Motion Detection Using Electric Field Theory

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Abstract: Motion detection is an important step in computer vision and image processing. Traditional motion detection systems are classified into two categories, namely, feature based and gradient based. In feature based motion detection, features in consecutive frames are detected and matched. Gradient based methods assume that the intensity varies linearly and locally. The method, which we propose, is neither feature nor gradient based but uses the electric field theory. The pixels in an image are modeled as point charges and motion is detected by using the variations between the two electric fields produced by the charges corresponding to the two images.

1. Introduction

The use of electric field theory has been applied to corner detection [1], skeletonization [2], image segmentation [3] and form detection [4]. In this paper, electric field theory is applied to detect motion.

Motion detection is an important step in computer vision and image processing. Most traditional motion detection systems can be classified into two categories, namely, feature based or gradient based [5]. In feature based motion detection, features in consecutive frames are detected and matched. In other words, the motion is estimated by finding the features detected in one frame and matched with features detected in another frame. However, in general, it is not easy to detect features and to match them. Furthermore, only motions of particular features are estimated. Gradient based methods, on the other hand, assume that the intensity varies linearly and locally in an image [6]. High accuracy is often required around the border of objects. Several problems may arise because of this assumption. In general, estimation of rotational motion is more difficult than that of translation. Some other motion detection methods employ the moment or the torque, and derive good estimation for the rotational motion [7][8][9]. However, in these methods, the correspondence of object regions before and after the motion has to be known beforehand.

The method, which we propose, is neither feature based nor gradient based but uses the electric field theory[10]. It does not have the problem associated with existing approaches. Pixels are interpreted as point charges such that the magnitude of each point charge is equivalent to the intensity of the pixel. Hence, we can obtain the electric field corresponding to the image. Motion in two consecutive images is detected by using the variations between the two electric fields produced by the distribution of charges induced by these images.

2. Electric field produced by a distribution of charges

A monochromatic digital image of size $M \times N$ pixels can be modeled as an array of point charges such that the magnitude of each point charge is proportional to the intensity of the pixel located at the corresponding position in the intensity image. Using this interpretation, changes in the image induce changes in the charges.

As shown in Fig.1, let the magnitude of point charge on (i,j) , $i=1,2,\cdots,M; j=1,2,\cdots,N$ is q_{ij} at time $t=0$ and the velocity is $\boldsymbol{v}_{ij}=(v_{xij},v_{yij})$. Since the electric field $\boldsymbol{E}(x,y,z,t)$ observed at any point (x,y,z) at time t is an amount of the electric field produced by each charge at the point, $\boldsymbol{E}(x,y,z,t)$ is calculated by

$$\boldsymbol{E}(x,y,z,t) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{ij}\{(x-i-v_{xij}t)\boldsymbol{i}+(y-j-v_{yij}t)\boldsymbol{j}+z\boldsymbol{k}\}}{\{(x-i-v_{xij}t)^2+(y-j-v_{yij}t)^2+z^2\}^{3/2}}, \quad (1)$$

where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are unit vectors of x, y and z coordinates respectively.

Without loss of generality, we consider two images observed at $t=0$ and $t=1$. Let the distribution of the charges at $t=0$ and $t=1$ are q_{0ij} and q_{1ij} respectively. The electric field at $t=1$ is calculated by substituting $q_{ij}=q_{0ij}$ and $t=1$ into Eq.(1). On the other hand, the electric field at $t=1$ is calculated with the distribution of charges at $t=1$, i.e., the electric field is calculated by substituting $q_{ij}=q_{1ij}$ and $t=0$ into Eq.(1). Needless to say, since the electric fields derived from both of the calculations should be same, we have a following equation.

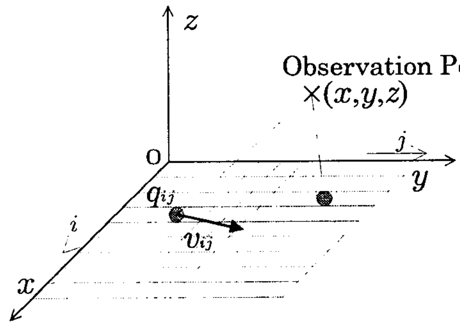


Figure 1. Electric field by moving charges on x-y plane.

$$\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{0ij}\{(x-i-v_{xij})\boldsymbol{i}+(y-j-v_{yij})\boldsymbol{j}+z\boldsymbol{k}\}}{\{(x-i-v_{xij})^2+(y-j-v_{yij})^2+z^2\}^{3/2}} = \sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{1ij}\{(x-i)\boldsymbol{i}+(y-j)\boldsymbol{j}+z\boldsymbol{k}\}}{\{(x-i)^2+(y-j)^2+z^2\}^{3/2}} \quad (2)$$

This equation is decomposed into three equations with respect to three components $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.

