. With this linear formulation, we can save considerable computation efforts compared with the classical p-median formulations which requires m^2 binary variables since $p \le m$.

2.3 Further reduction of binary variables

The linear formulation (L1) is further reduced in terms of the number of binary variables. The reduction is based on Kettani and Oral's work[3]. To reduce the number of binary variables in (L1), the following notation is introduced:

$P = \{1, \Lambda, p\}$

$p^{*} = [\log_2 p]^{+}$: the smallest integer greater than $\log_2 p$

$P^{*} = \{1, \Lambda, p^{*}\}$

$D = \{2^{d-1} \mid d \in P^{*}\}$

$\mathbf{w}_j = \{w_{j1}, \Lambda, w_{jp^{*}}\}$: binary representation vector of j

$A_d = \{j \mid w_{jd}, j \in P\}, d \in P^{*}$

$B_j = \{d \mid w_{jd}, d \in P^{*}\}, j \in P$

$C_j = \{l \mid w_l \ge w_j, l \ne j, l \in P\}, j \in P.$

Reduction of binary variables is applied to the set of constraints on machine cells and the binary restriction. Then, following Kettani and Oral's finding, the constraints (2) and (5) can be equivalently replaced by

$$1 \le \sum_{d \in P^{*}} 2^{d-1} u_{id} \le p, \; i = 1, \Lambda, m \qquad (9)$$

$$\sum_{k \in A_d} x_{ik} = u_{id}, \; i = 1, \Lambda, m, d \in P^{*} \qquad (10)$$

$$x_{ik} + \sum_{l \in C_k} x_{il} \ge \sum_{d \in B_k} (u_{id} - 1) + 1, \; i = 1, \Lambda, m, k \in P - D \qquad (11)$$

$$x_{ik} \ge 0, \quad h = 1, \Lambda, m; \; k = 1, \Lambda, p \qquad (12)$$

$$u_{id} = 0 \text{ or } 1, \quad i = 1, \Lambda, m; \; d = 1, \Lambda, p^{*}. \qquad (13)$$

With the equivalent constraints above, the new p-median median formulation further reduced in terms of the binary variables is as follows:

(L2)

Maximize (6)

Subject to (2)-(4), (7),(8), and (9)-(13).

Table 1 summarizes the total number of the variables and the constraints required in Wang and Roze's p-median formulation[11] and the reduced p-median formulations. It is obvious from the table that as compared with existing p-

median formulation the reduced formulations generally require fewer binary variables since $[\log_2 p]^+ \le p \le m$ in a typical CF problem. The number of extra continuous variables needed in the reduced formulations is also maintained at a minimum level.

	Wang & Roze	Reduced formulation L1	Reduced formulation L2
Variables			
binary	m^2	pm	$[\log_2 p]^+ m$
continous	0	$p(m-1)$	$p(2m-1)$
Constraint	$3m+1$	$(p+1)m$	$p(2m-1)$

Table 1. Number of variables and constraints required in formulations

3. Phase II: reassignment of machines and parts

To improve the incumbent cell configuration which is obtained using the new p-median mathematical model, subsequent reassignment of bottleneck machines and parts is essential. Reassignment of bottleneck machines and parts which does not rely on human judgement is based on simple rearrangement of rows and columns. The basic idea of reassignment of bottleneck machines and parts is to apply maximum density rule with regard to rows and columns of the incumbent rearranged incidence matrix until no further improvement on the number of exceptional elements is attained. Detailed steps of phase II for reassignment of bottleneck machines and parts are as follows:

Step 1.(Initialization) Start with the rearranged incidence matrix which is obtained by applying maximum density rule with regard to parts from the machine cell solution of phase I and compute the number of exceptional elements from the incumbent rearranged matrix.

Step 2.(Reassignment of bottleneck machines) For each bottleneck machine, compute the number of parts(i.e, 1's) which it processes in each cell. If the size of the cell to which that bottleneck machine belongs is greater than or equal to 2, reassign that machine to the smallest cell in which it processes most parts. Otherwise, reassign that bottleneck machine to the next smallest cell in which it processes most parts and merge the current part family corresponding to the cell to which that bottleneck machine belongs into the part family corresponding to that next smallest cell.

Step 3.(Reassignment of bottleneck parts) For each bottleneck part, compute the number of visits to each cell. If the size of the family to which that bottleneck part belongs is greater than or equal to 2, reassign that part to the smallest family corresponding to the cell in which it visits most machines. Otherwise, reassign that bottleneck part to the next smallest family corresponding to the cell in which it visits most machines and merge the current machine cell corresponding to the family to which that bottleneck part belongs into the machine cell corresponding to that next smallest family.

Step 4.(Stopping) Recompute the number of exceptional elements. If the number of exceptional elements decreases, stop. Otherwise, go to Step 2.

4. Numerical example

One published problem taken from Zolfaghari and Liang[12] was used to compare the efficiency of the existing p-median formulation and reduced p-median formulations. The selected data represents an incidence matrix with 50 machines, 150 parts and $|\mathbf{A}|=874$ which is the largest machine-part incidence matrix reported in the literature as far as the present author knows. The tests were made on an HP 9000/715 workstation using the CPLEX mixed integer optimization software. The number of cells required are set to 5, 6 and 7, respectively. The lower limits on cell size are set to 2 in all cases so as to avoid single-machine cell solutions and the upper limits on cell size are set to 16, 13 and 10, respectively, for $p=5,6$ and 7. The tests were made on two p-median formulations, Wang and Roze's formulation[11] and (L2).

Table 2 summarizes the computational results. In the table, not only the number of exceptional elements at the end of each phase but also the grouping efficiency corresponding to the number of exceptional elements found at the end of phase II is reported as the fractional value in parentheses for comparison purpose. Note that for increasing values of p the existing p-median formulation does not find any integer feasible solution under the time limit of 600 CPU seconds. On the other hand, the reduced formulation (L2) find good integer feasible solutions under the same CPU limit as Wang and Roze's formulation. After reassignment of machines and parts, the initial cell configuration is improved further.

Initial value of p	# of exceptional elements: Wang & Roze	L2 Phase I	L2 Phase II	Final value of p
5	A	139	52(.744b)	5
6	A	125	58(.750)	5
7	A	410	60(.819)	6

(a) indicates that the formulation (P1) finds no integer feasible solution under the time limit of 600 CPU seconds.
(b) represents the grouping efficiency corresponding to the number of exceptional elements found at the end of phase II of the formulation (L2).

Table 2. Solutions to Zolfaghari and Liang's problem

5. Concluding remarks

In this paper a two-phase methodology for solving large CF problems is developed. Phase I uses efficient p-

median formulations to find initial cell configuration. The p-median formulations proposed in phase I contain fewer binary variables as compared with existing p-median formulations. This helps to find good initial cell configuration within reasonable computer runtime for large CF problem. Phase II applies maximum density rule to improve the incumbent cell configuration in terms of the number of exceptional elements. The procedure consists of reassigning bottleneck machines and parts by applying the rule with regard to rows and columns of the incumbent rearranged incidence matrix until no further improvement on the number of exceptional elements is attained. The test results show that on large CF problems the p-median formulation proposed in phase I yields good integer solutions within reasonable time limit and phase II improves the initial cell configuration substantially.

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