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Transient Analysis of Hybrid Systems Composed of Lumped Elements and Frequency Dependent Lossy Disributed Interconnects

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Abstract: A method to analyze the high speed interconnects that are composed of frequency dependent lossy distributed lines is presented. Network modeling of hybrid systems is implemented by using the modified nodal admittance matrix in the Laplece transformation domain. The network response is computed by different two methods. One method is the asymptotic wavewform evaluation (AWE) method and other is numerical Laplace inversion method. The merits and demerits of two methods are discussed by applying to several concrete illustrative networks.

1 Introduction

Due to the large number of interconnects present in VLSI circuits, the simulation of interconnects is very important. Lumped circuit modeling is useful so long as the interconnect is short in length. For high speed operations, the interconnects are no longer electrically short and distributed transmission line analysis must be introduced. In high frequency applications the analysis of frequency dependent effect becomes very important. Several methods have been proposed for the analysis of the circuits that contain frequency dependent lossy interconnects. Asymptotic waveform evaluation (AWE) method is a recently developed technique for time domain analysis of distributed interconnects[1],[2]. AWE is a computationally efficient method that approximates the response of large system with a low order transfer function. The poles and residues of the transfer functions are computed by expanding the transmission line parameters of the frequency dependent lossy interconnects. But the accuracy of approximations is not always sufficient. The formulation for modeling frequency dependent lossy interconnects is derived in the Laplace transformation domain. So, we can use the numerical inversion method[3] in order to obtain transient responses. The objective of this paper is to present a simulation method for hybrid systems composed of lumped elements and frequency dependent lossy distributed interconnects.

2 Network modeling for lumped and distributed elements

For circuits composed of lumped elements, network is formulated by splitting the elements into two groups[4],[5]. One group is formulated by admittance description to obtain

$$Y_I(s)V_I(s) + A_I I_I(s) - b_I = 0 \tag{1}$$

by Kirchhoff's current law where Y_I is the admittance matrix for node voltage vector V_I. A_I maps the unkown current vector I_I and b_I is the current source vector.

Other group is formulated by impedance description to obtain

$$B_{II}V_{II}(s) + Z_{II}(s)I_{II}(s) - b_{II} = 0 \tag{2}$$

by Kirhihoff's voltage law where Z_{II} is the impedance matrix for node current vector I_{II}. B_{II} maps the unknown voltage vector V_{II} and b_{II} is the voltage source vector.

Among the voltage vectors and current vectors in two groups the following two relations must be held.

$$H_{V,I}V_I(s) \quad + \quad H_{V,II}V_{II}(s) = 0 \tag{3}$$
$$H_{I,I}I_I(s) \quad + \quad H_{I,II}I_{II}(s) = 0 \tag{4}$$

Equations (1),(2),(3) and (4) can be combined in a single matrix equation of the following form.

$$\begin{bmatrix} Y_I(s) & 0 & A_I & 0 \\ 0 & B_{II} & 0 & Z_{II}(s) \\ H_{V,I} & H_{V,II} & 0 & 0 \\ 0 & 0 & H_{I,I} & H_{I,II} \end{bmatrix} \begin{bmatrix} V_I(s) \\ V_{II}(s) \\ I_I(s) \\ I_{II}(s) \end{bmatrix} = \begin{bmatrix} b_I \\ b_{II} \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

3 Multi-conductor Lossy transmission line

For lossy multi-conductor transmission line, the voltage vector and current vector at the position x are given by the following telegrapher's equations in the s domain.

$$-\frac{dV(x,s)}{dx} = [sL+R(s)]I(x,s) \quad (6)$$

$$-\frac{dI(x,s)}{dx} = [sC+G(s)]V(x,s) \quad (7)$$

L, C are the per unit length inductance and capacitance matrices and $R(s), G(s)$ are frequency dependent lossy resistance and conductance matrices.

Equations (6) and (7) can be combined in a single matrix equation of the following form.

$$\frac{d}{dx}\begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix} + \begin{bmatrix} 0 & sL+R \\ sC+G & 0 \end{bmatrix}\begin{bmatrix} V(x,s) \\ I(x,s) \end{bmatrix} = 0 \quad (8)$$

Terminal voltage and current vectors of the line ends with length l can be expressed in the following form.

$$\begin{bmatrix} V(l,s) \\ I(l,s) \end{bmatrix} = T(s)\begin{bmatrix} V(0,s) \\ I(0,s) \end{bmatrix} \quad (9)$$

$$T(s) = \exp\left\{-\begin{bmatrix} 0 & sL+R(s) \\ sC+G(s) & 0 \end{bmatrix} l\right\} \quad (10)$$

4 Formulation of hybrid systems

Hybrid system is composed of lumped network of equation (5) and distributed networks of equation (9) and can be combined in the single matrix equation resulting in

$$Y(s)X(s) = E(s) \quad (11)$$

where $Y(s)$ is a modified nodal matrix, $X(s)$ is an unknown vector and $E(s)$ is input excitation vector. This equation is solved by the two different methods. One is AWE method and other is numerical Laplace inversion method.

