Proceedings of ITC-CSCC 2000, Pusan, Korea

A Visualization Method of Gravitational Lensing by Gravitational Sources in Three Dimensional Space

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Abstract: Computer graphics(CG) is one of the most useful tools by which we can easily understand visible/invisible natural phenomena. Namely, phenomena in the universe is attractive one because of its beauty and invisibility on the earth. For gravitational lensing phenomena, a ray tracing algorithm is proposed based on a lens-plane. In the method, position of gravitational sources is restricted. In this paper an improved visualization method is proposed, which can represent more complex situation than the previous method.

1. Introduction

Recently, computer graphics(CG) has been well-known as one of the most useful tools by which we can easily understand visible/invisible natural phenomena. Namely, since phenomena in the universe is attractive one because of its beauty and invisibility on the earth, the visualization for them by CG is very effective. Therefore, many visual simulation methods have been proposed[1] [2][3][4]. Among the phenomena, in this paper we consider the so-called gravitational lensing. It is occurred by gravitational sources such as black holes that light curves in their neighbor regions. As the result, we can see stars behind the gravitational sources as different from the ones without the sources. The greater the number of gravitational sources is, the more complex the observation image is. Also the gravitational lensing often causes the phenomena by which we observe the brighter image than the one without the sources and the observation image has time interval[5]. This phenomena is interesting to be visualized by CG.

For such phenomena, methods based on the so-called ray tracing algorithm have been proposed [1][4]. It is well-known that the ray tracing algorithm is one of the most useful rendering method to generate photo-realistic CG images[6]. Kishi proposed a method using a "lens-plane", on which all gravitational sources are located[4]. In the method, each ray bends only once on the plane, so that CG images can be generated in low computational cost. However, since the method requires all gravitational sources to be on the same lens-plane, the viewpoint is restricted as well as the location of the gravitational sources.

In order to loosen the restriction, in this paper we shall propose a visual simulation method based on the ray tracing algorithm considering plural lens-planes. By the proposed method images of the gravitational lensing can be generated from any viewpoint.

2. A Visualization of Gravitational Lensing

2.1 Outline

In this paper we consider a case when the distance between the observer and the gravitational sources and the distance between the object and them are sufficiently big, respectively. Then a ray curving from the light source to the observer can be approximated by two lines. In order to trace the ray by the ray tracing algorithm, in this paper, a lens-plane is defined, on which each gravitational source locates. When the ray from the observer intersects the lens-plane, new direction is determined by the gravitational source. From the above, the gravitational lensing can be represented based on the ray tracing algorithm. The proposed method consists of two parts; definition of the lens-plane and refraction of the ray. In the proceeding subsection, we shall explain precisely.

2.2 Definition of Lens-plane

Lens-plane $l_i, i = 1, 2, \ldots$, is defined for gravitational sources $g_i, i = 1, 2, \ldots$, as follows, respectively:

- l_i parallels the screen at ray tracing algorithm.
- g_i locates on l_i.

If l_i and l_j are same, remove one of them, so that a lens-plane may have plural sources (see **Figure 1**).

2.3 Refraction of Ray

Let $\boldsymbol{Q}_i$ be position where the ray $\boldsymbol{V}_{ray}$ intersects the lens-plane l_i and $\boldsymbol{P}_g$ be position of the gravitational source g_i on l_i. In order to determine new direction $\boldsymbol{V}_{new}$ after intersection, the following is proposed (see **Figure 2**):

Let $\boldsymbol{V}_g$ be:

$$\boldsymbol{V}_g = \frac{\boldsymbol{P}_g - \boldsymbol{Q}}{|\boldsymbol{P}_g - \boldsymbol{Q}|}. \quad (1)$$

