On Optical Power Distribution of Grating-Assisted Couplers with Three-Guides

Kwang-Chun Ho*, Yung-Kwon Kim**
*: Halla University, **: Kon-Kuk University
San 66, Heungup, Wonju, KangwonDo
Tel: +82-371-760-1247, Fax: +82-371-760-1251
kwangho@hit.halla.ac.kr

Abstract

The coupling properties of supermodes guided by grating-assisted directional couplers (GADCs) can be phrased in rigorous modal theory. Such a modal solution for TE modes expressed by simple electrical transmission-line networks is utilized to analyze the power distribution of GADCs with three guiding channels. In particular, the modal transmission-line theory can serve as a template for computational algorithms that systematically evaluate the coupling efficiency that are not readily obtained by other methods.

1. Introduction

Grating-assisted coupling structures are increasingly used for many applications in the field of photonics, especially, such as optical power distribution for wavelength division multiplexing and optical switching. One of the most widespread approaches adopted for evaluating these configurations is the coupled-wave theory [1], permitting simply physical intuition as well as analytical solutions for power coupling among modes in optical waveguides. Recently, although more developed approaches [2] to mode-coupling phenomena are presented for the solution of problems that involves parallel waveguides consisted of three-guides with gratings, the models may not be sufficient to explore in detail the design characteristics of optical power distributor supported by the superposition of multi-modes.

To achieve this objective, we propose and develop newly an equivalent network approach, which is based on modal transmission-line theory [3], to distribute equally the incident power through the output guiding channels in optical grating-assisted directional couplers with three-guides.

A typical grating-assisted geometry with three-guiding channels applicable to the proposed approach and the corresponding equivalent transmission-line network are illustrated in Fig. 1. As can see, the coupler has three-guides so that only three propagating supermodes are the significant meaning and dominate the power coupling of GADCs, being described by three equivalent networks in coupling region as shown in Fig. 1(b). Furthermore, InP/InGaAsP materials compose the coupler and their refractive indices are calculated by using single-effective-oscillator model [4]. Then, the input power incident into one of three-guides at z=0 generates three guiding supermodes in coupler and after travelling an optimized

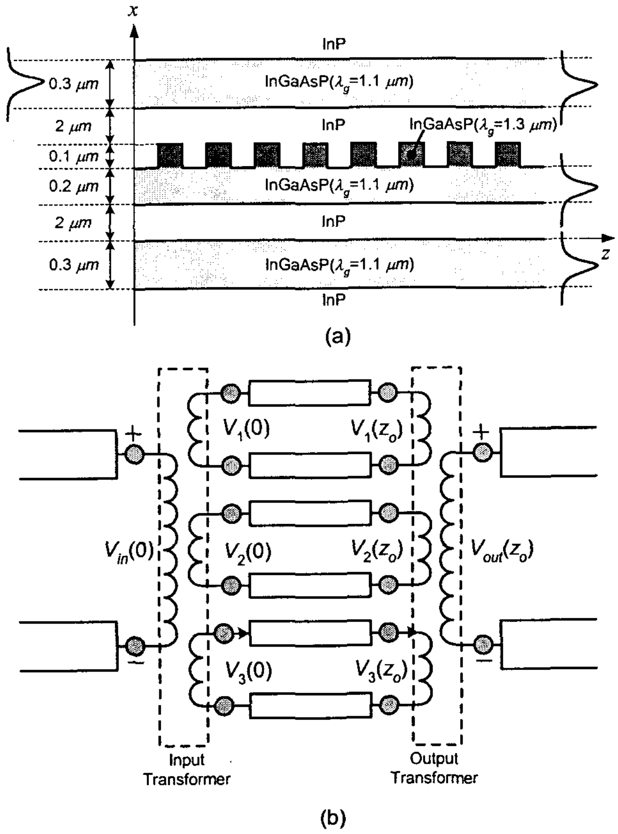


Fig. 1. Grating-assisted directional coupler with three-guides: (a) schematic configuration showing the power distribution and (b) the equivalent transmission-line network.

coupling length $z=z_0$ the coupler distributes equally the output power through three-guides.

