

# Noise Reduction Approach of Nonlinear Function for a Range Image using 2-D Kalman Filtering Method

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**Abstract:** A new 2-D block Kalman filtering method which uses a nonlinear function is presented to generate a more accurate filtered estimate of a range image that has been corrupted by additive noise. Novel 2-D block Kalman filtering method is constructed of the conventional method and nonlinear function which utilizes to control estimation error. We show that novel 2-D Kalman filtering method using a nonlinear function is effective at reducing the additive noise, not distorting shape edges.

## 1. Introduction

Recently, unmanned vehicles require an autonomous guidance system, which can guide to a target of destination automatically. In vehicle category, accurate guidance system is especially an important issue. The active imaging more system provides a 3-D image with a higher resolution. We expect to accurate guidance information, i.e., azimuth and elevation in 3-D image [1]. In addition, this imaging system gives range information that varies widely and sharply at sharp edges among the target, the foreground and the background. An edge characteristic of range information is very important in the target extraction from the noisy foreground and background. The imaging system for vehicles is always noisy because of the various factors. So, we consider that the noisy range image, which is obtained by using the Imaging Laser Radar System, is difficult in the accurate target extraction. Then, the conventional linear filtering method, i.e., Low Pass Filter, is difficult to reduce noise in the presence of a range image because of range information, i.e., sharp edges. So, the nonlinear filters, e.g., median filter,  $\epsilon$ -filter [2], and  $\epsilon$ -Kalman filter [3] which reduce additive noise has been reported not to influence sharp edges. These filters reduce additive noise, while ones do not influence sharp edges. In the case of vehicle with various images each time, the conventional mathematical and statistics model could not be assumed. Thus, the recursive filtering method such as  $\epsilon$ -Kalman filter is effective to the noise reduction in vehicle. However,  $\epsilon$ -Kalman filtering method has been only realized on a 1-D case. The 1-D case of the  $\epsilon$ -Kalman filter is difficult to extend to a 2-D case.

In this paper, we propose a novel approach of a nonlinear function for a range image using a 2-D Kalman filtering method. First, we denote the novel algorithm of the Kalman filtering method. The proposed method utilizes a nonlinear function to control estimation error of the conventional 2-D Kalman filtering method. Secondly, we discuss the proposed method using a test range image. Finally, we discuss the proposed method

using a real range image, which is obtained by using the Imaging Laser Radar System.

## 2. 2-D Block Kalman Filtering Method

### 2.1 Conventional Method

Azimi-Sadjadi et al. proposed the 2-D block Kalman filtering method using a full plane model [4], which causality is maintained within the filtering process by employing multiple concurrent block estimators. The local state-space model for the image process is given by  $S(n) = AS(n-1) + w(n)$  (1) where  $S(n)$  is the current state vector consisting of 8 blocks  $S_i(n)$ ,  $i \in [0,7]$ ;  $S(n-1)$  is the post state vector consisting of 8 blocks  $S_i(n-1)$ , and  $w(n)$  is a zero mean white driving noise vector of size  $8 \times 1$ . The spatial positions of  $S_i(n)$  at a given iteration "n" are shown in Fig 1. The peculiar numbering of these blocks is solely chosen to provide easier programming by getting all the blocks which are to be estimated, i.e., blocks 4, 5, 6 and 7 numerically close together and their support numerically adjacent. The blocks which are not filtered estimates are obtained by shifting the blocks within the state as the state advances to the right, so that the previously estimated blocks occupy the proper spatial positions within the state. Fig. 2 illustrates the state propagation along horizontal direction with each iteration

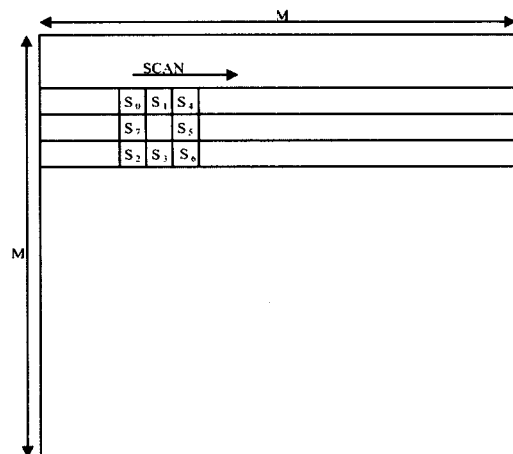


Fig. 1. The spatial positions of  $S_i(n)$ .

