The Fibonacci Edge Labeling on Fibonacci Trees

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Abstract: We present a novel graph labeling problem called Fibonacci edge labeling. The constraint in this labeling is placed on the allowable edge label which is the difference between the labels of endvertices of an edge. Each edge label should be (3m+2)-th Fibonacci numbers. We show that every Fibonacci tree can be labeled Fibonacci edge labeling. The labelings on the Fibonacci trees are applied to their embeddings into Fibonacci Circulants .

1. Introduction

Graphs provide a mathematical model for broad range of real world problems. And many interesting problems on graphs with the assignment of labels to the vertices or the edges of graphs. Clearly, without additional constraints, every graphs can be labeled in infinitely many ways. Thus utilzation of labeled graph model requires additional constraints which characterize the problem being investigated. These necessary constraints are naturally in studying the wide variety of seemingly unrelated practical applications for which graph provide underlying mathematical models.

Graph labelings can be categorized as vertex labelings and edge labelings. In a vertex labeling, there is an assignment of integers to the vertices of a graph and a function is designed to assign a value to each edge based on the two values assigned to the endvertices of the edge. A number of vertex labeling problems are defined by giving constraints on the edge values or on some functions of edge values. In an edge labeling, integers are initially assigned to the edges of a graph. Then, on each vertex, a value is induced based on the values assigned to the edges incident to that vertex.

Modern research dealing with the labeling with the labelings of graphs dates from the middle 1960's in which several classes of problems were defined in which edges were assigned values equaling the absolute difference between the value assigned to the endvertices. Harper[1,2] calculated the bandwidth of hypercube, which is the minimum edge values of other labeling problems on hypercube and he also determined optimal values of other labeling problems on hypercubes. Rosa, and independently Golomb[3], sought graphs having graceful labelings which is assignment to the vertices of distinct, nonnegative integers no larger than the size of the graph, which in turn induce distinct and consecutive edge values from 1 to the size of graph. Since that time there has been hundreds of graph labeling papers published. These include the continuation of work on the 'hard' problems in both bandwidth labeling. But they also include other classes of labeling problems, approached from a wide variety of viewpoints.

In this paper, several graph labeling problems are newly proposed and they concerned with linear arrangement problem. We introduce a new problem called Fibonacci edge labeling. Fibonacci edge labeling on a Fibonacci tree is defined to be a linear arragement such that the set of edge labels is a subset of Fibonacci numbers. The problems has its motivation in the Zechendorf's theorem. It is known that any natural number can be uniquely represented as a sum of Fibonacci numbers. It is embedded to the structure of a newly proposed network architecture, Fibonacci Circulants[4]. Determining whether a given tree is a subgraphs of the interconnection network topology is one of the important issue in the study of parallel computing [5, 6, 7, 8]. As computation graphs, trees are structures; underlying divide and conqure problem's solution spaces, like those algorithms for NP-complete problems. Fibonacci trees play an important role in broadcasting messages in parallel networks. Parallel algorithms can be efficiently performed on the interconnection network with Fibonacci trees as their subgraphs. This paper is organized as follows: In section 2, we represent represent related research; Section 3 presents Fibonacci edge labeling on Fibonacci trees; section 4 presents conclusion.

2. Related research

2.1 Edge Labelings

Most of the works done on graph labeling deal with vertex labelings and vertex labeling problems, linear arrangement problem has been drawn much attention. A linear arrangement of a graph G(V,E) is a one-to-one mapping π of the vertices V onto first $|V|$ positive integers $\{1,2,\cdots, |V|\}$. Linear arrangement problem subject to various constraints is an interesting combinatorial problem having many applications especially in the area of VLSI layout design. Among well-known linear arrangement problems, there are min-sum linear arrangement problem (sometimes called optimal linear arrangement problem), min-cut linear arrangement problem and bandwidth problem. The min-sum linear arrangement minimizes the sum of edge labels. The min-cut linear arrangement problem is to find a linear arrangement with minimum cross overing. And the bandwidth problem is to minimize the maximum edge label.