5 Asymptotic waveform evaluation method

We assume that modifide nodal matrix $Y(s)$ is described by the Taylor series at $s = 0$ as

$$Y(s) = \sum_{i=0}^{\infty} Y_i s^i \quad (12)$$

and unkown vector $X(s)$ is also approximated by

$$X(s) = \sum_{i=0}^{\infty} M_i s^i. \quad (13)$$

Substituting equations (12) and (13) into equation (11) we can obtain

$$[Y_0 + Y_1 s + Y_2 s^2 + \cdots][M_0 + M_1 s + M_2 s^2 + \cdots] = E(s). \quad (14)$$

By matching the corresponding powers of s the following recursive relationship can be derived.

$$M_0 = Y_0^{-1}E \quad (15)$$

$$M_n = Y_0^{-1}\left[-\sum_{r=1}^{n} Y_n M_{n-r}\right] \quad n \geq 1 \quad (16)$$

The approximate solution for j-th element of $X(s)$ is given

$$\hat{X}_j(s) = \frac{k_1}{s-p_1} + \frac{k_2}{s-p_2} + \cdots + \frac{k-q}{s-p_q} = -\sum_{l=1}^{q} \frac{k_l/p_l}{1-s/p_l} \quad (17)$$

where p_l and k_l are approximate poles and residues.

The poles of the system are approximated by computing the roots of characteristic equation given by

$$a_0 + a_1 s + a_2 s^2 + \cdots + a_{q-1}s^{q-1} + s^q = 0. \quad (18)$$

The coefficients for polynomial are found from

$$\begin{bmatrix} m_0 & m_1 & \cdots & m_{q-1} \\ m_1 & m_2 & \cdots & m_q \\ \vdots & \vdots & & \vdots \\ m_{q-1} & m_q & \cdots & m_{2(q-1)} \end{bmatrix}\begin{bmatrix} -a_0 \\ -a_1 \\ \vdots \\ -a_{q-1} \end{bmatrix} = \begin{bmatrix} m_q \\ m_{q+1} \\ \vdots \\ m_{2q-1} \end{bmatrix} \quad (19)$$

where

$$m_i = [M_i]_j \quad i = 0, 1, \cdots, 2q-1. \quad (20)$$

The residues of $\hat{X}_j(s)$ are determined by

$$\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_q \end{bmatrix} = -\begin{bmatrix} p_1^{-1} & p_2^{-1} & \cdots & p_q^{-1} \\ p_1^{-2} & p_2^{-2} & \cdots & p_q^{-2} \\ \vdots & \vdots & & \vdots \\ p_1^{-q} & p_2^{-q} & \cdots & p_q^{-q} \end{bmatrix}^{-1}\begin{bmatrix} m_0 \\ m_1 \\ \vdots \\ m_{q-1} \end{bmatrix}. \quad (21)$$

6 Numerical Laplace inversion method

In high frequency applications the interconnect impedance is partially determined by the conductor skin effect and the modified nodal matrix depens on the Laplace operator s. Using asymptotic waveform evaluation, modified nodal matrix is appproximated by Taylor series at $s = 0$. This approximation will be most accurate in the neighborhood of the dc region. Loss of accuracy will occur for high frequency region. The complex frequency hopping (CFH) is used to overcome this problem, but the approximated results are not always accurate. We use numerial Laplace inversion method to overcome this problem and obtain more accurate results.

When the j-the element $\hat{X}_j(s)$ of unknown vector $X(s)$ is obtained in s domain, we calculate its time solution $x_j(t)$ numerically by the following formula at given points t_n.

$$\hat{x}_j(t_n) = \frac{\exp(at_n)}{T}\left[\mathrm{Re}\sum_{k=0}^{K-1} \hat{X}\left(a + i\frac{k\pi}{T}\right)\exp\left(i\frac{2\pi nk}{K}\right) - \frac{\hat{X}(a)}{2}\right] \quad (22)$$

$$t_n = 2T\frac{n}{K} \quad n = 0, 1, 2, \cdots, K/2-1 \quad (23)$$

7 Frequency dependent loss due to skin effect

At lower frequencies the current flows uniformly over the cross section of the conductor. Therefore the resistance and the internal inductance of the conductor are constant and equal to their dc values . The skin depth of a conductor is inversely proportional to the square root of the frequency. At higher frequencies the skin depth becoms less than the cross section of conductor and the influence of the skin effect is no longer negligible. The per unit length frequency dependent loss of a conductor is given by