2. Modal Characteristics of GADCs

For the composite corrugation structure pictured in Fig. 1, the complex propagation constant (eigenvalues) $k_{zn}=k_{zo}+2n\pi/\Lambda$ with $k_{zo}=\beta+i\alpha$ (where n and Λ represent the space harmonics and the periodicity of grating, respectively) can be calculated by applying the transverse resonance condition of modal transmission-line theory [3]

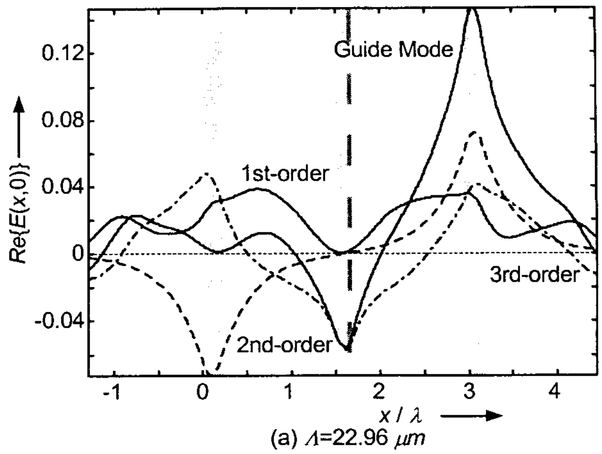
$$|\mathbf{Y}_{\mathbf{up}}+\mathbf{Y}_{\mathbf{dn}}|=0 \quad , \qquad (1)$$

where $\mathbf{Y}_{\mathbf{up}}$ and $\mathbf{Y}_{\mathbf{dn}}$ indicate the admittance square matrices looking up and down at an arbitrary j-th layer boundary on x-axis, respectively. The unknown eigenvalue k_{zn} is then related to all the functional quantities included in Eq. (1), and the rigorous three supermodes guided in GADCs are determined from the dispersion relation given in Eq. (1). Once determined the quantity k_{zn}, the fields E and H of the pertinent propagating waves at any point (x, z) inside the periodic interval $0 \le z \le z_o$ can be precisely defined for homogeneous regions and for inhomogeneous (periodic) regions [3, 5].

Such field distributions of the three rigorous modes are shown in Fig. 2, where the grating periods are intentionally selected and the aspect ratio is 0.5. As shown in that figure, we obtain two symmetric (no zero-crossings or two zero-crossings of the electric field distribution between the upper and lower guides) solutions, and one anti-symmetric (a single zero-crossing of the electric field distribution between the two guides) solution for rigorous modes.

Especially, as shown in Fig. 2, if we choose a suitable grating period (here Λ=18.99 μm at an operating wavelength λ=1.55 μm), the guiding mode given by the superposition of supermodes may almost concentrate in the upper guiding channel. This means that the power transmitted through output channels can be equally distributed at Λ=18.99 μm rather than at the other periodicity after propagating arbitrary coupling length z_o. It happens because the residual components concentrated in the center and lower guides causes the power distribution to deteriorate, which will be discussed in detail at Section 3.

Consequently, the three significant modes, which is called supermodes, decay or grow exponentially due to the loss or gain of the guiding structures and is transmitted



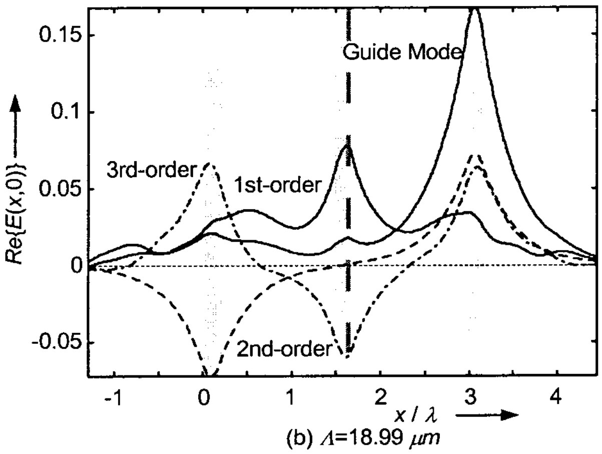


Fig. 2. Distribution of supermodes excited at input boundary for two different periodic (a) Λ=22.96 μm, and (b) Λ=18.99 μm selected.

through the output guiding channels at $z=z_o$.

To see the power transfer between three-guides of GADCs, we assume that a wave is incident into upper guiding channel. For TE modes propagating in homogeneous stratified waveguides, the transverse electric E_y and magnetic H_x fields at the input ($z<0$) and the output ($z>z_0$) regions are expressed as [10]

$$E_\xi(x,z)=V_\xi(z)e_\xi(x)\,,\; H_\xi(x,z)=-I_\xi(z)h_\xi(x), \quad (2)$$

where the modal voltage V_ξ and current I_ξ are related by

$$\frac{V_\xi}{I_\xi}=\frac{\omega\mu}{k_{z,\xi}}$$

with the propagation constant $k_{z,\xi}$ designated $\xi = in$ or out for the input or output region, respectively. Here, e_ξ and h_ξ denote the electric and magnetic modal functions in uniform stratified guides.