Above linear arrangement problems originated from various problem of practical interest. Among them., there has been much work on VLSI placement problem motivated by VLSI placement problem motivated by VLSI applications. The graph models a circuit with the vertices as active elements and the edges as the connecting wires. In some applications to VLSI design, the active elements are placed in rows or on a single line. Some simplifications are made, that is, on a line, distance of each element is assumed same and their required spacing is same. In this case, min-sum linear arrangment problem corresponds to finding layout having minimum number of tracks to route the wires above the line. And bandwidth problem is used to minmize the longest wire length which causes the maximum propagation delay of signals in the circuit.

2.2 Fibonacci Circulants

Recent advances in intergreated circuit technology make it possible to construct very large interconnection networks. Together with these advances, many interconnection network topologies have been proposed and investigated in the literature. Interconnection networks are often modeled as graphs. Hypercube Q_m of dimension m is a graph with 2^m vertices labeled $\{0,1,\cdots,2^m-1\}$; two vertices are joined by an edge if and only if their binary representation differ in exactly one bit position.

One important consideration for a network topology is whether there exists good mappings from various kinds of trees to the topology. This is due to the fact thar trees play a important role as data structure or an interconnection structure into another has been studied as graph embedding. In particular, there have been many papers on embedding trees in hypercubes. An embedding of a(guest) graph G into a(host) graph H is a one-to-one mapping ϕ of vertices of G into vertices of H, combined with an assignment of each edge e=(v, w) of G to a path between $\phi(v)$ and $\phi(w)$ in H. One of the most important measures of the quality of an embedding ϕ is dilation. The dilation of an edge e in G under the embedding ϕ is the length of the path in H to which e is assigned, and the dialation ϕ is the maximum dilation over all edges in G.

In this section, we proposed a new topology of communication networks. A circulant graphs with n vertices, denoted by $C_n(a_1, a_2, \cdots, a_k)$, $k \geq 1$, $0 < a_1 < a_2 < \cdots < a_k \leq \lfloor n/2, \rfloor$ is defined as follows: $V = (0, 1, 2, \cdots, n-1)$ and $(v, w) \in E$ if and only if there exists a_l, $1 \leq l \leq k$, such that $v + a_l \equiv w (\text{mod } n)$. The a_i, $1 \leq l \leq k$, is called a jump, and the sequence. The networks $FC(f_{3m+2}, 3)$, $m \geq 0$, is defined as follows : the node set $V = \{0, 1, \cdots, f_{3m+2}-1\}$, and the edge set $V = \{0, 1, \cdots, f_{3m+2}-1\}$, and the edge set $E = \{(v, w) \mid \text{there exists i}, 0 \leq l \leq m-1, \text{ such that } v + f_{3i+2} \equiv w \ (\text{mod } f_{3m+2})$. $FC(f_{3m+2}, 3)$ is a circulant graph $C_{f3m+2}(f_2, f_5, \cdots, f_{3m-1}+2)$. Each f_{3i+2}, $0 \leq l \leq m-1$, is called a jump. Examples of the network FC(21, 3) can be found in Fig 2.1

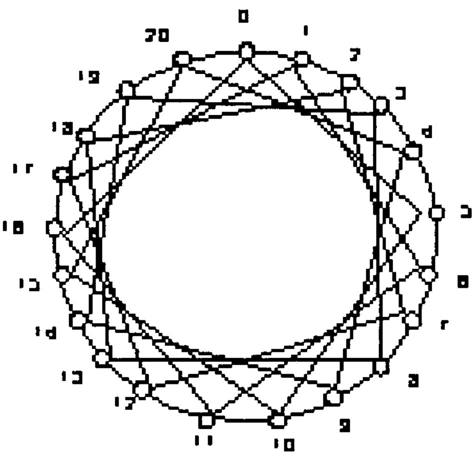


Fig 2.1 the example of the network FC(21,3)

Obviously, $FC(f_{3m+2}, 3)$ has a hamliltonian cycle unless $f_{3m+2} \leq 2$. $FC(f_{3m+2}, 3)$ is a $FC(f_{3m+2}, 3)$ is not dege symmetric, but two edge $(v, v + f_{3i+2})$ and $(w, w + f_{3i+2})$ are similar, that is an automorphism g of $FC(f_{3m+2}, 3)$ such that g(v)=u and $g(v + f_{3i+2}) = w + f_{3i+2}$. The degree $\delta(FC(f_{3m+2}, 3))$ of $FC(f_{3m+2}, 3)$ is 2m.