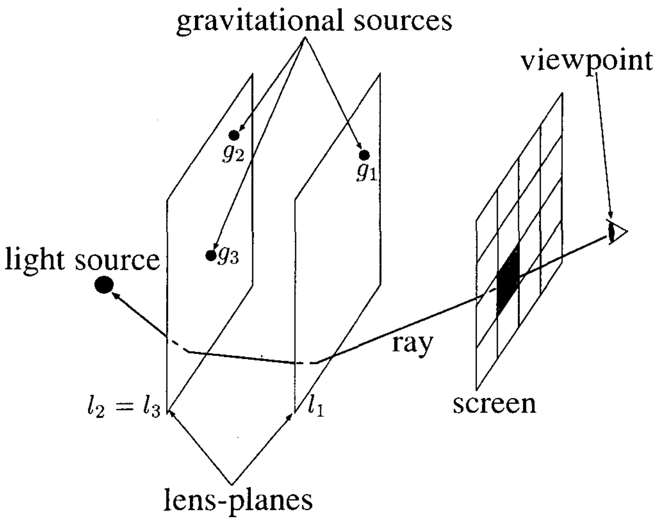

Figure 1. a ray bended by lens-planes

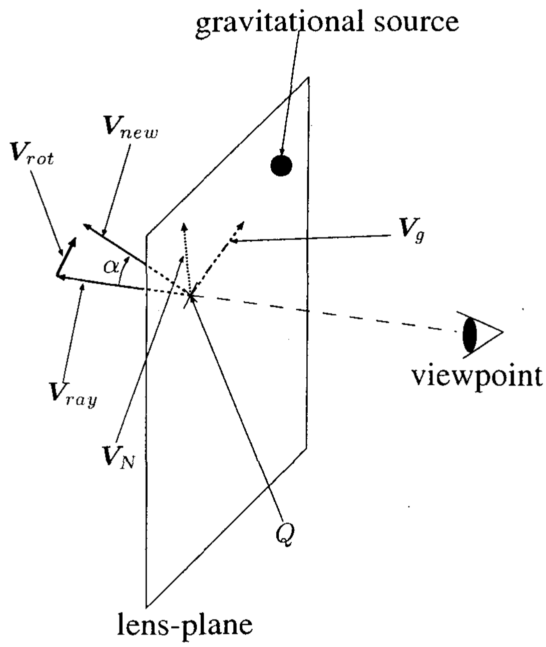

Figure 2. Refraction of Ray

Let $\boldsymbol{V}_N$ be outer product between $\boldsymbol{V}_g$ and $\boldsymbol{V}_{ray}$:

$$\boldsymbol{V}_N = \frac{\boldsymbol{V}_g \times \boldsymbol{V}_{ray}}{|\boldsymbol{V}_g \times \boldsymbol{V}_{ray}|}. \tag{2}$$

Also let $\boldsymbol{V}_{rot}$ be outer product between $\boldsymbol{V}_{ray}$ and $\boldsymbol{V}_N$:

$$\boldsymbol{V}_{rot} = \frac{\boldsymbol{V}_{ray} \times \boldsymbol{V}_N}{|\boldsymbol{V}_{ray} \times \boldsymbol{V}_N|}, \tag{3}$$

From general relativity, angle of refraction α[rad] is given as follows[7]:

$$\alpha = \frac{4GM}{bc^2}, \tag{4}$$

b[m] : the distance between D and the gravitational source,
c[m/s] : the velocity of light,
G[Nm^2/Kg^2] : the gravitational constant,
M[Kg] : the mass of the gravitational source.

From $\boldsymbol{V}_{rot}$ and α, displacement $\boldsymbol{v}$ is given as:

$$\boldsymbol{v} = \tan\alpha \frac{\boldsymbol{V}_{rot}}{|\boldsymbol{V}_{rot}|}. \tag{5}$$

Then, the new direction $\boldsymbol{V}_{new}$ is obtained as:

$$\boldsymbol{V}_{new} = \frac{\boldsymbol{v} + \boldsymbol{V}_{ray}}{|\boldsymbol{v} + \boldsymbol{V}_{ray}|}. \tag{6}$$

If there are other gravitational sources on the lens-plane, let $\boldsymbol{V}_{new}$ be $\boldsymbol{V}_{ray}$ and repeat bending of the ray for all of them. For example, in Figure 3, $\boldsymbol{V}_{ray}$ becomes $\boldsymbol{V}_{new1}$ by bending at α_1(α_1 is the angle of refraction for g_1), and $\boldsymbol{V}_{new1}$ becomes $\boldsymbol{V}_{new2}$ by bending at α_2(α_2 is the angle of refraction for g_2).