Then, if we neglect the reflections at the input and output junction boundaries, the total field in the grating-assisted coupling ($0 \le z \le z_0$) region can be written by a linear superposition of three propagating rigorous modes

$$E_c(x,z) = V_1(z)\sum_n e_{jn}^{(1)}(x)e^{i(2n\pi/\Lambda)z} + V_2(z)\sum_n e_{jn}^{(2)}(x)e^{i(2n\pi/\Lambda)z} + V_3(z)\sum_n e_{jn}^{(3)}(x)e^{i(2n\pi/\Lambda)z} \quad , \tag{3}$$

where the basis modal voltage is $V_\nu(z) = V_{0,\nu}\, e^{ik_{z0,\nu}z}$, for which the propagation constant $k_{z0,\nu}=\beta_\nu+i\alpha_\nu$ with ν=1, 2 or 3 designates three lowest-order modes, and $e_{jn}^{(\nu)}(x)$ represents the spatial variation of modes along x-direction.

The field incident into the junction boundary z=0 from the upper guide generates three rigorous modes, being guided by the periodic region where they propagate independently along the longitudinal z-direction. Then, the boundary conditions at z=0 with neglecting the facet reflections give us the following identity happened between the input (homogeneous) and coupling (inhomogeneous) regions.

$$V_{in}(0)e_{in}(x) \cong V_1(0)\sum_n e_{jn}^{(1)}(x) + V_2(0)\sum_n e_{jn}^{(2)}(x) + V_3(0)\sum_n e_{jn}^{(3)}(x) \quad . \tag{4}$$

Then, performing cross-product in Eq. (4) with

$$\sum_r k_{zr,\nu} e_{jr}^{(\nu)}(x) \quad \text{for } k_{zr,\nu} = k_{z0,\nu} + \frac{2r\pi}{\Lambda} \quad ,$$

and integrating over the cross section (cs) of guiding structure, the modal voltages at input boundary satisfying the field orthogonality condition [5] of rigorous modes are found to be

$$V_1(0) = A_1 V_{in}(0) \quad , \quad V_2(0) = A_2 V_{in}(0) \quad , \tag{5}$$

where the input transformation constant A_ν are given by

$$A_\nu = \frac{1}{C_\nu}\int_{cs}\left\{e_i(x)\sum_r k_{zr,\nu} e_{jr}^{(\nu)}(x)\right\}dS$$

with the appropriate normalization constant C_ν determined by the power normalized condition of GADCs considered.

The orthogonal rigorous modes excited at the input interface (z=0) propagate along the longitudinal z-direction, and the modal voltages $V_\nu(z_0)$ at an arbitrary accessible terminal $z = z_0$ decay exponentially in terms of the leakage losses α_ν. However, if the sign of α_ν changes from positive to negative, meaning that the coupler serves as an active device rather than passive one, the modal functions grow exponentially at the optical coupling region and are transmitted through the upper or lower channel. Then, the boundary condition at an arbitrary output terminal z=z_0 yields

$$V_{out}(z_0)e_{out}(x) \cong V_1(z_0)\sum_n e_{jn}^{(1)}(x)e^{i(2n\pi/\Lambda)z_0} + V_2(z_0)\sum_n e_{jn}^{(2)}(x)e^{i(2n\pi/\Lambda)z_0} + V_3(z_0)\sum_n e_{jn}^{(3)}(x)e^{i(2n\pi/\Lambda)z_0} \tag{6}$$

for the modal fields traveling at the output and coupling regions. Thus, applying the power normalization of output modal fields

$$\int_{cs} e_{out}(x)h_{out}^*(x)\,dS = 1 \tag{7}$$

to Eq. (6), the output modal voltage $V_{out}(z_0)$ can be expressed as

$$\frac{V_{out}(z_0)}{V_{in}(0)} = A_1B_1e^{ik_{z0,1}z_0} + A_2B_2e^{ik_{z0,2}z_0} + A_3B_3e^{ik_{z0,3}z_0} \equiv T_{te} \tag{8}$$

where T_{te} describes the coupled transfer factor between the input and output modal voltages, and the output transformation coefficient B_ν is

$$B_\nu = \int_{cs}\left\{h_{out}^*(x)\sum_n e_{jn}^{(\nu)}(x)e^{i(2n\pi/\Lambda)z_0}\right\}dS \quad .$$