3. Evaluation function to obtain the velocities

Combining above three equations, we derive a following evaluation function to obtain each velocity of charge.

$$\begin{aligned}
&F(\boldsymbol{v}_{11},\boldsymbol{v}_{12},\cdots,\boldsymbol{v}_{ij},\cdots,\boldsymbol{v}_{MN})\\
&=\left\{\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{0ij}(x-i-v_{xij})}{\{(x-i-v_{xij})^2+(y-j-v_{yij})^2+z^2\}^{3/2}} -\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{1ij}(x-i)}{\{(x-i)^2+(y-j)^2+z^2\}^{3/2}}\right\}^2\\
&+\left\{\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{0ij}(y-j-v_{yij})}{\{(x-i-v_{xij})^2+(y-j-v_{yij})^2+z^2\}^{3/2}} -\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{1ij}(y-j)}{\{(x-i)^2+(y-j)^2+z^2\}^{3/2}}\right\}^2\\
&+\left\{\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{0ij}z}{\{(x-i-v_{xij})^2+(y-j-v_{yij})^2+z^2\}^{3/2}} -\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{1ij}z}{\{(x-i)^2+(y-j)^2+z^2\}^{3/2}}\right\}^2
\end{aligned} \quad (3)$$

Finding v_{xij}, v_{yij} is equivalent to the minimization problem of $F(\boldsymbol{v}_{11},\cdots,\boldsymbol{v}_{MN})$ with respect to $\boldsymbol{v}_{ij},(i=1,\cdots,M; j=1,\cdots,N)$.

4. Minimization of the function F by an iterative procedure

The function (3) has minimum when

$$\frac{\partial F}{\partial \boldsymbol{v}_{ij}}=\left(\frac{\partial F}{\partial v_{xij}},\frac{\partial F}{\partial v_{yij}}\right)=\mathbf{o} \qquad i=1,\cdots,M;\ j=1,\cdots,N \quad (4)$$

is satisfied. However, since it is difficult to solve Eq.(4) explicitly, let us employ the method of steepest decent to obtain $\boldsymbol{v}_{ij}(i=1,\cdots,M, j=1,\cdots,N)$ which minimizes F, i.e.

$$\begin{aligned} v_{xij,L}&=v_{xij,L-1}-h\frac{\partial F}{\partial v_{xij}}\\ v_{yij,L}&=v_{yij,L-1}-h\frac{\partial F}{\partial v_{yij}}\end{aligned} \quad , \quad (5)$$

where L denotes the iteration number and h is a positive real number which controls the amount of correction.

Appling procedure (5) for all charges, we have

$$\boldsymbol{V}_L=\boldsymbol{V}_{L-1}-h\nabla_{\boldsymbol{V}}F|_{\boldsymbol{V}=\boldsymbol{V}_{L-1}}, \quad (6)$$

where $\boldsymbol{V}_L$ is a matrix constructed by the velocity vectors. In addition, $\nabla_{\boldsymbol{V}}F$ denotes the derivative of F with respect to V, and $|_{\boldsymbol{V}=\boldsymbol{V}_{L-1}}$ means the substitution of $\boldsymbol{V}=\boldsymbol{V}_{L-1}$ into $\nabla_{\boldsymbol{V}}F$, i.e.