3. Fibonacci edge labeling on Fibonacci trees.

Determining whether a given tree is a subgraph of the interconnection network topology is one of the important issue in the study of parallel computing. As computation graphs, trees are the structure underlying divide and conquer problem solving strategies and algorithms which require a problem's solution space, like those algorithms for NP-complete problems. A labeling on a graph G(V, E) is one-to-one mapping of the vertices V into distinct integers. We consider here the labeling in which the integers that assigned to the vertices are {1, 2, ⋯ | V | }.

Each edge of the graph have an include edge label by the labeling. The edge label of an edge is the absolute difference between the labels of end-vertices of the edge. Fibonacci edge labeling on Fibonacci tree is defined to be a linear arrangement such that the set of edge labels is a subset of $\{f_{3m+2},\ m \geq 0\}$ for some Fibonacci number. The Fibonacci edge labeling is a embedding method of Fibonacci circulants by (3_m+2)- th Fibonacci numbers.

Defintion3.1 Fibonacci tree FT_k

A Fibonacci tree of height 0 is a single node.
For all $k > 0$, a Fibonacci tree of height k is a tree formed by connecting the root of two subtrees of height k-1 and height k-2 with edges and the total node number is f_{k+3}-1. Form the definition3.1, some examples of Fibonacci trees are shown in Figure 3.1

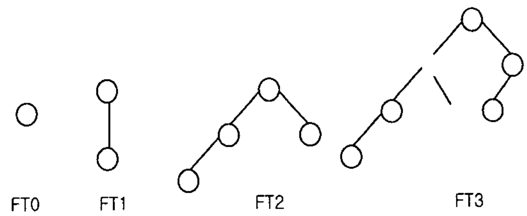


Figure 3.1 The examples of Fibonacci trees

Property 5.1
The number of nodes in Fibonacci tree FT_k is f_{k+3}-1.

Proof : This proof can be easily verified by the property of Fibonacci number.

Property5.2
The number of terminal nodes in FT_k is f_{k+1}

Proof : omitted

Property5.3
To delete terminal nodes of FT_k, the number of nodes in FT_k is that of FT_{k-1}.

Proof : omitted

Property 5.4 : $f_n = 4\, f_{n-3} + f_{n-6}$

Proof : omitted

Property 5.5
$|V(FT_{3m})| = (f_{3m}\text{-}1) + 5(f_{(3m-1)}) \quad + \quad f(_{(3m-1)}\text{-}1)$

proof : omitted

The labeling scheme on FT_m, $m \geq 0$ is as this :

Step1:In case of FT_{3m+1}, $m \geq 1$
F_{3m+2} terminal nodes of FT_{3m} are added to one node per each terminal node in order to be f_{3m+2} edge label.

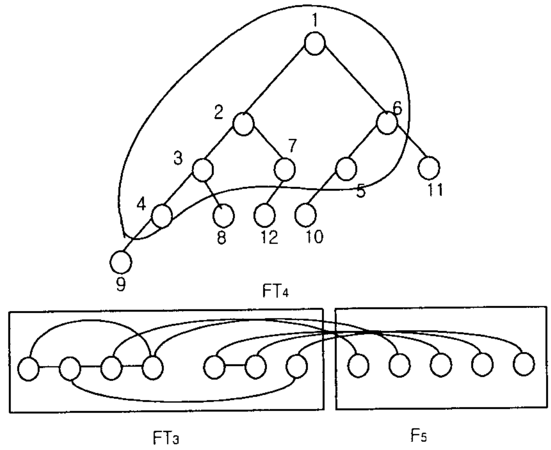


Fig 3.2 the example in case of FT_{3m+1}, $m \geq 1$

Step2: In case of FT_{3m+2}, $m \geq 1$
the vertex label of the root is FT_{3m+2} and the left subtree of the root is the inverse arrangement of FT_{3m-2}, FT_{3m-2}; the right subtree of the root is $FT_{3(m-1)}$.