$$Z_c(s) = R + B\sqrt{s}. \tag{24}$$

The telegrapher's equations in the s domain will change as follows

$$-\frac{dV(x,s)}{dx} = [sL + Z_c(s)]I(x,t) \tag{25}$$

$$-\frac{dI(x,s)}{dx} = sCV(x,s) \tag{26}$$

and $T(s)$ of equation (10) can be modified as

$$T(s) = \exp\left\{-\begin{bmatrix} 0 & sL + Z_c(s) \\ sC & 0 \end{bmatrix} l\right\}. \tag{27}$$

8 Illustrative examples

Example 1

A single phase lossy transmission line of length l shown in Fig.1 is analyzed. The circuit has source and load resistances of 50Ω each. The source is a ramp function with 50ps rise time and rises to 1V and per unit length parameters are listed in Fig.1[2].

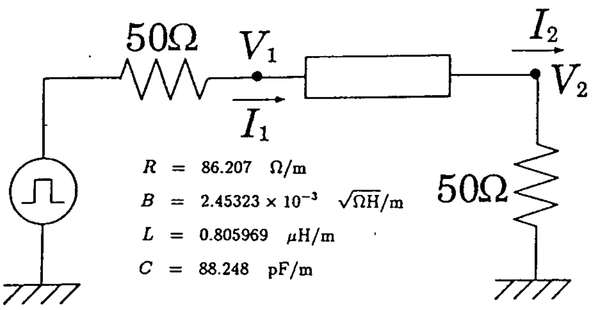

Fig.1 Circuit diagram of example 1

Calculated voltage response at far end is shown in Fig.2. This line has the delay time

$$T_d = \sqrt{LC} = 8.434\text{nsec} \tag{28}$$

per unit length, so wave front reaches at the far end with delay time 1.687ns. Calculated result by Laplace inversion shows this fact but results by AWE method do not show this time delay. Calculated results by two different methods did not show close agreement. It may be comcluded that Laplace inversion method gives more accurate result.

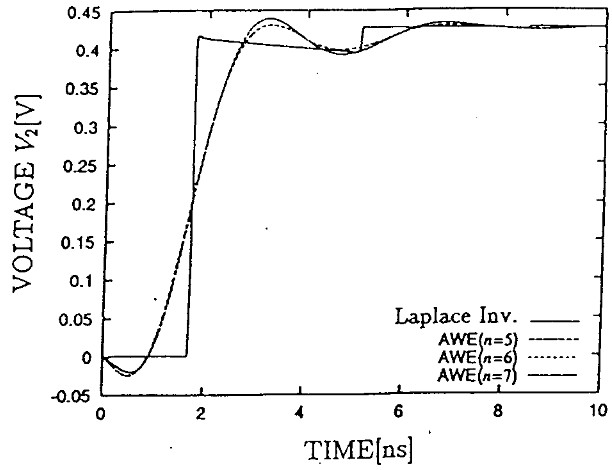

Fig.2 Voltage response at far end

Example 2

Three conductor lossy transmission line with length 15cm shown in Fig.3 was also analyzed. Source is the same wave form as examle 1.

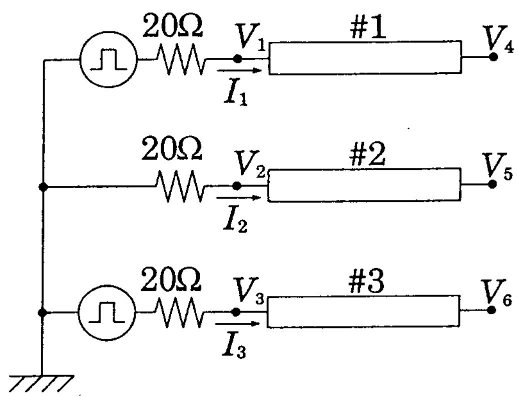

Fig.3 Circuit diagram of example 2

Per unit length parameters are[2]

$$R = \begin{pmatrix} 117 & 0 & 0 \\ 0 & 117 & 0 \\ 0 & 0 & 117 \end{pmatrix} \quad \Omega/\text{m} \tag{29}$$

$$B = \begin{pmatrix} 8.018 & 1.719 & 0.363 \\ 1.719 & 8.538 & 1.719 \\ 0.363 & 1.719 & 8.018 \end{pmatrix} \times 10^{-3} \quad \sqrt{\Omega\text{H}}/\text{m} \tag{30}$$

$$L = \begin{pmatrix} 327.7 & 67.6 & 14.8 \\ 67.6 & 323.6 & 67.6 \\ 14.8 & 67.6 & 327.7 \end{pmatrix} \quad \text{nH/m} \tag{31}$$

$$C = \begin{pmatrix} 134.8 & -28.8 & -3.2 \\ -28.8 & 146.1 & -28.8 \\ -3.2 & -28.8 & 134.8 \end{pmatrix} \quad \text{pF/m}. \tag{32}$$

Calculated results of V_4 at far end by two different methods are shown in Fig.4. Solid line shows the result by Laplace inversion and dotted line shows the result by AWE method with CFH. Close agreement was not obtained in this case.