$$\boldsymbol{V}_L=\begin{pmatrix} v_{x11} & v_{y11}\\ v_{x12} & v_{y12}\\ \vdots & \vdots\\ v_{xmn} & v_{ymn}\\ \vdots & \vdots\\ v_{xMN} & v_{yMN}\end{pmatrix}_L, \quad (7)$$

$$\nabla_{\boldsymbol{V}}F|_{\boldsymbol{V}=\boldsymbol{V}_{L-1}}=\begin{pmatrix} \left.\frac{\partial F}{v_{x11}}\right|_{\boldsymbol{V}=\boldsymbol{V}_{L-1}} & \left.\frac{\partial F}{v_{y11}}\right|_{\boldsymbol{V}=\boldsymbol{V}_{L-1}}\\ \left.\frac{\partial F}{v_{x12}}\right|_{\boldsymbol{V}=\boldsymbol{V}_{L-1}} & \left.\frac{\partial F}{v_{y12}}\right|_{\boldsymbol{V}=\boldsymbol{V}_{L-1}}\\ \vdots & \vdots\\ \left.\frac{\partial F}{v_{xmn}}\right|_{\boldsymbol{V}=\boldsymbol{V}_{L-1}} & \left.\frac{\partial F}{v_{ymn}}\right|_{\boldsymbol{V}=\boldsymbol{V}_{L-1}}\\ \vdots & \vdots\\ \left.\frac{\partial F}{v_{xMN}}\right|_{\boldsymbol{V}=\boldsymbol{V}_{L-1}} & \left.\frac{\partial F}{v_{yMN}}\right|_{\boldsymbol{V}=\boldsymbol{V}_{L-1}}\end{pmatrix}. \quad (8)$$

The derivatives of F with respect to v_{xmn} and v_{ymn} are calculated by

$$\begin{aligned}\frac{\partial F}{\partial v_{xmn}}&=2F_x\frac{2(x-m-v_{xmn})^2-(y-n-v_{ymn})^2-z^2}{\{(x-m-v_{xmn})^2+(y-n-v_{ymn})^2+z^2\}^{5/2}}q_{omn}\\ &+6F_y\frac{(x-m-v_{xmn})(y-n-v_{ymn})}{\{(x-m-v_{xmn})^2+(y-n-v_{ymn})^2+z^2\}^{5/2}}q_{omn}\\ &+6F_z\frac{(x-m-v_{xmn})z}{\{(x-m-v_{xmn})^2+(y-n-v_{ymn})^2+z^2\}^{5/2}}q_{omn}\end{aligned}, \quad (9)$$

$$\begin{aligned}\frac{\partial F}{\partial v_{ymn}}&=6F_x\frac{(x-m-v_{xmn})(y-n-v_{ymn})}{\{(x-m-v_{xmn})^2+(y-n-v_{ymn})^2+z^2\}^{5/2}}q_{omn}\\ &+2F_y\frac{-(x-m-v_{xmn})^2+2(y-n-v_{ymn})^2-z^2}{\{(x-m-v_{xmn})^2+(y-n-v_{ymn})^2+z^2\}^{5/2}}q_{omn}\\ &+6F_z\frac{(y-n-v_{ymn})z}{\{(x-m-v_{xmn})^2+(y-n-v_{ymn})^2+z^2\}^{5/2}}q_{omn}\end{aligned}, \quad (10)$$

where we replaced as follows.

$$\left.\begin{aligned}R_{0ij}&=\{(x-i-v_{xij})^2+(y-j-v_{yij})^2+z^2\}^{3/2}\\ R_{1ij}&=\{(x-i)^2+(y-j)^2+z^2\}^{3/2}\end{aligned}\right\}, \quad (11)$$

$$\left.\begin{aligned}F_x&=\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{0ij}(x-i-v_{xij})}{R_{0ij}}-\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{1ij}(x-i)}{R_{1ij}}\\ F_y&=\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{0ij}(y-j-v_{yij})}{R_{0ij}}-\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{1ij}(y-j)}{R_{1ij}}\\ F_z&=\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{0ij}z}{R_{0ij}}-\sum_{i=1}^{M}\sum_{j=1}^{N}\frac{q_{1ij}z}{R_{1ij}}\end{aligned}\right\}. \quad (12)$$

Consequently by procedure (6) and equations (7)(8)(9)(10), we obtain the velocity of the charge located at (m,n) at $t=0$.

5. Selection of observation point

Although the observation point can be located at arbitrary position theoretically, we set the observation point above each point charge for the accuracy of calculation. If we set only one observation point for all charges, the estimation accuracy about the motion of charges far from the observation points is reduced. Setting an observation point for each charge individually, we can neglect the difference in the accuracy. Furthermore we move the observation point according to the estimated velocity in the iterative procedure. For example, if we have an estimated result $v_{xmn,L}$ and $v_{ymn,L}$ after L iterations, we locate the observation point $(m+v_{xmn,L}, n+v_{ymn,L}, z)$ for the charge q_{0mn} as shown in Fig.2. In this way, the observation point is always located above the charge of interest.