2.4 The Proposed Algorithm

The proposed algorithm is as follows:

STEP1 For initial data, set location of viewpoint, celestial bodies, screen and gravitational sources. And let celestial luminance be $\boldsymbol{I}_c$.

STEP2 Define lens-plane for each gravitational source. If any lens-planes are same, then define one plane for the related gravitational sources.

STEP3 For arbitrary pixel on the given screen, define initial ray, initialize the luminance($\boldsymbol{I} = \mathbf{o}$) and execute the ray tracing algorithm.

STEP4 Calculate the intersection for all objects. If the ray intersects no object, let $\boldsymbol{I}_c$ be:

$$\boldsymbol{I}_c = \mathbf{o}, \tag{7}$$

and go to **STEP5**. Else if the ray intersects lens-plane, go to **STEP6**. Else if the ray intersects celestial body, go to **STEP7**.

STEP5 Add $\boldsymbol{I}_c$ to $\boldsymbol{I}$:

$$\boldsymbol{I} = \boldsymbol{I} + \boldsymbol{I}_c, \tag{8}$$

where $\boldsymbol{I}$ is color of the pixel. Then proceed to the next pixel.

STEP6 Calculate the refraction angle α by formula(4) and define a new ray $\boldsymbol{V}_{new}$ from ray refraction algorithm and $\boldsymbol{V}_{ray}$.

STEP7 Calculate I_c at the intersection. Then go to **STEP5**.

STEP8 For all pixels on the screen, execute from **STEP3** to **STEP7**.

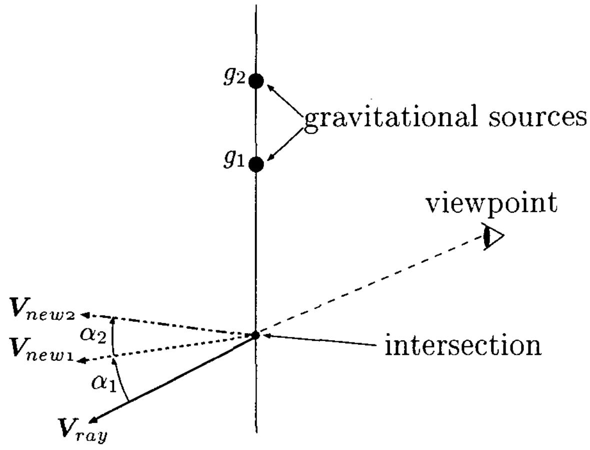

Figure 3.

3. Simulation

In all simulation, the mass of gravitational source is 1.989×10^{31}[Kg]. The sphere in the right side in each oblong image shows the location of gravitational source and is not related to the simulation.

Figure 4. an image of the earth without gravitational source

Figure 5. an image of earth influenced by gravitational lensing (1)

Figure 6. an image of earth influenced by gravitational lensing (2)

For reference, Figure 4 shows an image without gravitational source. Figure 5 and Figure 6 show images of gravitational lensing by three gravitational sources. In Figure 5, gravitational sources are in a line but they are not in Figure 6. We can see a small ring inside the large one in Figure 5. Such ring is called as "Einstein ring". Also in Figure 6, we can see an arc without gap unlike Kishi's method. Figure 8 shows horizontal movement of three gravitational sources and Figure 7 shows an image without them. We can see a small ring and a small arc in the center of image. It is concluded that the proposed method can represent more complex action of light and more correct image than the previous method.

Let N and N_g be the number of rays(pixels) which intersect the celestial body without gravitational source and with gravitational source, respectively. Then amplification factor A is obtained as follows:

$$A = \frac{N_g}{N}. \tag{9}$$

Table 1 shows computational time and amplification factor in each simulation under the 440MHz Ultra SPARK II i system. We can see that the nearer to ring the image is, the bigger the amplification factor is.