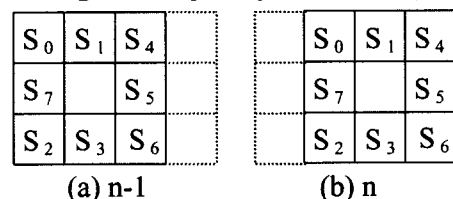


Fig. 2. The state propagation.

The supports for blocks 4, 5, 6 and 7 are given in Fig. 2. This results in an A matrix of size  $8 \times 8$ .

The observation equation in block form is

$$\mathbf{x}(n) = \mathbf{C}\mathbf{S}(n-1) + \mathbf{v}(n) \quad (2)$$

where  $\mathbf{x}(n)$  is the corrupted image or observation vector of size  $4 \times 1$ ;  $\mathbf{v}(n)$  is the observation noise vector of the same size as  $\mathbf{x}(n)$  containing a scalar zero mean white Gaussian additive noise  $v(n)$  with variance  $\sigma_v^2$ , and H is a  $4 \times 8$  matrix containing the elements of the point spread function (PSF) of the non-causal blur. The Kalman filter equations for the system in Eqs. (1) and (2) are

$$\mathbf{K}_n(n) = \frac{\mathbf{P}(n|n-1)\mathbf{C}^T}{\mathbf{C}\mathbf{P}(n|n-1)\mathbf{C}^T + \mathbf{Q}_v} \quad (3)$$

$$\tilde{\mathbf{x}}(n) = \mathbf{x}(n) - \hat{\mathbf{x}}(n|n-1) \quad (4)$$

$$\hat{\mathbf{S}}(n|n) = \hat{\mathbf{S}}(n|n-1) + \mathbf{K}_n(n)\tilde{\mathbf{x}}(n) \quad (5)$$

$$\mathbf{P}(n|n) = (\mathbf{I} - \mathbf{K}_n(n)\mathbf{C})\mathbf{P}(n|n-1) \quad (6)$$

$$\mathbf{P}(n+1|n) = \mathbf{A}\mathbf{P}(n|n)\mathbf{A}^T + \mathbf{Q}_w \quad (7)$$

$$\hat{\mathbf{S}}(n+1|n) = \mathbf{A}\hat{\mathbf{S}}(n|n) \quad (8)$$

where  $\hat{\mathbf{S}}(n|n)$  is the a priori (before updating) estimation vector;  $\hat{\mathbf{S}}(n|n-1)$  is the a posteriori (after updating) estimation vector;  $\tilde{\mathbf{x}}(n)$  is the estimation error,  $\hat{\mathbf{x}}(n|n-1)$  is the a posteriori estimation state vector, and  $\mathbf{P}(n|n)$  is the a priori error covariance matrix defined by

$$\mathbf{P}(n|n) = E[(\mathbf{S}(n) - \hat{\mathbf{S}}(n|n))(\mathbf{S}(n) - \hat{\mathbf{S}}(n|n))^T] \quad (9)$$

$\mathbf{P}(n|n-1)$  is the a posteriori error covariance matrix defined by

$$\mathbf{P}(n|n-1) = E[(\mathbf{S}(n) - \hat{\mathbf{S}}(n|n-1))(\mathbf{S}(n) - \hat{\mathbf{S}}(n|n-1))^T] \quad (10)$$

$\mathbf{K}_n(n)$  is the Kalman gain matrix,  $\mathbf{Q}_w$  and  $\mathbf{Q}_v$  are correlation matrices of the independent process w and v. The state is reinitialized at the beginning of each strip. After a strip is processed, one advances a block row and starts a new strip without using any filtered estimates from the prior strip. Thus recursion only occurs along a strip. The model parameters to be estimated are  $\mathbf{A}$ , the correlation matrices  $\mathbf{Q}_w$  and  $\mathbf{Q}_v$ . The estimation method of the model parameters refers to a part of Azimi-Sadjadi et al. and Jo et al. [5].