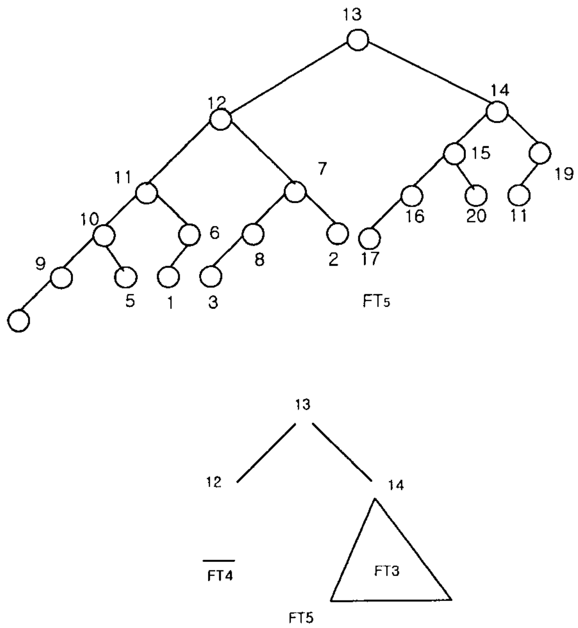
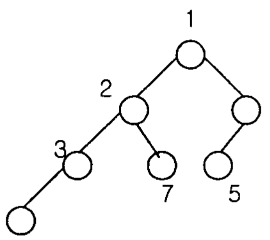


Fig 3.3 the example in case of FT_{3m+2}, $n \geq 1$

Step3: In case of FT_{3m+2}, $m \geq 2$,
from the basic frame FT_3, node1 is expanded f_{3m}-1 nodes

Node 5 is expanded $f_{((3m-1)-1)}$ nodes, and the other is expanded $f_{(3m-1)}$ nodes.

There are two series α_m, β_m, $n \geq 0$, $n = m + 3$
in case $\alpha_1 = 3$, $\alpha_{2n+3} = \alpha_{2n+1} + f_{3n+8} + (-1)^n$
in case $\alpha_2 = 4$, $\alpha_{2n+4} = \alpha_{2n+2} + f_{3n+8} + (-1)^{n+1}$
in case $\beta_1 = 6$, $\beta_{2n+3} = \beta_{2n+2} + f_{3n+8} + (-1)^{n+1}$
in case $\beta_2 = 5$, $\beta_{2n+4} = \beta_{2n+2} + f_{3n+8} + (1)^{n+1}$



FT3

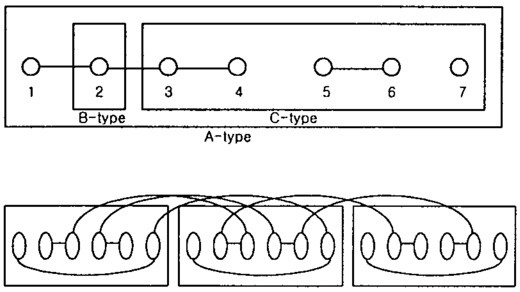


Fig 3.4 the example in case of FT_{3m+2}, $m \geq 2$

4. Conclusion

This paper proposes a Fibonacci edge labeling on Fibonacci trees. They are derived from linear arrangement of a graph G(V, E) is a one-to-one mapping π of the vertices V onto first |V| positive integers {1, 2, ... , |V|}. Linear arrangement problem subject to various constraints is an interesting combinatorical problem having many applications especially in the area of VLSI layout design. Fibonacci edge labeling has its motivation in the structrue of a newly proposed network embedding method between Fibonacci trees and Fibonacci circulants. Trees play an important role in broadcasting messages in parallel networks. Parallel algorithms can be efficiently performed on the interconnection network with Fibonacci trees as their subgraphs.

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