In addition, parameter z, which is the height of the observation point, must be set according to the magnitude of motion. Although giving smaller z and we will have more accurate results, it is difficult to follow the bigger motion in the image. Hence we change z gradually in the procedure. Firstly we set z a large value and estimate the global motion in the image. Then z is reduced gradually in order to get a more accurate result.

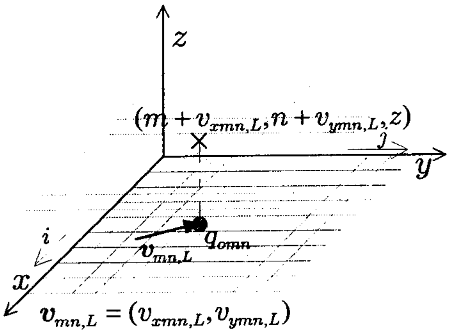


Figure 2. Relation between a charge and the observation point.

6. Application to real images

In the human vision system, any motion of dark objects can be recognized. However with our method which treats the intensity of each pixel as a point charge, it is difficult to recognize the motion of dark objects. If we apply our method directly to such images including motion of dark objects, the method detects motion of the lighter objects only.

Furthermore we should consider the number of pixels to be processed. Since the calculating procedure includes the convolution with respect to all the pixels in the image, the calculation time increases proportional to the square of the number of pixels in an image. Therefore it is desirable to reduce the number of point charges to be processed. If we apply the method only to the edge pixels, not only can the calculation times be reduced; the problem associated with dark objects can be resolved. Furthermore, it is more efficient to calculate the velocities only for edges that move in the image sequence. For detecting such moving edges, it is efficient to use Coincident Edge Detection method [11].

7. Experiments

We applied the proposed method for detecting motions in several image sequences. In the figures in our experiments, ○ denotes a point charge in the first image and × denotes that in the second image.

7.1 Motion detection of a line

First of all, we experimented with the motion of a line. We started the proposed iterative procedure with z=100. The iteration is 100 for each step, i.e. z is reduced by 0.5 every 100 iterations.

- Rotation

Fig.3 shows a case where a line segment rotates around the midpoint of the segment. We obtain good results.

- Motion with variation

Fig.4 shows the results for motion with shift and bend. We find that the proposed method is applicable for motion including variation

- Motion of a split line

In Fig.5, a line segment is shifting and splitting into 2 line segments. If we stop the procedure at z=25.0 (Fig.5(a)), the part without the line segment has detected motion. If we continue the procedure till z=6.25, each pixel in the first frame is forced to correspond to a pixel in the second frame correctly (Fig.5(b)).

7.2 Rotation of a rectangle edge

Fig. 6 shows the results of analyzing rotational motion of a rectangle edge. In Fig. 6(a), the rectangle is rotated by 10 degrees. In Fig. 6(b), the rectangle is rotated by 30 degrees. Although all of the pixels in the first frame are matched with those in the second frame, we can see some gaps between them, i.e. pixels in the corners in the first frame are not matched with those in the second frame. In general, the more the object is rotated, the more is the gap.

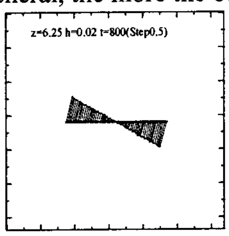


Figure3. Rotation.

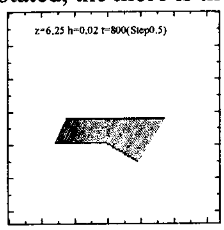


Figure4. Motion with variation.

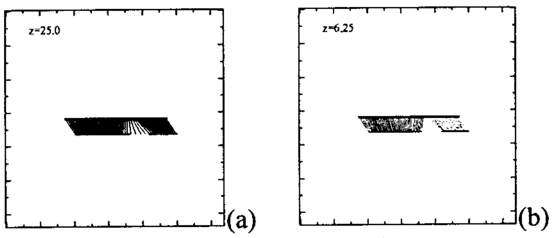


Figure 5. Motion of a split line.

