On Reducing Estimation Error Caused by Variable Sampling Rate

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Abstract: In this paper, we show that a variation in sampling rate give rise to system performance degradation and propose a method to effectively reduce the error. We first capture the variation as a first order autoregressive (AR) model and project it as an additional sensor measurement noise. By considering that the sensor measurements include correlated noise, we perform a decorrelation process and then apply a standard Kalman filter (SKF) to estimate the target-state. As a result of the two-step procedure, we achieve a significant reduction in the target state estimation error.

1. Introduction

To achieve the best possible solution in target tracking problem has been a great deal of interest in both defense and civilian application [1][2]. Several issues have been regarded as the principal topics in relation to the target tracking such as the number of targets, the target's maneuvering, and the environmental effects on tracking systems [3]. Yet, past research has overlooked a tangible error source, the variation in sampling rate. There has been a lack of concern on the error that can occur during measurement data acquisition process. Preceding studies have tended to take the measurement data for granted and assumed that it is acquired exactly at a predetermined sampling time. However, this is an ideal case. In practice, the presence of variation in sampling rate is real and can cause a significant error. For example, the sensor's physical factors such as thermal noise, acoustic anomalies, electromagnetic interference, etc. can lead to perturbations in sampling rate. In this paper, we postulate that one of the sources of error is caused by the variation in sampling rate when taking measurements. We then explore the influences in variation of sampling rate to target tracking error that may have been overlooked as an item for possible improvement.

The perturbation in sampling rate simply adds to the measurement noise in the form of correlated noise. With this measurement modeling, we show how variations in sampling rate can degrade the performance of a target tracking system. We then propose a corrective filter focused to solve the problem using a decorrelation process.

This paper is organized as follows. In Section 2, we introduce the AR model that captures a variation in sampling rate and suggest a new sensor measurement model incorporating its perturbing effect. Section 3 presents the decorrelation process as a solution that reduces the unexpected error caused by the perturbation in sampling rate. This work is applied to tracking a fast moving target (e.g. fixed wing aircraft) with a constant velocity (Section 4). Section 5 presents the conclusions of this paper and suggests the remaining issues for further investigation.

2. Modeling of Variations

In general, sensor measurements are acquired at a predetermined sampling interval. This assumption, however, is based on an ideal case. In real applications, variations in sampling rate can take place abruptly and these variations give rise to unexpected measurement error. When a moving target is being observed with a sampling rate T, the possible observation error caused by a variation Δ in T is shown in Fig 1. In Fig 1, α denotes the target's constant velocity.

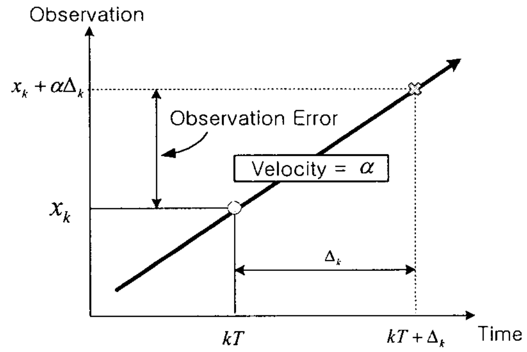

Fig. 1 Variations in sampling rate

We can assume that the variations in sampling rate are originated from the same source in a specific observation window. As a result of this assumption, we can represent the variation at time k as a first order AR model as follows:

$$\Delta_k = \lambda\Delta_{k-1} + \eta_k \tag{1}$$

where Δ_k is a variation of sampling rate at time k
λ is a correlation coefficient
η_k is zero-mean white Gaussian noise
with covariance $E[\eta_k \eta_l^T] = R_\eta \delta_{kl}$.

Sensor measurement data is directly related with sampling rate. It is a reasonable postulation that the variations in sampling rate are projected linearly to variations in observation data. With this postulation, let Δ_k be a variation in sampling rate and p_k be a variation in observation data caused by sampling time changes at time k. Then the relationship between sampling rate and sensor measurement data can be described as follows:

$$p_k = \alpha\Delta_k \tag{2}$$

$$p_k = \lambda p_{k-1} + \alpha\eta_k \quad (3)$$

where α is a velocity of a target.

The unexpected variations in sampling rate and their effect in sensor measurement data are shown in Fig 2. The undesirable effects of perturbations in sampling rate can be captured by the modified sensor measurement data Z_k, in the form of an additional correlated noise term. Provided that the target is moving with a constant velocity and the sensor measurement comprises the positions of target, the observed sensor measurements can be modeled by:

$$\begin{aligned} Z_k &= HX_k + p_k + v_k \\ &= HX_k + \alpha\Delta_k + v_k \end{aligned} \quad (4)$$

$$X_k = \begin{bmatrix} x_k \\ v_k \end{bmatrix} = \begin{bmatrix} \text{Postion of a target} \\ \text{Velocity of a target} \end{bmatrix} \quad (5)$$

where Z_k is a sensor measurement at time k,
X_k is a target state vector,
H is a measurement matrix,
α is a target velocity,
Δ_k is perturbation in a sampling rate at time k,
v_k is a measurement noise.