## 2.2 Nonlinear Approach and Theoretical Analysis

Sharp edges of the range image are considered for the non-continuous point of the observation vector. The edge vector is the parallel vector added to the estimation error in Eq. (4) in the orthogonal principle. Thus, the observation vector considered edges is given by

$$\text{The Observation Vector} = \mathbf{x}(n) + \xi(n) \quad (11)$$

where,  $\xi(n)$  is the edge vector. By using Eq. (11) and the orthogonal principle, the error of Eq. (4) is given by

$$\tilde{\mathbf{x}}(n) = \mathbf{x}(n) + \xi(n) - \hat{\mathbf{x}}(n|n-1) \quad (12)$$

Then, the state-space of Eq. (12) is shown in Fig. 3. If  $\mathbf{x}(n) \ll \xi(n)$ , the estimation error is a bigger value.

To reduce the estimation error, we consider the difference between filtered block 7 of the current state vector and  $\mathbf{x}(n)$  (Eq. (12)). We have

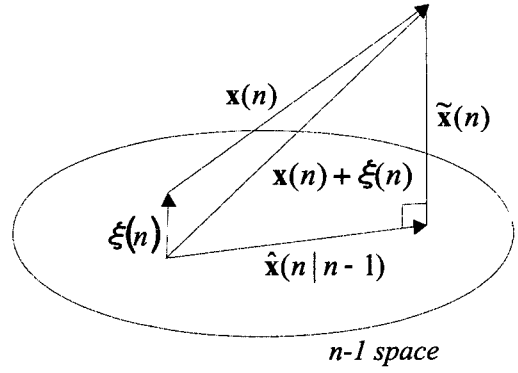


Fig. 3. State Space of Eq. (12).

$$\begin{aligned} & \mathbf{x}(n) + \xi(n) - (x_7(n) + \xi_7(n)) - (\hat{\mathbf{x}}(n|n-1) - (x_7(n) + \xi_7(n))) \\ & = \begin{bmatrix} x_0(n) - x_7(n) + \xi_0(n) - \xi_7(n) \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} -\hat{x}_0(n|n-1) + x_7(n) + \xi_7(n) \\ \vdots \\ -\hat{x}_7(n|n-1) + x_7(n) + \xi_7(n) \end{bmatrix} \end{aligned} \quad (13)$$

Here, we introduce the nonlinear function in Eq. (13), i.e.,  $|F(X)| \leq \Delta \quad -\infty < X < \infty$ . (14)

Thus, using Eqs. (13) and (14), we have

$$\tilde{\mathbf{x}}(n) = F \begin{bmatrix} x_0(n) - x_7(n) + \xi_0(n) - \xi_7(n) \\ \vdots \\ 0 \end{bmatrix} + F \begin{bmatrix} -\hat{x}_0(n|n-1) + x_7(n) + \xi_7(n) \\ \vdots \\ -\hat{x}_7(n|n-1) + x_7(n) + \xi_7(n) \end{bmatrix} \quad (15)$$

Eq. (15) is introduced by the nonlinear function. However, if  $\xi(n)=0$ , we have

$$\tilde{\mathbf{x}}(n) = \begin{bmatrix} x_0(n) - \hat{x}_0(n|n-1) \\ \vdots \\ x_7(n) - \hat{x}_7(n|n-1) \end{bmatrix} \quad (16)$$

which is denoted the conventional estimation error in Eq. (4). If  $\xi_7(n) \gg 0$ , we have

$$\tilde{\mathbf{x}}(n) = \begin{bmatrix} 0 \\ \vdots \\ \Delta \end{bmatrix} \quad (17)$$

which is denoted that the estimation error  $\tilde{x}_7(n)$  is given  $\Delta$  by using the nonlinear function. The other estimation error block row which is the smaller than estimation error  $\tilde{x}_7(n)$  is denoted to be 0. If  $\xi(n) \gg 0$ , we have

$$\tilde{\mathbf{x}}(n) = \begin{bmatrix} x_0(n) - x_7(n) + \Delta \\ \vdots \\ \Delta \end{bmatrix} \quad (18)$$

which is denoted by calculating the correlation of  $x_7(n)$ . If  $\xi(n) \gg 0$  other than  $\xi_7(n)$ , we have

$$\tilde{\mathbf{x}}(n) = \begin{bmatrix} x_7(n) - \hat{x}_0(n|n-1) \\ \vdots \\ x_7(n) - \hat{x}_7(n|n-1) \end{bmatrix} \quad (19)$$

$\tilde{x}_7(n)$  is used for the conventional estimation error. On the other hand, other estimation error is used for difference between the estimated  $\hat{\mathbf{x}}(n|n-1)$  and  $x_7(n)$ .