Table 1. Computational Time and Amplification Factor

Figure	①	②	③
5	4.46	2.65	800 × 400
6	2.50	2.39	800 × 400
8	1.92 2.67 3.89 4.34	total 10.11	each image size 800 × 400

① Amplification Factor
② Computational Time[s]
③ Image Size[horizontal pixels × vertical pixels]

4. Conclusion

In this paper, we proposed a visualization method of gravitational lensing by gravitational sources in three dimensional space by computer graphics. Kishi's method can represent gravitational lensing by gravitational sources which are located on a plane without complex calculation. The proposed method in this paper capacitates us execute the visual simulation whose input data is more complex than the one at his method and visualize the gravitational lensing by gravitational sources from arbitrary viewpoint unlike his method. Considering the use in education, it is concluded that if given data is near the one in previous simulation, this method can be applied for computational time.

In a future paper, the following should be considered:

- Reduction of computational time
- Representation of the phenomena which we can see the back of object

Acknowledgement

This research is partially supported by the Research Project on "High Quality Solid Modeling and Visualization in Computer Graphics", at the Institute of Science and Engineering, Chuo University.

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Figure 7. no gravitational source

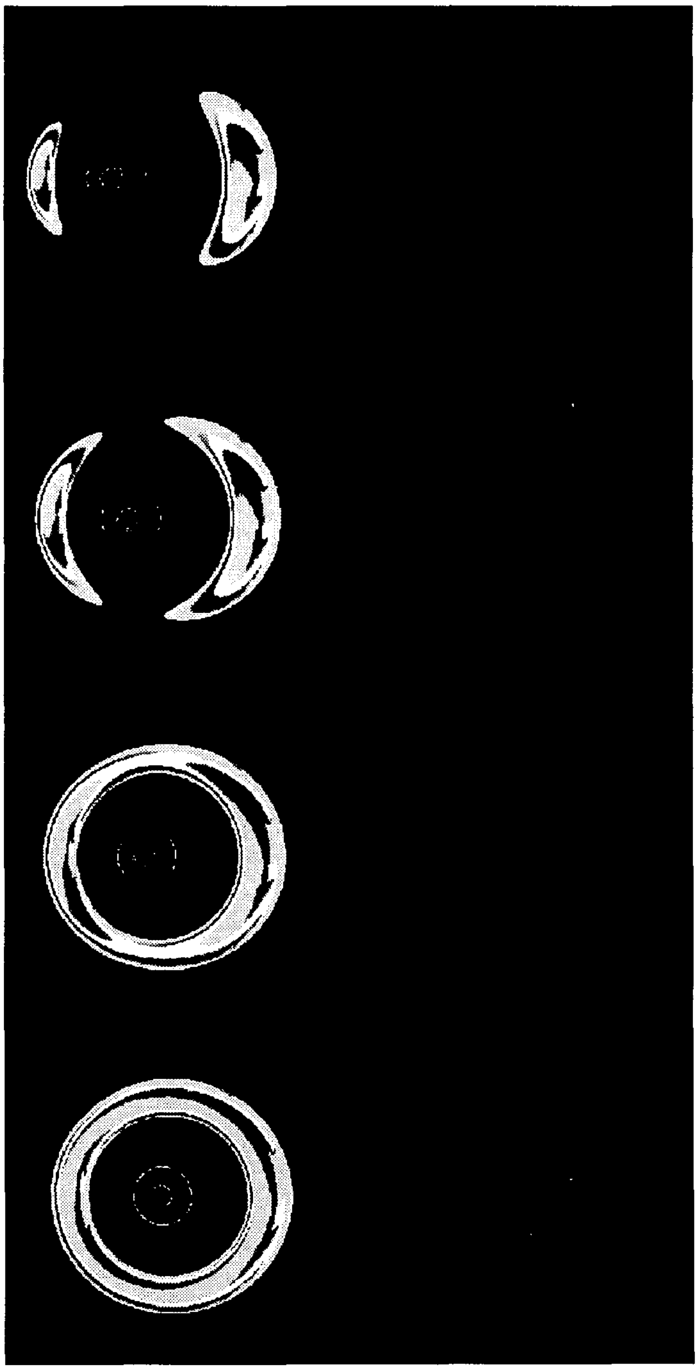

Figure 8. movement of three gravitational sources