Liveness and Conjecture in Petri Nets*

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Abstract. Beyond free choice net system this paper presents some liveness knowledge in asymmetric net system including necessary and sufficient condition for an asymmetric net system being live and having liveness monotonicity, and an algorithm, polynomial time complexity, for such deciding. Also two conjectures about system livensss are in the contribution.

1.Motivation.

In the early 60's Carl Adam Petri achieved his initial conceptual breakthrough ([1]) and founded the field by creating the mathematical tool called net, later called Petri net in the net community and then in the scientific community. Petri net captured simply, precisely and elegantly the essence in information processing and was widely used in communication systems, parallel computing, computer integrate manufacturing systems,...,etc. In the Petri net applications besides the system modeling, the system behaviors analyzing, including system livesness, is very important.

As a Petri net is a static description for a dynamic system, it is quite easy to get the system states graph called the Petri net reachbility graph (RG, [2]) from the net. In fact the RG of a Petri net is also a static description for the system but in detail and the RG is very complicated comparing with the original net. From the RG many system properties, such as system invariants, liveness, boundedness, fariness,...,can be discovered directly. Unfortunately those discoveries via RG are NP hard and in general case no polynomial time complexity algorithms for that analyses. Fortunately net people have found other ways, called structure theory, and have taken some polynomial algorithms for analyzing some subclass of Petri nets in the past and recent years ([3]-[10]).

We concentrate the system liveness analysis in this paper, and in the reason of practical uses especially for such subclass of Petri net that there may exist polynomial algorithms for the purpose. The paper is organized as follows. Section 2 gives the basic concepts and some related results. Section 3 presents some liveness knowledge discovered in asymmetric choice net system and a liveness conjecture. Section 4 concludes the paper and gives another liveness conjecture, both conjectures are for system liveness but in different point of view.

2.Basic Concepts and Related Results

As Petri net is a popular mathematical tool[2] for system modeling, analyzing and simulating, we recall very few concepts about net and some related results to begin.

Theorem 2.1([3]).
Let a Petri net system Σ be a state machine(SM,[3]). The

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SM is live iff it is strongly connected and there are at least one token in the SM.
Obviously there is an algorithm, polynomial time complexity, for an SM liveness analysis by graph theory.

Theorem 2.2([4]).
Let a Petri net system Σ be a marked graph(MG, [4]). The MG is live iff it is strongly connected and there are at least one token in every directed cycle of the MG.

By the sufficient and necessary condition in the theorem 2.2, a polynomial algorithm can be found in similar way with SM.

Definition 2.1
Let N=(P, T;F) be a Petri net.

(1) $H \subseteq P$ is a siphon of N iff $H \neq \phi$ and ${}^{\bullet}H \subseteq H^{\bullet}$. A siphon is minimal iff it does not contain a siphon as a proper subset.

(2) $R \subseteq P$ is a trap of N iff $R \neq \phi$ and $R^{\bullet} \subseteq {}^{\bullet}R$. A trap is minimal iff it does not contain a trap as a proper subset.

The concepts of siphon and trap are useful in our study.

Corollary 2.1
Let $\Sigma = (N, M)$ be a Petri net system with the marking M, H be a siphon of a Petri net N and R be a trap of N.
(1) The number of tokens in H for any (N, M) does not increase, i.e. $\forall M' \in [M>, M'(H) \leq M(H)$;
(2) The number of tokens in R for any (N, M) does not decrease, i.e. $\forall M' \in [M>, M'(H) \geq M(H)$.

Theorem 2.3 ([3]).
Let a Petri net system Σ be a free choice net system (FC system,[3]). It is live iff every minimal siphon of Σ includes a marked trap.

Theorem 2.4 ([6]).
Let a Petri net system Σ be an FC system. There exists polynomial algorithm to analyze the liveness of Σ.
Now we consider bigger (than FC system) subclass to discover more knowledge about system liveness.

3.Conjecture and Some Study in Asymmetric Choice Net System

Definition3.1

(1) A Petri net N is structurally live iff $\exists M, (N, M)$ is live.
(2) A Petri net system (N,M) has liveness monotonicity iff (N,M) is live then $\forall M' > M, (N, M')$ is live.

Corollary 3.1
Let a Petri net system Σ be an SM or MG or FC system. It is structurally live and has livenss monotoncity.
Proof. directly by theorem 2.1, 2.2, 2.3 and definition 3.1.
□

By our observations the property of liveness monotonicity is something very important and naturally we get the following conjecture.

Conjecture 3.1.
Let Σ be a Petri net system. There exists a polynomial algorithm to analyze the liveness of Σ iff Σ does satisfy the liveness monotonicity.

Definition 3.2
A petri net system (N, M) is called Asymmetric Choice net system (AC system in short) iff $\forall p, q \in P, p^{\bullet} \cap q^{\bullet} \neq \phi \Rightarrow p^{\bullet} \subseteq q^{\bullet} or\ q^{\bullet} \subseteq p^{\bullet}$. It is easy to discover that AC system includes FC system, FC system includes SM and MG, SM∩MG≠ Φ and SM≠ MG(see fig3.1)

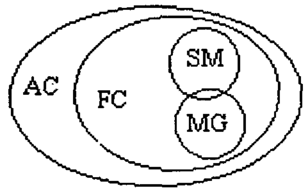

Fig 3.1

In general, an AC system does not satisfy liveness monotonicity.(see fig3.2).