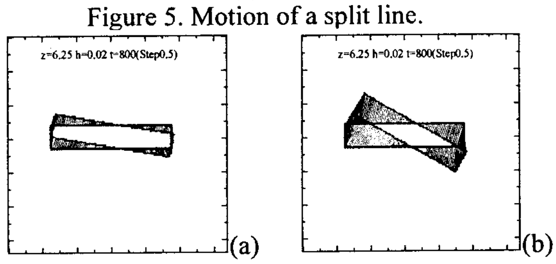


Figure 6. Rotation of a rectangle edge.

7.3 Variation of shapes

Fig.7 shows a result when the shape of object changes. In Fig.7 (a), circler markers show an object which is rectangle region in the first image, and cross markers show a quadrangle object in the second image. Fig.7 (b) shows the motion of each pixel and the triangle markers show the positions which the pixels moved to. The motions of the pixels in the rectangle region are converged into the quadrangle in the second image.

7.4 Motion detection from a real image sequence

Motion in a real image sequence is analyzed. Each image is of size 128×128 pixels. Two sample frames are shown in Fig. 8 (a) (first image) and Fig. 8 (b) (second image). Firstly, moving edges are detected using the Coincidence Edge Detection method. The proposed approach is then applied to the moving edges

Fig. 8 (c) shows the result. It is worthy to note that although there is some noise around the lower left corner, the estimated motion of the human body is not affected by the noise. In other words, with our proposed method, local motion can be detected.

8. Conclusions

In this paper, we propose a method for motion detection using the electric field theory. A major advantage of this method is that we do not have to detect features and do not have to find their correspondence. Furthermore the proposed method is less affected by the condition of gradients of images than the gradient based motion detection.

The experimental results indicate that the proposed method is effective to detect motion with shift, rotation, and scaling. We also found that the method deals with any variations of objects with motion. However it is difficult to estimate precise motion for large rotation. Although some existing methods derive good estimation for the rotational motion, the correspondence of regions before and after the motion has to be known beforehand [7][8][9] Our proposed method can be used as a preprocessing step for these methods.

For real images, in order to reduce processing time, it is desired to detect moving edges first. Therefore we combine our proposed method with the Coincident Edge Detection method with very encouraging results. The experimental results for a real image demonstrate the effectiveness of our proposed approach.

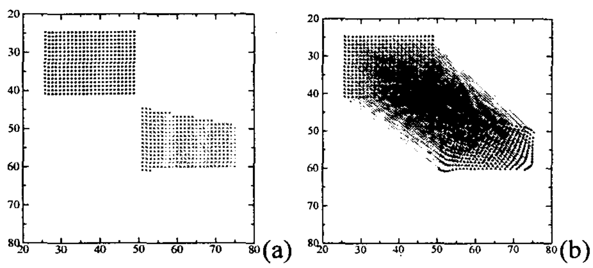
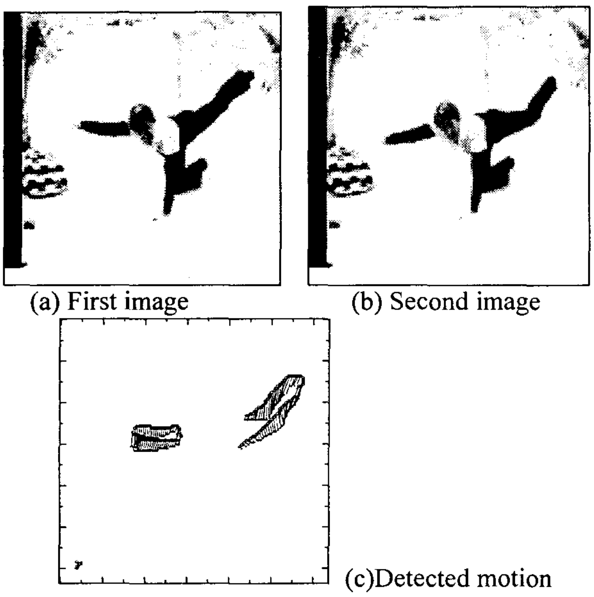


Figure 7. Motion with a variation of shape.


(a) First image (b) Second image

(c)Detected motion

Figure 8. Motion detection from a real image sequence.

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