## 3. Computer Simulation

In this section the effectiveness of the proposed 2-D Kalman filtering method using the nonlinear function is examined on the range image by using computer

simulation. The corrupted image is obtained by adding a white Gaussian noise to the original image, i.e., the test and real range image.

### 3.1 Test Range Image

The effectiveness of the proposed 2-D Kalman filtering method is examined on the test range image. The corrupted range image is obtained by adding a scalar zero white Gaussian noise with variance 1 to the original range image. The resultant corrupted image is shown in Fig. 4. In Fig. 4, the target is the square object on the flat plane and the range information of the image denotes the cross range between the imaging system and the target.

Here, a part of the proposed Kalman filtering algorithm introduces the Jo's estimate method that estimates the model parameters  $\mathbf{A}$ , the correlation matrices  $\mathbf{Q}_w$  and  $\mathbf{Q}_v$ . Thus, we consider that proposed method does not need to obtain the experimental parameters such as the conventional method and the proposed method is effective at reducing noise in the image information obtained by using the imaging system for a vehicle.

Fig. 5 shows the test range image filtered using the proposed method. In Fig. 5, the model parameters and the correlation matrices are estimated from the observation image, i.e., the test range image. Visual evaluation of the image in Fig. 5 shows that the filter is effective at reducing the amount of the noise. To compare the result with the proposed method, we are shown row 63 in the original image and the filtered image in Fig. 6. In the each plot, the solid line represents the filtered image and the dashed line represents the original image. The filtered image in Fig. 6 shows that the filter is effective at reducing the noise. Then, the proposed method does not distort sharp edges in the test range image.

### 3.2 Real Range Image

To discuss the proposed 2-D Kalman filtering method which reducing the noise, we obtain the real range image using the Imaging Laser Radar System, which is used for high power LD. The Imaging Laser Radar System, which is an active system to transmit laser beam, detect a reflectance light and scan the space using a mirror, can obtain the range image between imaging system and target using measuring the interval delay time from the transmitter to the receiver at the scanning point.

The resultant obtained image is shown in Fig. 7. Fig. 7 is obtained from measuring the diffusion square plane and natural object by using the Imaging Laser Radar System. The corrupted image is obtained by adding a scalar zero mean white Gaussian noise with variance 1 to the original image. Fig. 7 shows that the background takes a complex form of natural objects, and discrimination of the target and the background is difficult for visual evaluation.

Fig. 8 shows the real range image filtered using the proposed method. In Fig. 8, the model parameters and the correlation matrices are estimated from the observation image, i.e., the real range image. Visual evaluation of the image in Fig. 8 shows that the filter is effective at reducing the amount of the noise. To compare the result with the proposed method, we are

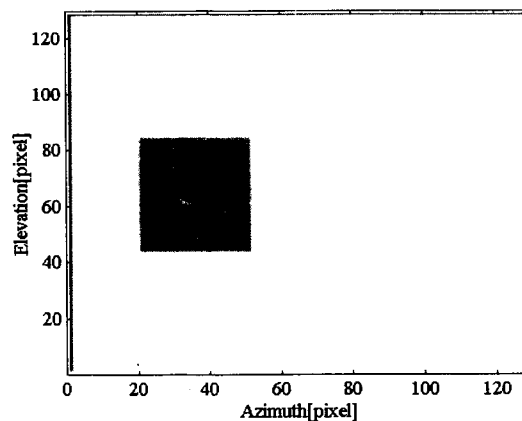


Fig. 4. Test range image.

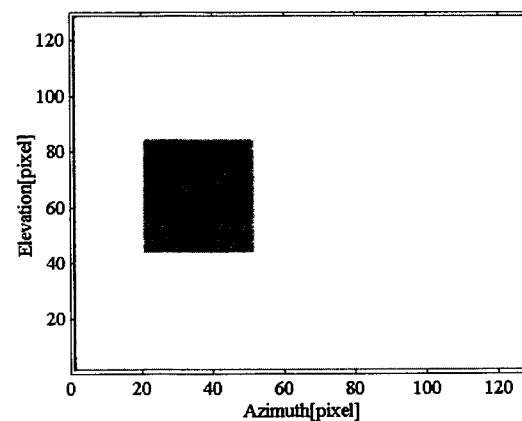


Fig. 5 Filtered test range image using the proposed method.