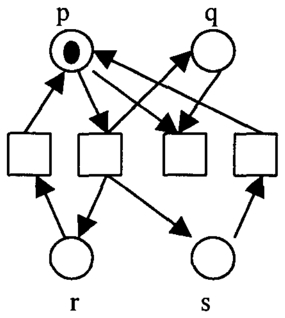

Fig 3.2 It is live but no liveness monotonicity

Theorm 3.1

Let Σ_0=(N,M_0) be an AC system. It is live and satisfies liveness monotoncity iff every minimal siphon in Σ_0 contains a marked trap.

Proof. $\Leftarrow$

As every siphon in (N,M_0) has a marked trap, then $\forall M_i$, $M_i \geqslant M_0$, every siphon in (N, M_i) must also contain a marked trap. From [2], we can deduce (N, M_i) is live.

$\Rightarrow$

Need to prove, if there exists a minimal siphon H s.t. every trap in H under M_0 is unmarked, then $\exists M \geqslant M_0$, (N,M) is not live.

To find the maximum trap in H by the following 4 steps.

Step 1. H'$\leftarrow$H, Σ' is the subsystem generated by H', i$\leftarrow$0.

Step 2. If $\exists t \in T'$, s.t. $t^\bullet=\Phi$, let $^\bullet t=\{p\}$,

then t is denoted by t_{i+1},p is denoted by p_{i+1};

else stop

Step 3. Let H'$\leftarrow$H'-$\{p_{i+1}\}$, Σ' is the subsystem generated by H', i$\leftarrow$i+1.

Step 4. If H' $\neq \phi$ then stop else goto step2.

In step 2, the reason of 'let $^\bullet t=\{p\}$' is the fact of $|{}^\bullet t \cap H|=1$. As $|H|$ is finite, definitely the steps will go to 'stop' at step2 or step4. Let the value of i be m when 'stop'. Because p in step2 does not in any trap, the H' we get is the maximum trap (may be empty).

By the following 6 steps to seek an $M \geqslant M_0$ and a firing sequence σ s.t. M [σ > M' and H is unmarked in M'.

Step 1. i$\leftarrow$m

$\sigma' \leftarrow \phi$

M'$\leftarrow M_0$

M_0'$\leftarrow M_0$

$\Sigma' \leftarrow \Sigma$

j$\leftarrow$0

Step 2. If i=0 then stop

Step 3. While $\exists p \in {}^\bullet t_i$, $p \notin H$, and M'(p)<M'(p_i)

do

M'(p)$\leftarrow$M'(p)+1

M_0'(p)$\leftarrow M_0$'(p)+1

end of do

Step 4. While t_i is enabled

do

fire t_i

j$\leftarrow$j+1

end of do

Step 5. from M'[t_i,...t_i>M_i (fire t_i j times) get M_i.

Step 6. $\sigma' \leftarrow \sigma' t_i \ldots t_i$(j times)

i$\leftarrow$i-1

M'$\leftarrow M_i$

j$\leftarrow$0

goto step 2.

After step 4, $M_i(p_i)$=0. As the t_i selection, firing t_i can only add token to p_j(j<i), not add token to p_k(k $\geqslant$i) or $p \in H'$. Obviously M'(p_i)=0 and M'(H')=0 where H-H'=$\{p_i|i=1,\ldots m\}$. When above procedure stops, M'(H)=0,i.e. H is unmarked at M'. Let M=M_0', $\sigma=\sigma'$. As M>M_0 and (N, M) is not live, it contradicts the liveness monotonicity. □

To illustrate theorem 3.1, see fig 3.3 and fig 3.2.

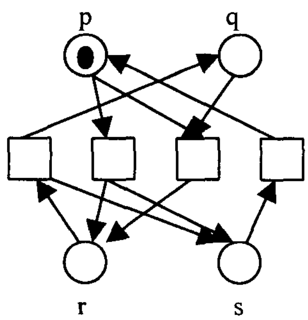

Fig3.3 It is live and has liveness monotonicity

Algorithm 3.1(outline)

Input an AC system(N,M_0)

Output Yes, it is live and has liveness monotonicity.

No, otherwise

A1. check N for being connected, if N is not connected then stop with 'No'.

A2. find all minimal siphon Hi(1,…,h)

A3. for each siphon to find a marked trap R, if there is one siphon H_j in which no marked trap then stop with 'No'.

A4. 'Yes' □

Theorem 3.2

Algorithm 3.1 is a polynomial one.

Proof.

List the worst case time complexity of the algorithm each steps as follows.

(A1) check the net for being connected. $O(|P|+|T|+|F|)$

(A2) find all minimal siphon Hi. $O(|P|^2(|P|+|T|+|F|))$

(A3) to find a marked trap R. $O(|P|^2(|P|+|T|+|F|))$

(A4) at most once.

Altogether $O(|P|+|T|+|F|+|P|^2(|P|+|T|+|F|))$. Let $n = \max(|P|,|T|)$, $|P|=|T|=n$ and $|F|=n^2$ the worst case time complexity of the algorithm is $O(n^4)$. □

4.Conclusion

Based on our observation in state machine, marked graph and free choice net system the paper discovered some liveness knowledge in asymmetric choice net system but not enough because the property of liveness is packed with liveness monotonicity in analyzing.

Frankly we are interested in deciding liveness of a system when it is faced to us. Unfortunately theorem 3.1 and algorithm 3.1 do not finish our dreams which would be the future works. Fortunately follow the conjecture 3.1 and theorem 3.1 we can guess there may exist some subclass in asymmetric choice net system, for which liveness algorithm is polynomial. In the line several papers will come.

Before close our argument the following conjecture would be attractive.

Conjecture 4.1

The biggest subclass of Petri net system is in asymmetric choice net system, for which liveness algorithm is polynomial.

The conjecture 3.1 and conjecture 4.1 made, are making and will make us in hard work.

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