The measurement noise is zero-mean white Gaussian and its covariance is $E[v_k v_l^T] = R_v\delta_{kl}$, while η_k and v_k are assumed to be uncorrelated.

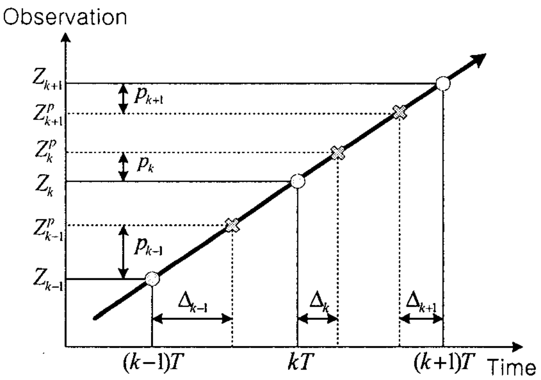

Fig. 2 Unexpected observation error

3. Decorrelation Process

When applying the SKF for the state estimation of a target with the sensor measurements described above, we need to decorrelate the noise of the sensor measurements. Without considering the correlation aspect of measurement noises, the SKF becomes unsuitable for estimating the target states in its present form. To make the SKF become effective in the process with undesirable correlated noise, we apply a decorrelation process before filtering [4, 5, 6]. As a first step to decorrelate the measurement noise, we define Y_k as the modified measurement as follows:

$$Y_k = Z_k - \lambda Z_{k-1} \quad (6)$$

The sensor measurements in the SKF can be expressed as:

$$Z_k = HX_k + v_k \quad (7)$$

where H is a measurement matrix, and
X_k is a state vector of a target at time k.

Rearranging the above equations (4) ~ (7), we obtain the new measurement equation as:

$$\begin{aligned} Y_k &= Z_k - \lambda Z_{k-1} \\ &= (HX_k + \alpha\Delta_k + v_k) \\ &\qquad - \lambda(HX_{k-1} + \alpha\Delta_{k-1} + v_{k-1}) \\ &= H(I - \lambda\Phi^{-1})X_k + \alpha(\Delta_k - \lambda\Delta_{k-1}) \\ &\qquad + v_k - \lambda v_{k-1} + \lambda H\Phi^{-1}\Psi\xi_{k-1} \\ &= H(I - \lambda\Phi^{-1})X_k + \alpha\eta_k + v_k - \lambda v_{k-1} \\ &\qquad + \lambda H\Phi^{-1}\Psi\xi_{k-1} \end{aligned} \quad (8)$$

Then

$$Y_k = \bar{H}X_k + \bar{v}_k \quad (9)$$

where

$$\bar{H} \equiv H(I - \lambda\Phi^{-1}) \quad (10)$$

$$\bar{v}_k \equiv \lambda H\Phi^{-1}\Psi\xi_{k-1} + \alpha\eta_k + v_k - \lambda v_{k-1} \quad (11)$$

Note that Φ represents the state transition matrix, Ψ a noise gain matrix, and ξ_{k-1} a process noise. The process noise is zero-mean white Gaussian noise with known covariance such that $E[\xi_k\xi_l^T] = R_\xi\delta_{kl}$. Also the process noise is uncorrelated with measurement noise. The new measurement noise $\bar{v}_k$ is desired to be white, but it is currently correlated with the process noise ξ_{k-1}. In most practical situations, the new measurement noise can be regarded as white with little degradation in performance since the first term is very small. As shown in above equations, after decorrelating the correlated measurement noise, we can obtain the uncorrelated noise. As a result, the SKF can be applied to the case with sensor measurements having additional noise term due to perturbation, by simply substituting Z_k, H, and v_k with Y_k, $\bar{H}$, and $\bar{v}_k$ respectively.

4. Computer Simulations

Computer simulations are performed to demonstrate the influence of variations in sampling rate and to compare the system performance between the SKF with and without the decorrelation process. For experiments, we assume a scenario with a target (fast moving fixed wing aircraft) moving at constant velocity 100 m/sec and the sampling rate is one every 0.05 second. The entire time duration in which the sampling rate variations occur is 60 seconds (1200 samples). The variance in measurement noise and the variance in process noise are 100 m^2 and 1 m^2 respectively. The correlation coefficient is assumed to be $\lambda = 0.6$ and 0.8. The Monte Carlo simulations with 50 runs in each simulation are performed. The performance of the filtering is evaluated by calculating the root mean square error (RMSE) as follows:

$$RMSE(k) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(X_k - \hat{X}_k^i)^2} \qquad (12)$$

$\hat{X}_k^i$ is the i th estimate of target-state at time k in Monte Carlo simulations and N is the total number of simulations. Figure 3 and 4 show how the unexpected perturbations in sampling rate influence on the performances of the tracking systems.