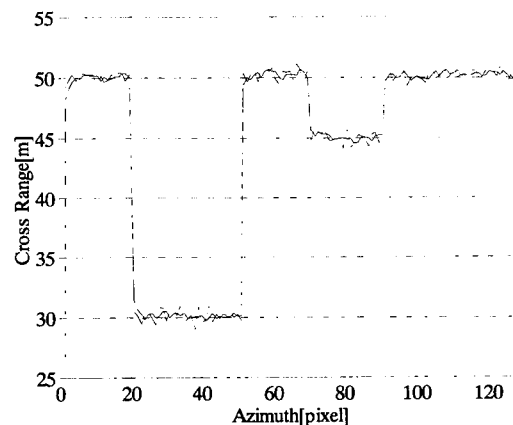


Fig. 6. Row 63 in the test range image and the filtered image.

shown row 63 in the original image and the filtered image in Fig. 9. In the each plot, the solid line represents the filtered image and the dashed line represents the original image. The filtered image in Fig. 9 shows that the filter is effective at reducing the noise. Then, the proposed method does not distort sharp edges in the real range image.

## 4. Conclusion

In this paper, we propose the noise reduction approach of the nonlinear function for the range image using the 2-D Kalman filtering method. The proposed method, which

introduces the nonlinear function to control the estimate error, denotes reducing the estimation error of the edge vector. The effectiveness of the proposed method is examined on the test range image. As a result, the proposed method is effective at reducing the noise, not distorting edges in the test range image. Furthermore, the effectiveness of the proposed method is examined on the real range image that is obtained by using the Imaging Laser Radar System. Then, the proposed method is effective at reducing the noise, not distorting a complex form of natural objects in the real range image.

## References

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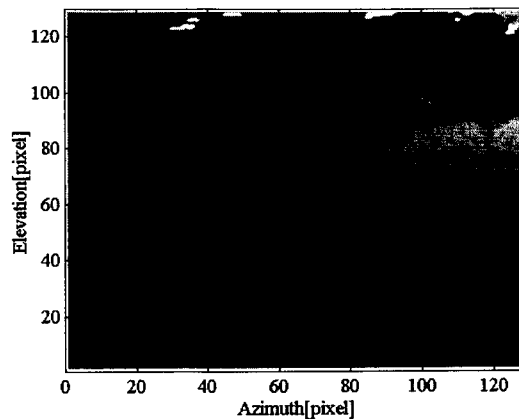


Fig. 7. Real range image.

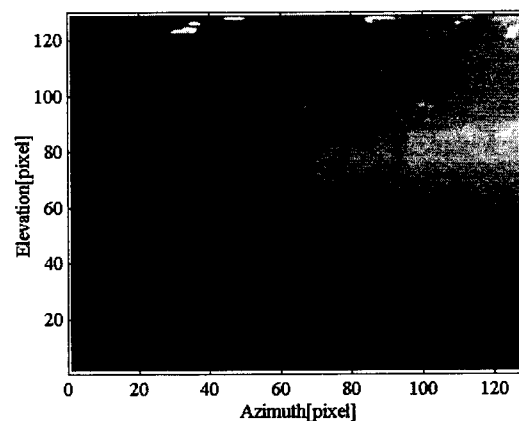


Fig. 8 Filtered real range image using the proposed method.

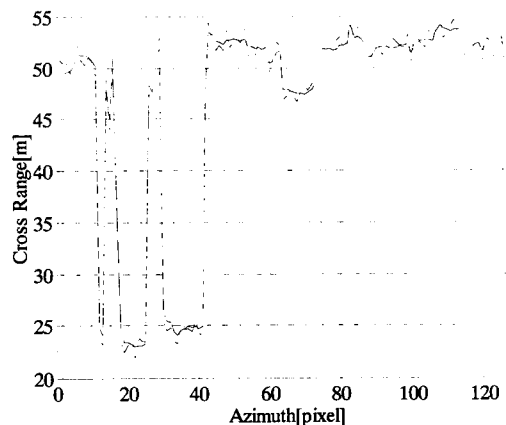


Fig. 9. Row 63 in the real range image and the filtered image.