The equivalent transmission-line network illustrating pictorially the electromagnetic analysis procedure presented above for GADCs is given in Fig. 1(b). Consequently, we can define a convenient formalism to analyze the power transfer of TE modes, which is called coupling efficiency η. The coupling efficiency represents the ratio of the output power ($V_{out}I_{out}^*$) to the input power ($V_{in}I_{in}^*$), which is determined from Eq. (8)

$$\eta_{TE} = \frac{P_{out}}{P_{in}} = \frac{\mathrm{Re}(k_{z,out})}{\mathrm{Re}(k_{z,in})}|T_{te}|^2 \quad , \tag{9}$$

where $k_{z,in}$ and $k_{z,out}$ are the propagation constants at the input and output regions, respectively.

3. Numerical Results and Discussion

To search the condition for identical power distribution between three guiding channels of GADCs, we explore the coupling efficiency η as a function of the propagation distance z under the selected grating periods.

As shown in Fig. 3, there exists no coupling length to distribute equally the power incident into GADCs in the case of Λ=22.96 μm, but the most of power (over 90%) is transferred to the lower guide at $z \approx 700$ μm.

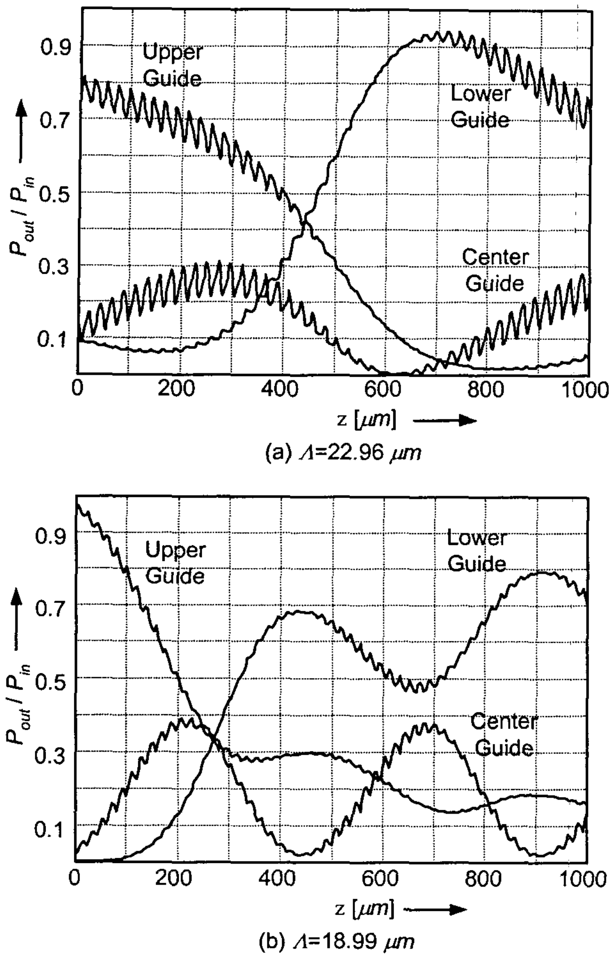


Fig. 3. Coupling efficiency as a function of propagation distance *z*, where power is incident into upper guide.

This fact indicates that we can obtain the effective power transfer between the two outer guides governed by the interference of supermodes, whose fluctuation is sensitively controlled by the grating period.

On the contrary, under such a suitable grating period as Λ=18.99 μm, the incident power is partitioned through three output channels contained about 33% power, respectively, at coupling length $z \approx 270$ μm.

Throughout this paper, we do not refer to how the condition of grating period is determined, in fact, this condition is attributed to an excitation ratio defined newly for supermodes generated at the input junction boundary z=0. The detailed information will be reported as full paper later.

These features can not be readily explained by conventional simplified approaches. Because it is difficult to get the condition of equal power distribution dependent on the synchronism of three kinds of supermodes generated in coupler, which is the criterion of conventional approaches to define the coupling efficiency of GADCs. However, it is consistent with the analytical arguments describing the evolution of the modal characteristic diagrams for rigorous modes.

Conclusively, in this paper we have analyzed the power transfer in GADCs comprised by three-guides by using modal transmission-line theory based on a rigorous solution of boundary-value problems. From the numerical analysis, we have found that the incident power is equally distributed at the output guiding channels, provided that the GADCs operates at the optimized grating period that has never referred in the previous published papers [1, 2].

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