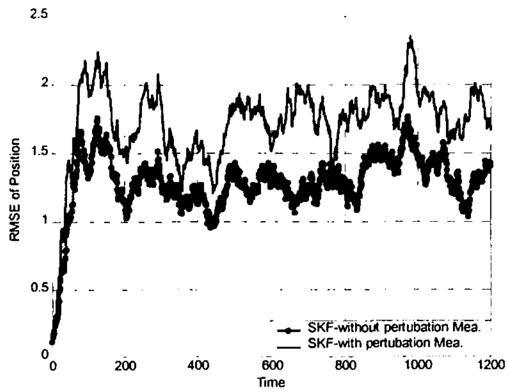

Fig. 3 RMSE of Position (λ = 0.6)

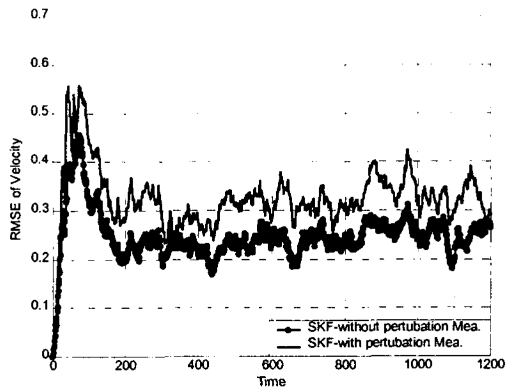

Fig. 4 RMSE of Velocity (λ = 0.6)

The system performance comparisons between with and without decorrelation process are shown in Figures 5, 6, 7, and 8 with λ being 0.6 and 0.8 each, respectively. The results clearly show that the SKF with decorrelation process produce reduced RMSE than those of the SKF without decorrelation process. In the SKF algorithm where variations in sampling rate are not considered, the target-state estimation error is increased. The results shown by the figures indicate that we can reduce the effects of the undesirable additional noise caused by variations in sampling rate, by performing a decorrelation process as described.

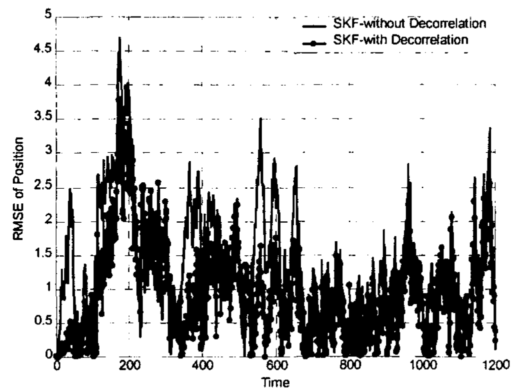

Fig. 5 RMSE of Position (λ = 0.6)

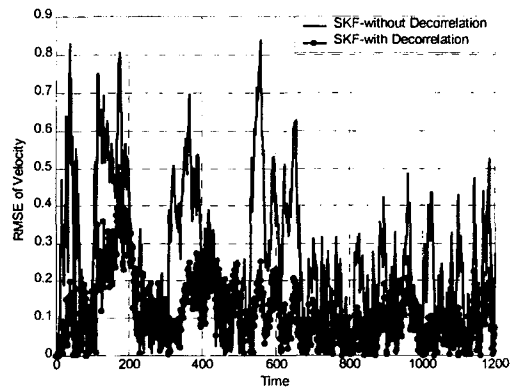

Fig. 6 RMSE of Velocity (λ = 0.6)

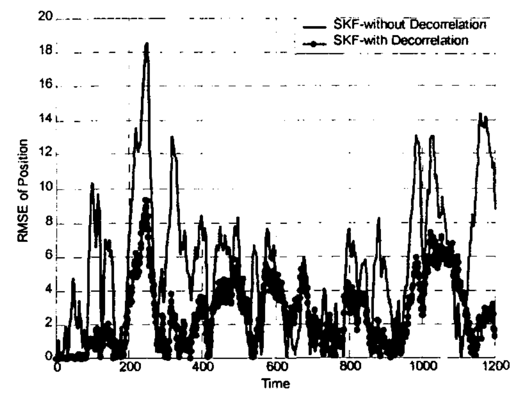

Fig. 7 RMSE of Position (λ = 0.8)

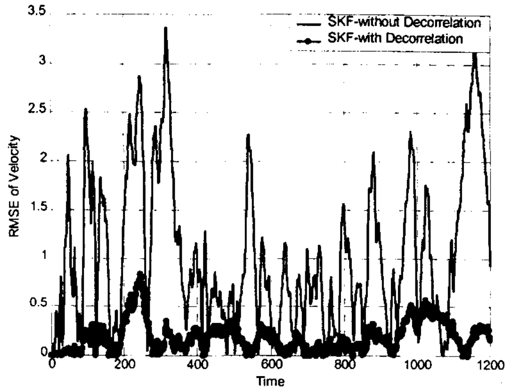

Fig. 7 RMSE of Velocity (λ = 0.8)

5. Conclusions

We explored the influences in the variation of sampling rate to target tracking error by showing how variations in sampling rate can degrade the performance of a target tracking system. We projected the variations to sensor measurement data and defined a modified measurement model. We then designed a corrective filter that performs a decorrelation process, which significantly reduced the RMSE. Our simulation results showed that the variations in sampling rate indeed causes the system performance degradation. We also verified that the resulting tracking error can be reduced by performing a decorrelation process.

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