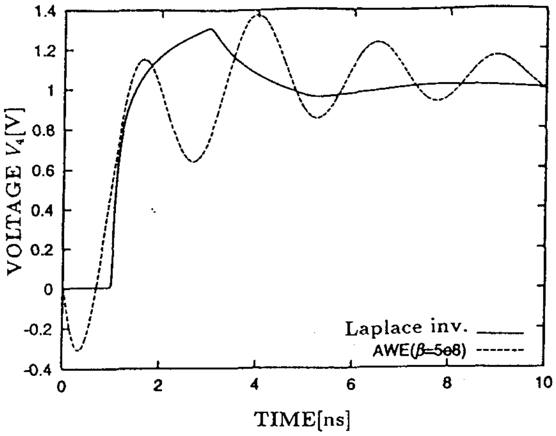

Fig.4 Voltage response of V_4

Example 3

Two conductor nonuniform transmission line shown in Fig.5 was analyzed. Source is the same waveform as example 1.

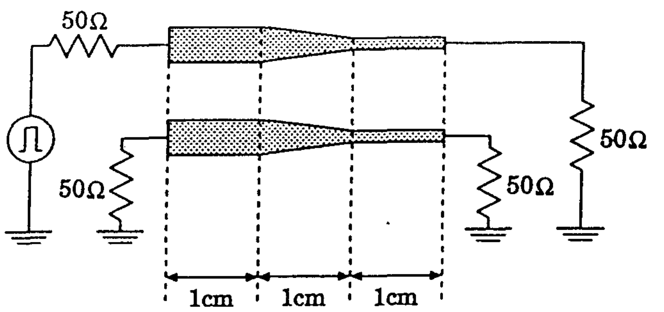

Fig.5 Nonuniform transmission line

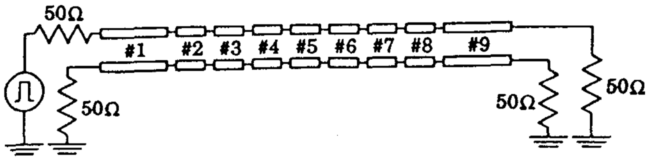

Fig.6 Cascade connection of uniform lines

Nonuniform line is approximated by the cascade connection of 9 short uniform lines as shown in Fig.6. Per unit length parameters are linted in Table 1. Calculated result at excited far end is shown in Fig.7.

9 Conclusion

In this brief, we gave a formulation for transient analysis of hybrid systems composed of lumped elements and frequency dependent lossy distributed interconnects. Two methods such as AWE approximation and numerical Laplace inversion have been presented. From the calculated examples it may be concluded that Laplace inversion method gives more accurate results.

Table 1 Per unit length parameters

	[cm]	$l_{11} = l_{22}$ [nH/cm]	$l_{12} = l_{21}$ [nH/cm]	$c_{11} = c_{22}$ [pF/cm]	$c_{12} = c_{21}$ [pF/cm]
#1	1.0	1.96	0.23	1.84	-0.09
#2	0.14	2.00	0.225	1.80	-0.0815
#3	0.15	2.13	0.21	1.68	-0.0615
#4	0.14	2.325	0.195	1.52	-0.0425
#5	0.14	2.56	0.18	1.36	-0.0295
#6	0.14	2.85	0.17	1.2	-0.0205
#7	0.15	3.225	0.165	1.04	-0.0145
#8	0.14	3.845	0.16	0.92	-0.0105
#9	1.0	3.71	0.16	0.88	-0.009

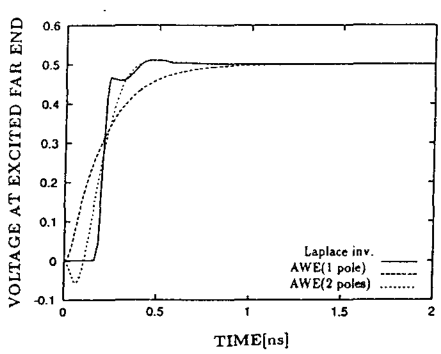

Fig.7 Voltage responce at excited far end

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